

**Numerical Results for the experiment in the paper  
titled: *Three-term conjugate gradient-like method for  
pseudomonotone operator equations with application  
in regularized logistic regression model***

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Table 1: Numerical Results for Problem 1

Problem 1		IHCGPM				ITIA				TDIA			
DIM	SP	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM
1000	$\tau_0^1$	10	22	0.35888	3.55E-07	8	18	0.034397	6.81E-07	11	23	0.06513	8.83E-07
	$\tau_0^2$	11	24	0.025489	7.11E-07	10	22	0.013149	3.94E-07	13	28	0.013614	5.66E-07
	$\tau_0^3$	9	20	0.008449	5.75E-07	10	22	0.010171	1.82E-07	10	22	0.009176	9.92E-07
	$\tau_0^4$	9	20	0.008521	2.18E-07	6	14	0.006318	5.21E-07	7	16	0.005943	8.6E-07
	$\tau_0^5$	13	28	0.024851	6.87E-07	9	20	0.019911	1.47E-07	11	24	0.01664	7.16E-07
	$\tau_0^6$	10	22	0.012016	1.8E-07	10	22	0.016108	1.9E-07	5	11	0.008887	3.21E-07
	$\tau_0^7$	12	26	0.047234	3.1E-07	10	21	0.010511	3.57E-07	11	24	0.009643	7.26E-07
5000	$\tau_0^1$	10	21	0.051387	2.81E-07	10	22	0.054594	3.81E-07	11	24	0.045416	3.26E-07
	$\tau_0^2$	11	24	0.096758	3.23E-07	9	20	0.042731	1.76E-07	12	25	0.035901	7.81E-07
	$\tau_0^3$	10	22	0.042971	3.92E-07	7	15	0.023939	8.38E-07	9	20	0.043508	5.36E-07
	$\tau_0^4$	8	18	0.021647	1.93E-07	7	15	0.018221	1.24E-07	6	14	0.020244	4.24E-07
	$\tau_0^5$	11	24	0.15199	7.69E-07	7	15	0.047576	4.76E-07	9	20	0.049234	6.25E-07
	$\tau_0^6$	12	26	0.080116	3.38E-08	11	23	0.050702	9E-08	13	27	0.053977	8.49E-07
	$\tau_0^7$	12	26	0.10535	2.35E-07	8	18	0.030962	8.98E-07	6	13	0.032464	2.88E-07
10000	$\tau_0^1$	10	22	0.096859	6.28E-07	7	15	0.064152	7.04E-08	7	16	0.060824	8.58E-07
	$\tau_0^2$	11	24	0.30791	3.6E-07	11	23	0.083735	9.84E-07	8	18	0.071509	9.3E-07
	$\tau_0^3$	10	22	0.090111	1.53E-07	8	17	0.062959	2.19E-07	5	11	0.041498	7.87E-07
	$\tau_0^4$	9	20	0.033461	3.08E-07	6	14	0.054821	1.44E-07	4	10	0.030553	8.55E-07
	$\tau_0^5$	14	30	0.045416	6.38E-08	9	19	0.078219	2.18E-07	13	27	0.097209	7.15E-07
	$\tau_0^6$	12	26	0.035901	4.42E-07	9	19	0.085604	5.46E-08	11	24	0.10266	6.24E-07
	$\tau_0^7$	11	24	0.043508	4.51E-07	8	17	0.068281	2.95E-07	11	24	0.096556	6.52E-07
50000	$\tau_0^1$	12	26	0.020244	3.28E-07	10	21	0.31962	7.03E-07	10	21	0.25905	7.77E-07
	$\tau_0^2$	14	30	0.049234	1.04E-07	7	15	0.26137	3.62E-07	10	22	0.36011	8.96E-07
	$\tau_0^3$	12	26	0.053977	2.02E-07	7	15	0.22353	9.36E-09	7	15	0.18069	6.66E-07
	$\tau_0^4$	10	22	0.032464	1.32E-07	7	16	0.21371	8.75E-07	7	16	0.17482	6.95E-07
	$\tau_0^5$	15	32	0.060824	5.14E-08	7	15	0.30229	8.26E-07	12	25	0.35315	8.37E-07
	$\tau_0^6$	13	28	0.071509	2.87E-07	8	17	0.44814	1.26E-07	11	24	0.3573	9.27E-07
	$\tau_0^7$	12	26	0.041498	7.97E-07	9	20	0.52051	3.84E-07	9	20	0.33287	7.64E-07
100000	$\tau_0^1$	13	28	0.030553	2.04E-07	8	18	0.55108	4.68E-07	5	12	0.48443	4.46E-07
	$\tau_0^2$	14	30	0.097209	1.1E-07	7	15	0.52566	3.92E-07	10	22	1.1234	6.44E-07
	$\tau_0^3$	13	28	2.3517	1.4E-07	8	17	0.59173	7.13E-08	9	19	0.46517	7.62E-07
	$\tau_0^4$	10	22	0.48683	2.2E-07	7	15	0.59459	7.22E-07	5	12	0.27258	4.49E-07
	$\tau_0^5$	15	32	15.918	5.66E-08	10	21	1.0287	9.93E-07	9	19	0.52557	8.53E-07
	$\tau_0^6$	13	28	4.6537	3.15E-07	8	17	0.46292	3.36E-08	6	14	0.41196	4.1E-07
	$\tau_0^7$	13	28	5.9674	3.87E-07	8	18	1.1395	4.44E-07	10	22	0.72478	3.78E-07

Table 2: Numerical Results for Problem 2

Problem 2		IHCgPM				ITIA				TDIA			
DIM	SP	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM
1000	$\tau_0^1$	12	25	0.12735	5.79E-07	5	11	0.009845	9.58E-07	6	13	0.005147	3.34E-09
	$\tau_0^2$	10	22	0.013169	3.93E-07	6	13	0.008316	2.23E-07	8	17	0.008984	4.34E-09
	$\tau_0^3$	11	24	0.007307	7.9E-07	5	11	0.005891	1.01E-07	6	13	0.005112	3.52E-09
	$\tau_0^4$	10	22	0.007866	5.26E-07	4	9	0.057206	3.69E-08	3	7	0.006305	9.26E-08
	$\tau_0^5$	9	20	0.017551	3.95E-07	6	13	0.013478	6.06E-08	8	17	0.008393	3.33E-09
	$\tau_0^6$	10	21	0.009799	1.1E-08	6	13	0.009842	1.29E-08	6	14	0.01143	8.18E-07
	$\tau_0^7$	7	16	0.007509	1.03E-07	5	12	0.005988	1.68E-07	6	13	0.009873	2.76E-08
5000	$\tau_0^1$	8	18	0.032043	4.43E-09	6	13	0.024431	1.67E-08	7	15	0.02452	1.52E-09
	$\tau_0^2$	7	16	0.11577	8.92E-07	8	17	0.044125	4.05E-09	8	17	0.035845	1.85E-07
	$\tau_0^3$	11	24	0.061342	9.42E-07	5	11	0.016014	4.63E-08	6	13	0.026522	1.3E-09
	$\tau_0^4$	11	23	0.041046	6.87E-07	4	9	0.012437	7.33E-09	4	9	0.01292	2.38E-08
	$\tau_0^5$	7	16	0.25742	3.88E-07	8	17	0.046273	5.93E-09	9	19	0.035111	5.85E-10
	$\tau_0^6$	10	22	0.050713	2.42E-07	6	13	0.03095	4.69E-08	8	17	0.032734	2.38E-07
	$\tau_0^7$	10	21	0.052271	3.8E-07	6	13	0.028944	2.52E-09	8	17	0.026938	7.5E-10
10000	$\tau_0^1$	3	8	0.067932	9.83E-07	6	13	0.038921	8.3E-09	7	15	0.064209	1.11E-09
	$\tau_0^2$	7	16	0.28918	2.11E-07	8	17	0.070709	6.3E-09	9	19	0.079878	9.98E-07
	$\tau_0^3$	8	18	0.06513	5.25E-08	6	13	0.046426	6.6E-09	6	13	0.043085	9.99E-08
	$\tau_0^4$	11	23	0.013614	9.75E-07	4	9	0.026523	1.33E-07	4	9	0.027363	2.79E-07
	$\tau_0^5$	8	17	0.009176	8.73E-07	8	17	0.078745	1.31E-08	9	19	0.082508	4.18E-10
	$\tau_0^6$	8	18	0.005943	6.88E-07	7	15	0.056991	2.03E-08	7	15	0.058476	9.81E-07
	$\tau_0^7$	9	19	0.01664	4.74E-07	6	13	0.11283	6.23E-08	7	15	0.051242	1.49E-09
50000	$\tau_0^1$	8	18	0.008887	1.11E-07	6	13	0.38028	1.33E-07	8	17	0.22456	1.6E-10
	$\tau_0^2$	13	28	0.009643	7.38E-07	9	19	0.2857	7.29E-10	10	21	0.77068	5.35E-07
	$\tau_0^3$	8	18	0.045416	4.3E-07	6	13	0.16181	6.78E-10	8	17	0.39209	1.6E-07
	$\tau_0^4$	5	11	0.035901	4.2E-08	4	9	0.13836	3.63E-07	5	11	0.10929	6.75E-11
	$\tau_0^5$	6	14	0.043508	6.51E-07	10	21	0.75374	8.06E-10	10	22	0.45798	8.15E-07
	$\tau_0^6$	7	16	0.020244	1.23E-07	8	17	0.36068	1.17E-09	7	15	0.41627	6.86E-07
	$\tau_0^7$	7	16	0.049234	4.57E-08	8	17	0.2522	2.43E-10	9	19	0.32444	3.55E-07
100000	$\tau_0^1$	12	26	0.053977	5.07E-07	7	15	0.57784	1.55E-09	9	19	0.52985	9.26E-07
	$\tau_0^2$	14	29	0.032464	5.36E-07	9	19	1.0185	9.59E-11	11	23	1.2686	3.62E-11
	$\tau_0^3$	12	25	0.060824	6.04E-07	6	13	0.42436	8.63E-10	8	17	0.43059	9.51E-11
	$\tau_0^4$	5	11	0.071509	1.17E-08	5	11	0.24221	1.68E-10	5	12	0.25322	7.75E-07
	$\tau_0^5$	6	14	0.041498	7.63E-07	10	21	1.344	2.33E-10	11	23	0.79908	3.08E-07
	$\tau_0^6$	7	16	0.030553	2.6E-08	8	17	0.4816	2.91E-09	8	18	0.54435	5.42E-07
	$\tau_0^7$	13	27	0.097209	9.47E-07	8	17	0.5302	4.56E-10	10	21	1.1686	8.3E-11

Table 3: Numerical Results for Problem 3

Problem 3		IHCgPM				ITIA				TDIA			
DIM	SP	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM
1000	$\tau_0^1$	12	26	0.024392	6.47E-07	9	19	0.016351	2.35E-09	7	15	0.004858	6E-09
	$\tau_0^2$	12	26	0.010392	7.03E-07	9	19	0.02402	2.87E-10	9	19	0.007972	9.81E-10
	$\tau_0^3$	12	26	0.005253	5.17E-07	8	17	0.010035	1.76E-10	7	15	0.005876	7.53E-11
	$\tau_0^4$	10	21	0.002492	2.89E-07	6	13	0.004737	1.18E-09	4	9	0.003278	1.96E-09
	$\tau_0^5$	10	22	0.014711	2.21E-07	10	21	0.009508	3.94E-10	8	17	0.016585	1.71E-10
	$\tau_0^6$	13	27	0.00683	1.73E-07	8	17	0.008284	2.7E-09	8	17	0.007217	4.94E-09
	$\tau_0^7$	13	27	0.016646	2.17E-07	9	19	0.01494	7.28E-09	8	17	0.005859	3.03E-10
5000	$\tau_0^1$	13	27	0.033073	2.41E-07	8	17	0.055645	2.76E-08	9	19	0.039538	1.45E-10
	$\tau_0^2$	11	23	0.25832	6E-07	9	19	0.080902	1.94E-09	11	23	0.041234	1.52E-10
	$\tau_0^3$	13	27	0.05063	1.93E-07	10	21	0.047952	1.77E-09	8	17	0.022768	7.69E-11
	$\tau_0^4$	10	21	0.012698	6.45E-07	6	13	0.021118	1.23E-08	5	11	0.0168	1.55E-12
	$\tau_0^5$	11	23	0.10494	9.21E-07	11	23	0.040693	4.55E-10	10	21	0.062106	9.27E-10
	$\tau_0^6$	8	18	0.060595	7.2E-07	9	19	0.025203	1.68E-08	10	21	0.043181	1.16E-10
	$\tau_0^7$	6	13	0.072206	5.47E-07	11	23	0.042524	2.29E-09	9	19	0.038656	1.66E-10
10000	$\tau_0^1$	10	22	0.090455	4.95E-07	9	19	0.14575	1.54E-08	9	19	0.061554	2.16E-11
	$\tau_0^2$	11	24	0.3348	8.06E-07	10	21	0.17362	1.42E-08	11	23	0.084587	4.3E-11
	$\tau_0^3$	13	27	0.14068	2.69E-07	8	17	0.058917	4.37E-08	9	19	0.062917	9.86E-11
	$\tau_0^4$	10	21	0.020842	9.13E-07	7	15	0.047099	8.99E-12	5	11	0.026995	4.04E-11
	$\tau_0^5$	11	24	0.32574	9.92E-07	11	23	0.091009	2.28E-09	11	23	0.077779	1.34E-10
	$\tau_0^6$	10	21	0.13076	5.57E-07	9	19	0.071303	2.59E-09	10	21	0.16512	5.25E-10
	$\tau_0^7$	11	23	0.14455	5.07E-07	10	21	0.12239	9.15E-10	8	17	0.13011	1.75E-09
50000	$\tau_0^1$	11	23	1.0931	9.4E-07	9	19	0.50641	1.81E-08	10	21	0.23237	5.1E-09
	$\tau_0^2$	12	26	2.9256	7.22E-07	12	25	0.38416	2.21E-09	11	23	0.30409	2.47E-11
	$\tau_0^3$	10	22	0.69192	6.81E-07	9	19	0.24887	2.59E-09	9	19	0.3859	4.16E-10
	$\tau_0^4$	12	26	0.055855	3.66E-07	8	17	0.17801	6.52E-09	7	15	0.31248	2.15E-10
	$\tau_0^5$	12	26	3.6888	7.71E-07	11	23	0.63947	4.75E-09	12	25	0.39434	3.93E-10
	$\tau_0^6$	12	25	1.7742	6.74E-07	10	21	0.35144	9.81E-09	11	23	0.29414	7.75E-11
	$\tau_0^7$	12	25	1.3407	9.37E-07	11	23	0.37988	1.77E-08	11	23	0.49	4.32E-12
100000	$\tau_0^1$	12	25	2.0544	5.57E-07	12	25	1.0817	2.63E-10	10	21	0.61909	7.58E-11
	$\tau_0^2$	13	27	7.409	5.42E-07	14	29	0.88249	2.02E-10	11	23	0.61169	2.48E-10
	$\tau_0^3$	11	24	1.3535	6.53E-07	11	23	0.78891	5.61E-10	10	21	0.88187	5.26E-10
	$\tau_0^4$	12	26	0.088528	5.17E-07	8	17	0.53426	2.07E-10	7	15	0.23357	8.19E-11
	$\tau_0^5$	13	27	10.046	5.69E-07	10	21	0.66867	1.43E-09	12	25	0.70157	2.31E-09
	$\tau_0^6$	12	26	3.2295	5.76E-07	10	21	0.66187	5.62E-09	11	23	1.2194	6.86E-10
	$\tau_0^7$	12	26	4.7519	7.47E-07	11	23	1.1255	3.45E-10	12	25	0.64256	9.63E-11

Table 4: Numerical Results for Problem 4

Problem 4		IHCgPM				ITIA				TDIA			
DIM	SP	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM
1000	$\tau_0^1$	8	17	0.024049	9.54E-07	6	13	0.00734	6.98E-07	7	15	0.005073	7.98E-07
	$\tau_0^2$	11	24	0.009533	5.6E-07	7	15	0.005745	9.17E-11	7	15	0.005815	8.54E-07
	$\tau_0^3$	9	19	0.002615	9.39E-07	5	11	0.003703	7.83E-07	6	13	0.01028	7.12E-11
	$\tau_0^4$	5	11	0.002962	3.49E-09	4	9	0.010514	1.34E-09	3	7	0.007323	6.79E-07
	$\tau_0^5$	11	24	0.013189	9.07E-07	7	15	0.0185	2.72E-09	8	17	0.007248	3.45E-12
	$\tau_0^6$	11	24	0.007966	9.02E-07	7	15	0.016512	5.4E-08	7	15	0.00495	7.33E-12
	$\tau_0^7$	11	24	0.015397	5.6E-07	7	15	0.009143	5.52E-07	7	15	0.00845	1.63E-11
5000	$\tau_0^1$	9	20	0.036217	8.58E-07	6	13	0.02561	1.53E-08	7	15	0.022821	7.88E-08
	$\tau_0^2$	12	26	0.22362	8.12E-07	9	19	0.039827	4.54E-10	9	19	0.054261	3.69E-13
	$\tau_0^3$	9	20	0.056977	8.75E-07	7	15	0.019763	6.62E-08	6	13	0.034033	1.37E-07
	$\tau_0^4$	5	11	0.012138	7.81E-09	4	9	0.022751	9.46E-08	4	9	0.031647	8.44E-08
	$\tau_0^5$	12	26	0.108	9.36E-07	9	19	0.078911	4.51E-12	9	19	0.048636	3.09E-07
	$\tau_0^6$	12	26	0.047791	4.39E-07	6	13	0.050737	1.24E-07	7	16	0.021606	7.41E-07
	$\tau_0^7$	11	24	0.054321	7.71E-07	7	15	0.024345	6.41E-09	8	17	0.027008	7.35E-07
10000	$\tau_0^1$	6	14	0.074468	1.74E-07	7	15	0.041633	2.79E-10	9	19	0.042547	5.71E-07
	$\tau_0^2$	13	27	0.19376	6.41E-07	9	19	0.055116	5.83E-10	9	19	0.058428	4.86E-07
	$\tau_0^3$	10	21	0.07048	6.19E-07	6	13	0.035024	8.31E-07	8	17	0.041593	3.22E-12
	$\tau_0^4$	5	11	0.022339	1.1E-08	4	9	0.043358	1.7E-07	4	9	0.023373	3.58E-07
	$\tau_0^5$	13	27	0.62366	7.14E-07	10	21	0.14986	2.23E-11	8	17	0.059131	2.13E-07
	$\tau_0^6$	9	20	0.082631	5.3E-07	9	19	0.060947	8.9E-10	8	17	0.054596	1.52E-12
	$\tau_0^7$	9	19	0.11981	8.86E-07	9	19	0.065527	2.08E-09	8	17	0.055688	1.15E-11
50000	$\tau_0^1$	9	20	0.43539	7.74E-07	8	17	0.17326	4.66E-10	8	18	0.20689	4.78E-07
	$\tau_0^2$	13	28	2.619	7.8E-07	10	21	0.34406	6.33E-07	9	20	0.29602	5.07E-07
	$\tau_0^3$	4	10	0.39266	1.34E-08	8	17	0.34718	4.3E-10	9	19	0.21009	1.02E-12
	$\tau_0^4$	5	11	0.077781	2.47E-08	6	13	0.22781	1.05E-08	6	14	0.14495	4.32E-07
	$\tau_0^5$	13	28	3.1617	8.46E-07	11	23	0.52669	4.63E-10	10	21	0.27684	7.06E-07
	$\tau_0^6$	8	17	1.141	6.21E-07	9	19	0.18076	1.96E-09	9	19	0.20004	2.78E-11
	$\tau_0^7$	6	14	1.2882	7.46E-07	10	21	0.22403	3.16E-11	9	19	0.24921	7.39E-07
100000	$\tau_0^1$	11	23	1.7539	6.27E-07	8	17	0.59476	1.3E-09	10	21	0.41552	4.31E-07
	$\tau_0^2$	14	29	6.3728	5.58E-07	10	21	0.73169	4.01E-11	10	21	0.53555	8.24E-07
	$\tau_0^3$	10	22	1.7504	8.97E-07	9	19	0.39852	2.4E-11	9	19	0.39308	6.67E-13
	$\tau_0^4$	5	11	0.075755	3.49E-08	6	13	0.21629	4.89E-10	6	13	0.32516	5.82E-07
	$\tau_0^5$	14	29	8.7035	6.03E-07	12	25	1.1417	3.37E-12	11	23	0.59436	1.26E-12
	$\tau_0^6$	8	18	2.8166	6.6E-07	9	19	0.65677	2.62E-10	11	23	0.47475	2.81E-13
	$\tau_0^7$	11	24	3.5816	8.85E-07	10	21	0.47334	6.4E-10	9	19	0.54009	1.23E-12

Table 5: Numerical Results for Problem 5

Problem 5		IHCgPM				ITIA				TDIA			
DIM	SP	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM
1000	$\tau_0^1$	8	18	0.02373	9.54E-07	15	32	0.020208	8.41E-07	10	22	0.019101	4.31E-07
	$\tau_0^2$	10	21	0.019474	4.44E-07	15	32	0.027667	7.37E-07	8	18	0.012595	8.56E-07
	$\tau_0^3$	5	12	0.011466	3.57E-09	14	30	0.015003	7.55E-07	10	22	0.013479	4.74E-07
	$\tau_0^4$	6	14	0.006505	8.28E-07	13	28	0.028853	5.9E-07	6	14	0.008152	3.1E-07
	$\tau_0^5$	8	18	0.041715	5.52E-07	17	36	0.090642	4.94E-07	12	26	0.019067	4.95E-07
	$\tau_0^6$	9	20	0.023495	6.47E-07	16	34	0.020108	5.31E-07	11	24	0.031273	2.97E-07
	$\tau_0^7$	10	21	0.029304	1.74E-07	15	32	0.03017	6.41E-07	11	24	0.015289	2.73E-07
5000	$\tau_0^1$	9	19	0.036532	7.42E-07	16	34	0.096009	6.76E-07	11	24	0.070287	4.61E-07
	$\tau_0^2$	8	17	0.11159	7.66E-07	7	16	0.076105	4.98E-07	9	20	0.06898	4.67E-07
	$\tau_0^3$	5	12	0.040774	7.99E-09	15	32	0.19562	6.64E-07	11	24	0.077573	3.78E-07
	$\tau_0^4$	8	18	0.019453	9.54E-07	15	32	0.19621	9.25E-07	7	16	0.040825	3.2E-07
	$\tau_0^5$	7	16	0.2328	4.31E-08	14	30	0.10919	7.54E-07	11	24	0.07413	7.33E-07
	$\tau_0^6$	10	21	0.19039	2.41E-07	17	36	0.10543	6.84E-07	12	26	0.095727	4.4E-07
	$\tau_0^7$	10	21	0.21668	3.89E-07	18	38	0.08686	5.28E-07	11	24	0.064011	4.14E-07
10000	$\tau_0^1$	9	20	0.13034	7.85E-07	14	30	0.23306	7.83E-07	10	22	0.11556	3.76E-07
	$\tau_0^2$	6	13	0.31305	9.5E-07	15	32	0.43477	3.11E-07	9	20	0.13128	6.69E-07
	$\tau_0^3$	5	12	0.084353	1.13E-08	15	32	0.33324	7.8E-07	11	24	0.10961	3.3E-07
	$\tau_0^4$	9	20	0.051207	3.85E-07	13	28	0.21081	6E-07	6	14	0.054115	8.52E-07
	$\tau_0^5$	8	17	0.88676	8.05E-07	14	30	0.19138	3.49E-07	12	26	0.19089	3.32E-07
	$\tau_0^6$	10	21	0.43894	3.41E-07	16	34	0.17693	8.79E-07	11	24	0.14345	7.04E-07
	$\tau_0^7$	13	28	0.19593	3.5E-07	17	36	0.35777	5.14E-07	11	24	0.14328	3.17E-07
50000	$\tau_0^1$	10	21	0.9061	2.93E-07	16	34	1.3879	7.03E-07	10	22	0.48889	2.88E-07
	$\tau_0^2$	8	18	5.093	6.15E-07	14	30	0.86315	6.46E-07	13	28	0.70117	4.22E-07
	$\tau_0^3$	5	12	1.0565	2.53E-08	17	36	0.76874	7.71E-07	11	24	0.66876	5.44E-07
	$\tau_0^4$	9	20	0.16256	8.61E-07	14	30	1.0787	8.95E-07	7	16	0.25562	2.98E-07
	$\tau_0^5$	9	19	6.0155	2.45E-07	15	32	1.2725	2.69E-07	11	24	0.59918	3.81E-07
	$\tau_0^6$	9	20	2.1011	5.38E-07	14	30	0.63562	5.36E-07	9	20	0.62639	7.19E-07
	$\tau_0^7$	8	17	2.951	2.38E-07	13	28	1.0863	7.98E-07	12	26	0.58899	4.52E-07
100000	$\tau_0^1$	9	19	3.0086	5.67E-07	17	36	1.2115	5.72E-07	12	26	0.9979	5.76E-07
	$\tau_0^2$	9	19	13.316	1.91E-07	13	28	2.0901	9.68E-07	12	26	1.2742	7.97E-07
	$\tau_0^3$	5	12	2.4507	3.57E-08	15	32	1.3687	7.77E-07	11	24	0.76709	2.77E-07
	$\tau_0^4$	10	22	0.46429	9.05E-07	14	30	1.271	5.95E-07	6	14	0.40907	6.33E-07
	$\tau_0^5$	9	19	15.1035	3.81E-07	12	26	1.9979	4.95E-07	13	28	1.2736	9.92E-07
	$\tau_0^6$	8	17	6.4031	3.43E-07	11	24	1.1461	4.72E-07	9	20	0.91691	9.51E-07
	$\tau_0^7$	8	17	8.1289	2.71E-07	16	34	1.5361	8.59E-07	12	26	1.0213	4E-07

Table 6: Numerical Results for Problem 6

Problem 6		IHCGM				ITIA				TDIA			
DIM	SP	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM
1000	$\tau_0^1$	27	56	0.094541	5.48E-07	28	58	0.030984	7.32E-07	43	88	0.048998	9.84E-07
	$\tau_0^2$	53	108	0.10987	7.85E-07	33	68	0.093699	6.64E-07	57	116	0.063836	8.95E-07
	$\tau_0^3$	30	62	0.051856	7.27E-07	28	58	0.13255	9.77E-07	41	84	0.044647	9.62E-07
	$\tau_0^4$	32	66	0.074039	5.08E-07	24	50	0.076594	7.24E-07	37	76	0.099043	9.51E-07
	$\tau_0^5$	49	100	0.20911	6.24E-07	37	76	0.13441	7.42E-07	53	108	0.10291	7.61E-07
	$\tau_0^6$	36	74	0.053551	9.39E-07	26	54	0.10621	9.88E-07	49	100	0.090872	7.9E-07
	$\tau_0^7$	45	92	0.049648	9.39E-07	41	84	0.062012	7.03E-07	42	86	0.05017	9.86E-07
5000	$\tau_0^1$	36	74	0.19046	7.52E-07	24	50	0.13157	9.77E-07	44	90	0.2184	9.67E-07
	$\tau_0^2$	63	128	0.52398	8.01E-07	53	108	0.4299	9.64E-07	56	114	0.44276	8.42E-07
	$\tau_0^3$	34	70	0.49518	8.5E-07	29	60	0.42334	7.09E-07	44	90	0.30757	9.33E-07
	$\tau_0^4$	25	52	0.18029	9.42E-07	25	52	0.13017	7.08E-07	37	76	0.2539	8.75E-07
	$\tau_0^5$	61	124	0.80285	9.36E-07	54	110	0.30093	9.47E-07	58	118	0.42889	9.75E-07
	$\tau_0^6$	40	82	0.18726	6.58E-07	43	88	0.25985	9.36E-07	47	96	0.4278	8.81E-07
	$\tau_0^7$	45	92	0.29834	9.85E-07	49	100	0.60154	8.72E-07	52	106	0.35419	8.16E-07
10000	$\tau_0^1$	38	78	0.46664	7.52E-07	33	68	0.37825	7.08E-07	51	104	0.71536	8.14E-07
	$\tau_0^2$	63	128	1.6932	6.7E-07	54	110	0.72824	9.21E-07	53	108	0.85592	8.53E-07
	$\tau_0^3$	36	74	0.93102	5.76E-07	44	90	1.0673	6.99E-07	49	100	0.6322	8.52E-07
	$\tau_0^4$	21	44	0.15069	9.81E-07	25	52	0.58922	9.73E-07	38	78	0.4678	7.29E-07
	$\tau_0^5$	63	128	1.1686	7.4E-07	53	108	0.64167	9.14E-07	57	116	0.83598	9.72E-07
	$\tau_0^6$	45	92	0.9801	9.51E-07	48	98	1.2085	9.16E-07	51	104	0.71351	8.62E-07
	$\tau_0^7$	53	107	0.81015	9.54E-07	52	106	0.63174	7.87E-07	54	110	0.86846	6.85E-07
50000	$\tau_0^1$	54	110	0.82476	9.82E-07	51	104	3.3626	9.27E-07	53	108	2.3252	9.64E-07
	$\tau_0^2$	63	128	0.93723	6.78E-07	62	126	3.6967	9.9E-07	57	116	2.6584	6.83E-07
	$\tau_0^3$	52	105	0.63323	9.89E-07	47	96	2.2349	8.2E-07	51	104	2.2093	8.33E-07
	$\tau_0^4$	34	70	0.95684	5.71E-07	20	42	1.227	6.29E-07	40	82	1.6397	9.09E-07
	$\tau_0^5$	65	132	0.67187	7.69E-07	60	122	4.0209	7.05E-07	61	124	2.7886	8.09E-07
	$\tau_0^6$	33	68	1.0854	7.66E-07	52	106	3.0323	6.85E-07	58	118	2.5015	7.86E-07
	$\tau_0^7$	62	126	2.9895	6.6E-07	54	110	3.1269	8.71E-07	56	114	2.513	6.86E-07
100000	$\tau_0^1$	58	118	2.8946	7.24E-07	48	98	4.685	6.74E-07	55	112	4.4725	8.13E-07
	$\tau_0^2$	63	128	2.7666	9.26E-07	65	132	7.2629	8E-07	57	116	4.8021	9.72E-07
	$\tau_0^3$	55	112	2.0135	9.65E-07	52	106	6.2247	7.82E-07	51	104	3.9811	9.28E-07
	$\tau_0^4$	28	57	3.566	9.68E-07	28	58	2.8649	8.24E-07	39	80	3.0267	9.87E-07
	$\tau_0^5$	65	132	2.9063	5.76E-07	65	132	7.329	7.73E-07	62	126	5.3669	9.57E-07
	$\tau_0^6$	55	112	2.8838	8.99E-07	51	104	5.3788	8.9E-07	57	116	4.6719	7.89E-07
	$\tau_0^7$	65	131	5.3236	9.91E-07	57	116	6.051	8.18E-07	54	110	4.5942	9.75E-07

Table 7: Numerical Results for Problem 7

Problem 7		IHCGPM				ITIA				TDIA			
DIM	SP	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM
1000	$\tau_0^1$	17	36	0.016153	8.23E-07	16	33	0.012917	7.2E-09	16	33	0.015417	1.32E-09
	$\tau_0^2$	17	36	0.011014	6.77E-07	15	31	0.010324	1.59E-10	16	33	0.016153	2.12E-11
	$\tau_0^3$	17	36	0.018584	8.33E-07	16	33	0.039878	6.01E-09	16	33	0.011014	6.42E-10
	$\tau_0^4$	18	37	0.020471	9.92E-07	16	33	0.012521	2.52E-09	16	33	0.018584	1.65E-10
	$\tau_0^5$	17	36	0.023355	5.95E-07	14	29	0.009784	1.1E-08	16	33	0.020471	4.04E-10
	$\tau_0^6$	17	36	0.04749	7.9E-07	16	33	0.014143	2.2E-10	16	33	0.019192	2.74E-09
	$\tau_0^7$	17	36	0.028866	7.66E-07	16	33	0.0107	2.89E-10	16	33	0.01406	3.65E-09
5000	$\tau_0^1$	19	40	0.73188	7.09E-07	18	37	0.059429	1.65E-09	18	37	0.057791	1.17E-09
	$\tau_0^2$	19	40	0.30578	5.64E-07	18	37	0.056039	7.05E-09	18	37	0.099396	1.6E-11
	$\tau_0^3$	19	40	0.29202	7.2E-07	18	37	0.085273	5.75E-10	18	37	0.090205	4.69E-10
	$\tau_0^4$	19	40	0.44277	7.58E-07	19	39	0.0712	3.18E-09	18	37	0.060754	1.97E-11
	$\tau_0^5$	18	38	0.56652	9.05E-07	17	35	0.052679	1.14E-08	17	35	0.055515	3.43E-09
	$\tau_0^6$	19	40	0.64968	6.75E-07	18	37	0.065186	1.07E-10	18	37	0.076135	2.25E-09
	$\tau_0^7$	19	40	0.29796	6.51E-07	17	35	0.1385	9.2E-10	18	37	0.082133	2.27E-10
10000	$\tau_0^1$	20	42	0.7022	5.61E-07	18	37	0.27023	1.91E-10	19	39	0.12705	2.35E-10
	$\tau_0^2$	19	40	1.5497	8.54E-07	18	37	0.13958	6.26E-09	18	37	0.15277	1.14E-09
	$\tau_0^3$	20	42	0.89798	5.71E-07	19	39	0.11246	9.12E-09	19	39	0.1775	5.86E-10
	$\tau_0^4$	20	42	1.6998	6.1E-07	18	37	0.13108	1.47E-10	18	37	0.12168	9.6E-10
	$\tau_0^5$	19	40	0.66209	7.8E-07	19	39	0.28446	6.56E-11	17	35	0.12522	4.4E-10
	$\tau_0^6$	20	42	0.93479	5.27E-07	18	37	0.26474	4.29E-09	18	37	0.13154	2.02E-10
	$\tau_0^7$	20	42	1.3347	5.04E-07	18	37	0.28029	2.95E-10	19	39	0.19129	6.35E-10
50000	$\tau_0^1$	21	44	9.5274	7.92E-07	20	41	0.89191	1.08E-09	21	43	0.67148	6.83E-11
	$\tau_0^2$	21	44	8.6881	6.51E-07	20	41	0.59898	3.38E-10	20	41	0.63691	1.17E-10
	$\tau_0^3$	21	44	9.644	8.03E-07	20	41	1.1563	2.95E-11	20	41	0.73731	5.71E-10
	$\tau_0^4$	21	44	10.4573	8.4E-07	19	39	1.0667	5.31E-10	20	41	0.57463	5.67E-10
	$\tau_0^5$	21	44	9.5858	5.74E-07	20	41	1.0795	1.88E-09	21	43	0.82331	4.36E-10
	$\tau_0^6$	21	44	9.3745	7.6E-07	21	43	0.54649	2.58E-09	19	39	0.58532	3.11E-09
	$\tau_0^7$	21	44	9.2884	7.36E-07	20	41	0.92451	3.49E-09	20	41	0.73188	1.73E-10
100000	$\tau_0^1$	22	46	29.9813	6.39E-07	21	43	1.2596	1.01E-09	21	43	1.1395	1.56E-10
	$\tau_0^2$	22	45	24.6287	9.93E-07	21	43	1.9317	1.16E-09	20	41	1.1452	4.07E-10
	$\tau_0^3$	22	46	27.2226	6.5E-07	21	43	1.5996	2.78E-10	20	41	1.37	5.79E-10
	$\tau_0^4$	22	46	28.5276	6.88E-07	20	41	1.391	3.02E-10	22	45	1.2678	1.26E-11
	$\tau_0^5$	21	44	21.3708	8.62E-07	21	43	1.1804	2.39E-09	22	45	1.2557	5.28E-10
	$\tau_0^6$	22	46	25.2144	6.05E-07	20	41	1.9144	5.9E-10	21	43	1.1463	4.74E-11
	$\tau_0^7$	22	46	24.4831	5.81E-07	22	45	1.6075	3.04E-11	21	43	1.0911	2.77E-09



Table 8: Numerical Results for Problem 8

Problem 8		IHCGM				ITIA				TDIA			
DIM	SP	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM
1000	$\tau_0^1$	11	24	0.038072	8.37E-07	6	13	0.009309	4.96E-07	8	17	0.008228	5.01E-07
	$\tau_0^2$	12	26	0.025283	6.14E-07	7	15	0.005935	9.54E-09	10	21	0.011122	1.43E-12
	$\tau_0^3$	11	24	0.017343	7.82E-07	7	15	0.006829	2.24E-08	7	15	0.007502	8.9E-07
	$\tau_0^4$	11	24	0.01586	5.52E-07	6	14	0.013337	7.3E-07	9	19	0.0105	5.55E-07
	$\tau_0^5$	12	26	0.022872	5.89E-07	8	17	0.02097	6.7E-09	8	17	0.013977	1.88E-07
	$\tau_0^6$	11	24	0.014567	9.78E-07	7	15	0.014319	6.35E-08	7	15	0.005948	8.98E-08
	$\tau_0^7$	12	25	0.03116	5.26E-07	6	13	0.011773	9.93E-07	9	19	0.009192	1.93E-12
5000	$\tau_0^1$	12	26	0.11026	8.45E-07	8	17	0.037275	4.31E-09	10	21	0.045716	1.2E-11
	$\tau_0^2$	13	28	0.25139	6.73E-07	8	17	0.03277	8.85E-10	10	21	0.041088	3.9E-12
	$\tau_0^3$	12	26	0.12076	8.25E-07	8	17	0.031234	1.44E-09	8	18	0.058377	6.83E-07
	$\tau_0^4$	12	26	0.26094	7.37E-07	7	15	0.069107	2.12E-09	8	17	0.044047	4.82E-07
	$\tau_0^5$	13	28	0.56893	6.55E-07	10	21	0.082833	2.03E-10	9	19	0.044145	4.07E-07
	$\tau_0^6$	12	26	0.26949	8.92E-07	9	19	0.042785	5.39E-09	8	17	0.030039	8.68E-13
	$\tau_0^7$	12	26	0.28245	9.15E-07	8	17	0.024124	1.83E-10	10	21	0.030798	8.73E-07
10000	$\tau_0^1$	13	27	0.3007	6.43E-07	10	21	0.074461	7.69E-11	11	23	0.079373	7.33E-13
	$\tau_0^2$	14	29	0.47036	5.17E-07	10	21	0.067861	1.23E-10	10	21	0.077622	5.72E-13
	$\tau_0^3$	13	27	0.38827	6.31E-07	9	19	0.06387	1.52E-09	10	21	0.096657	8.18E-12
	$\tau_0^4$	13	27	0.51449	5.78E-07	8	17	0.12621	5.03E-09	8	17	0.071898	5.69E-07
	$\tau_0^5$	14	29	0.65498	5.04E-07	10	21	0.15438	1.01E-10	10	21	0.081275	2.63E-12
	$\tau_0^6$	13	27	0.30577	6.71E-07	9	19	0.064757	2.37E-09	9	19	0.066101	3.78E-12
	$\tau_0^7$	13	27	0.39816	6.84E-07	9	19	0.067647	5.2E-11	11	23	0.082759	6.48E-12
50000	$\tau_0^1$	13	28	4.1621	7.6E-07	12	25	0.33963	3.01E-12	9	19	0.28735	1.88E-10
	$\tau_0^2$	14	30	5.4648	6.17E-07	11	23	0.62494	4.37E-11	11	23	0.37556	3.3E-07
	$\tau_0^3$	13	28	3.6567	7.49E-07	10	21	0.45542	4.42E-10	10	21	0.39305	2.99E-12
	$\tau_0^4$	13	28	2.4874	6.98E-07	9	19	0.29751	1.94E-10	9	19	0.27619	7.27E-07
	$\tau_0^5$	14	30	6.7525	6.01E-07	12	25	0.31273	9.32E-12	12	25	0.3685	3.34E-13
	$\tau_0^6$	13	28	4.5042	7.87E-07	11	23	0.63927	1.66E-10	10	21	0.3278	2.7E-11
	$\tau_0^7$	13	28	4.2301	7.99E-07	12	25	0.73514	4.89E-11	10	22	0.3123	5.24E-07
100000	$\tau_0^1$	14	29	9.1082	5.42E-07	11	23	0.6508	1.16E-12	10	21	0.75023	7.62E-07
	$\tau_0^2$	14	30	14.968	8.8E-07	11	23	0.59857	5.82E-11	11	23	0.69806	7.58E-07
	$\tau_0^3$	14	29	8.9347	5.34E-07	10	21	1.0649	1.38E-12	11	23	0.76809	4.56E-07
	$\tau_0^4$	13	28	7.4175	9.98E-07	10	21	1.01	2.11E-10	11	23	0.60053	8.03E-07
	$\tau_0^5$	14	30	16.6039	8.58E-07	12	25	0.93419	5.81E-12	11	24	0.74626	3.79E-07
	$\tau_0^6$	14	29	10.4818	5.6E-07	11	23	0.58237	7.72E-12	12	25	0.67642	6.7E-13
	$\tau_0^7$	14	29	12.5407	5.69E-07	11	23	1.1466	1.16E-12	10	21	0.59433	4.21E-08

Table 9: Numerical Results for Problem 9

Problem 9		IHCGM				ITIA				TDIA			
DIM	SP	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM
1000	$\tau_0^1$	104	210	0.10665	9.88E-07	40	82	0.08063	9.33E-07	77	156	0.079488	9.5E-07
	$\tau_0^2$	66	134	0.069908	7.32E-07	52	106	0.094297	9.99E-07	58	118	0.049194	9.61E-07
	$\tau_0^3$	106	214	0.082018	9.49E-07	51	104	0.089751	7.59E-07	74	150	0.057811	9.29E-07
	$\tau_0^4$	67	136	0.062508	8.86E-07	36	74	0.029208	9.21E-07	52	106	0.038367	9.11E-07
	$\tau_0^5$	74	150	0.047254	8.26E-07	60	122	0.045614	8.77E-07	87	176	0.086417	8.04E-07
	$\tau_0^6$	71	144	0.097436	9.73E-07	45	92	0.051434	9.89E-07	80	162	0.077609	8.69E-07
	$\tau_0^7$	72	146	0.15756	7.33E-07	54	110	0.040789	8.62E-07	81	164	0.067178	9.32E-07
5000	$\tau_0^1$	132	266	1.1102	9.74E-07	59	120	0.33351	8.29E-07	64	130	0.201	8.34E-07
	$\tau_0^2$	107	216	0.49932	8.22E-07	43	88	0.15842	9.52E-07	71	144	0.31936	9.75E-07
	$\tau_0^3$	129	260	0.50937	9.77E-07	41	84	0.18325	9.29E-07	74	150	0.31551	9.3E-07
	$\tau_0^4$	89	180	0.41313	8.25E-07	37	76	0.34442	9.45E-07	53	108	0.3687	8.49E-07
	$\tau_0^5$	59	120	0.67928	9.1E-07	60	122	0.55794	9.28E-07	83	168	0.50162	9.57E-07
	$\tau_0^6$	115	232	1.0575	1E-06	51	104	0.19016	7.74E-07	60	122	0.2391	9.61E-07
	$\tau_0^7$	93	188	0.76425	9.68E-07	52	106	0.20683	9.44E-07	82	166	0.34399	8.78E-07
10000	$\tau_0^1$	135	272	1.5988	8.46E-07	65	132	1.4728	9.6E-07	58	118	0.81015	9.78E-07
	$\tau_0^2$	126	254	1.5118	8.97E-07	49	100	1.1572	8.94E-07	70	142	0.82476	9.06E-07
	$\tau_0^3$	131	264	1.9093	9.95E-07	51	104	1.0578	8.66E-07	74	150	0.93723	9.61E-07
	$\tau_0^4$	89	180	1.2661	9.8E-07	52	106	0.54779	8.3E-07	54	110	0.63323	9.15E-07
	$\tau_0^5$	113	228	1.8603	8.2E-07	59	120	1.044	9.13E-07	82	166	0.95684	9.86E-07
	$\tau_0^6$	133	268	1.3052	9.99E-07	50	102	0.76811	8.75E-07	58	118	0.67187	9.07E-07
	$\tau_0^7$	113	228	2.5329	8.02E-07	53	108	0.55075	8.52E-07	81	164	1.0854	9.95E-07
50000	$\tau_0^1$	134	270	9.4061	9.5E-07	62	126	3.6344	8.86E-07	65	132	2.9895	9.84E-07
	$\tau_0^2$	130	262	6.6346	9.15E-07	56	114	3.6538	9.04E-07	70	142	2.8946	9.48E-07
	$\tau_0^3$	134	270	9.4822	8.74E-07	68	138	3.9582	9.03E-07	68	138	2.7666	9.29E-07
	$\tau_0^4$	89	180	9.5926	9.87E-07	51	104	2.4013	9.69E-07	45	92	2.0135	8.39E-07
	$\tau_0^5$	143	288	5.2601	9.92E-07	56	114	2.7256	9.87E-07	86	174	3.566	8.8E-07
	$\tau_0^6$	139	280	8.9448	9.76E-07	63	128	3.4135	9.4E-07	73	148	2.9063	9.28E-07
	$\tau_0^7$	137	276	8.6965	9.48E-07	73	148	3.9722	8.61E-07	70	142	2.8838	9.46E-07
100000	$\tau_0^1$	134	270	25.563	9.75E-07	70	142	6.6404	8.13E-07	73	148	5.3236	1E-06
	$\tau_0^2$	131	264	14.8077	9.81E-07	58	118	6.1853	9.02E-07	73	148	5.195	8.05E-07
	$\tau_0^3$	132	266	26.829	9.28E-07	72	146	6.4431	9.57E-07	69	140	5.0041	8.56E-07
	$\tau_0^4$	90	182	28.5692	9.46E-07	48	98	4.559	9.57E-07	43	88	3.3987	8.7E-07
	$\tau_0^5$	146	294	11.5796	8.16E-07	68	138	6.2845	9.6E-07	87	176	6.0342	8.63E-07
	$\tau_0^6$	140	282	23.2307	8.98E-07	63	128	6.5238	8.97E-07	65	132	4.7354	8.66E-07
	$\tau_0^7$	140	282	21.2308	8.23E-07	76	154	7.4666	8.12E-07	72	146	4.9623	8E-07

Table 10: Numerical Results for Problem 10

Problem 10		IHCgPM				ITIA				TDIA			
DIM	SP	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM
1000	$\tau_0^1$	7	16	0.055087	4.16E-08	10	21	0.009701	5.95E-07	8	17	0.008123	2.25E-07
	$\tau_0^2$	10	22	0.015904	8.89E-07	9	19	0.008634	1.58E-07	9	19	0.009054	2.19E-08
	$\tau_0^3$	10	22	0.004907	5.77E-07	8	18	0.009494	4.69E-07	9	19	0.007573	6.31E-07
	$\tau_0^4$	7	15	0.005828	7.7E-07	6	13	0.006329	4.28E-07	4	9	0.003902	7.24E-07
	$\tau_0^5$	10	22	0.016231	7.87E-07	15	31	0.02555	4.53E-07	11	23	0.028402	6.11E-09
	$\tau_0^6$	7	16	0.008494	6.31E-07	11	23	0.009793	6.94E-07	9	19	0.01017	3.39E-07
	$\tau_0^7$	7	16	0.016433	5.93E-07	15	31	0.013093	1.87E-08	8	18	0.011872	3.93E-07
5000	$\tau_0^1$	10	22	0.062705	5.02E-07	12	25	0.078777	4.05E-08	10	21	0.078699	1.21E-07
	$\tau_0^2$	10	22	0.20397	3.51E-07	9	20	0.076347	1.66E-07	16	33	0.1174	1.32E-07
	$\tau_0^3$	10	21	0.051989	6.9E-07	14	29	0.20198	8.3E-07	8	18	0.052405	7.98E-07
	$\tau_0^4$	7	16	0.034025	6.03E-09	22	45	0.2471	7.45E-07	5	12	0.057304	4.54E-07
	$\tau_0^5$	8	18	0.19182	1.28E-08	10	21	0.17112	7.53E-07	21	43	0.13617	1.87E-08
	$\tau_0^6$	9	19	0.21221	8.53E-07	10	21	0.16264	3.29E-08	12	25	0.077774	6.74E-07
	$\tau_0^7$	8	18	0.17988	7.97E-07	9	19	0.061834	9.02E-07	8	17	0.065503	5.69E-07
10000	$\tau_0^1$	10	21	0.11148	5.09E-07	9	19	0.13111	4.04E-08	13	28	0.14828	5.55E-07
	$\tau_0^2$	11	23	0.43698	9.08E-07	10	22	0.16485	7.91E-07	20	41	0.23981	7.57E-08
	$\tau_0^3$	7	16	0.1081	8.03E-07	14	29	0.34484	3.59E-07	13	27	0.2163	2.38E-07
	$\tau_0^4$	7	16	0.10285	2.31E-08	7	15	0.10673	3.91E-08	7	15	0.058052	8.34E-08
	$\tau_0^5$	9	20	1.2012	1.15E-07	11	23	0.1542	2.04E-07	14	30	0.21384	5.45E-07
	$\tau_0^6$	7	16	0.18293	3.02E-08	8	18	0.13365	3.76E-07	12	25	0.17328	7.55E-07
	$\tau_0^7$	8	18	0.2349	7.9E-07	15	31	0.17148	1.76E-07	9	20	0.14652	6.12E-07
50000	$\tau_0^1$	9	20	1.9725	6.03E-07	11	24	1.2022	1.01E-07	9	20	0.71812	9.07E-07
	$\tau_0^2$	11	24	5.0966	4.07E-07	13	28	0.88672	6.35E-07	9	20	0.6862	4.83E-07
	$\tau_0^3$	9	20	1.2834	8.09E-07	9	19	0.44661	9.64E-07	10	21	0.8345	5.34E-07
	$\tau_0^4$	9	19	0.15853	7.5E-07	9	19	0.51437	1.47E-07	8	17	0.37828	8.11E-07
	$\tau_0^5$	11	24	7.1769	8.1E-08	15	32	1.8807	3.63E-07	12	25	0.77302	1.8E-07
	$\tau_0^6$	10	21	2.1706	2.78E-07	12	26	0.52343	5.37E-07	12	26	0.76663	7.43E-07
	$\tau_0^7$	10	22	3.1726	6.48E-07	11	23	0.54474	2.13E-07	9	20	0.58327	5.07E-07
100000	$\tau_0^1$	10	22	4.397	6.98E-07	11	24	1.8476	8.07E-07	9	20	1.2599	4.93E-07
	$\tau_0^2$	12	26	14.6006	3.76E-07	11	23	1.7738	8.3E-07	12	25	1.4571	9.22E-07
	$\tau_0^3$	10	21	3.0861	2.15E-07	10	21	1.6068	5.76E-07	8	18	0.81051	7.42E-07
	$\tau_0^4$	8	18	0.40281	9.82E-07	25	52	2.3182	6.88E-07	9	19	0.97282	4.47E-07
	$\tau_0^5$	12	26	20.0751	2.25E-07	11	24	1.2214	6.2E-07	11	23	1.3219	4.26E-07
	$\tau_0^6$	11	24	6.7526	3.13E-08	11	23	2.1487	6.29E-07	10	22	1.1593	4.66E-07
	$\tau_0^7$	11	24	9.0745	3.01E-08	13	28	1.4092	4.57E-07	15	31	1.4413	4.19E-07

Table 11: Numerical Results for Problem 11

Problem 11		IHCGPM				ITIA				TDIA			
DIM	SP	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM
1000	$\tau_0^1$	11	24	0.019269	7.62E-07	5	11	0.020959	3E-09	5	11	0.006506	6.83E-07
	$\tau_0^2$	6	13	0.006811	4.4E-09	6	13	0.003787	5.9E-08	6	14	0.003984	9.9E-07
	$\tau_0^3$	11	24	0.003628	5.96E-07	5	11	0.004543	2.28E-09	5	12	0.003237	8.86E-07
	$\tau_0^4$	10	22	0.003133	5.34E-07	4	9	0.002445	1.7E-09	3	7	0.00189	6.6E-07
	$\tau_0^5$	10	21	0.00852	6.61E-07	6	13	0.005355	2.8E-10	7	15	0.004398	7.04E-08
	$\tau_0^6$	12	25	0.004707	6.96E-07	5	11	0.007905	7.69E-08	6	13	0.008258	5.53E-12
	$\tau_0^7$	11	24	0.009809	7.02E-07	6	13	0.004386	4.67E-09	7	15	0.005093	1.82E-12
5000	$\tau_0^1$	11	24	0.024685	4.47E-07	6	13	0.017609	8.42E-08	6	13	0.037219	7.26E-07
	$\tau_0^2$	8	18	0.068953	1.12E-07	7	15	0.016779	7.18E-09	8	17	0.038352	2.41E-13
	$\tau_0^3$	12	25	0.020824	7.08E-07	6	13	0.025728	1.5E-09	7	15	0.014606	2.92E-12
	$\tau_0^4$	11	23	0.007992	6.79E-07	4	9	0.007137	1.15E-07	4	9	0.009986	6.16E-08
	$\tau_0^5$	12	25	0.11886	7.91E-07	6	13	0.014072	1.1E-10	9	19	0.017335	3.09E-12
	$\tau_0^6$	8	18	0.056363	9.18E-07	6	13	0.031509	2.66E-09	7	15	0.020018	1.64E-12
	$\tau_0^7$	10	21	0.1057	5.29E-07	7	15	0.014144	5.12E-09	8	17	0.014342	2.43E-12
10000	$\tau_0^1$	11	23	0.13911	7.89E-07	7	15	0.046802	1.18E-08	6	13	0.025508	7.7E-07
	$\tau_0^2$	12	25	0.20667	9.1E-07	6	13	0.044856	2.6E-07	8	17	0.054282	1.37E-12
	$\tau_0^3$	5	11	0.047271	1.7E-07	6	13	0.033737	4.4E-09	7	15	0.053338	8.61E-07
	$\tau_0^4$	11	23	0.027811	9.6E-07	4	9	0.024463	1.98E-07	4	9	0.020689	3.34E-07
	$\tau_0^5$	12	26	0.33162	6.98E-07	5	11	0.042376	8.39E-08	9	20	0.15335	5.8E-07
	$\tau_0^6$	10	22	0.20379	5.71E-07	6	13	0.038846	4.32E-09	6	14	0.095344	5.05E-07
	$\tau_0^7$	11	23	0.2751	9.97E-07	8	17	0.061416	7.81E-09	7	15	0.038079	5.16E-08
50000	$\tau_0^1$	11	24	0.54052	5.54E-07	7	15	0.17481	8.59E-09	7	16	0.2819	4.59E-07
	$\tau_0^2$	13	27	2.9461	6.85E-07	6	13	0.14394	9.86E-07	10	21	0.22077	6.98E-07
	$\tau_0^3$	10	22	0.36083	9.11E-07	7	15	0.12973	1.18E-08	8	17	0.25995	2.79E-07
	$\tau_0^4$	5	11	0.11917	5.14E-09	4	9	0.065682	1.58E-07	5	11	0.17493	2.98E-12
	$\tau_0^5$	13	27	3.55	9.49E-07	9	19	0.23876	4.46E-09	10	21	0.2553	7.87E-07
	$\tau_0^6$	12	25	1.0885	9.1E-07	6	13	0.15428	1.33E-10	8	17	0.16788	5.11E-11
	$\tau_0^7$	12	26	1.7188	6.63E-07	7	15	0.23602	5.87E-11	9	19	0.23605	7.6E-07
100000	$\tau_0^1$	12	25	2.1483	6.7E-07	6	13	0.33328	2.77E-12	8	17	0.92769	3.1E-12
	$\tau_0^2$	13	28	8.0584	5.02E-07	9	19	0.6505	5.46E-11	10	21	0.53162	6.6E-07
	$\tau_0^3$	11	24	1.2596	7.75E-07	8	17	0.48872	2.91E-09	8	17	0.58746	2.71E-12
	$\tau_0^4$	5	11	0.21787	7.27E-09	5	11	0.19224	1.18E-09	5	12	0.41185	7.02E-07
	$\tau_0^5$	13	28	11.4267	6.86E-07	8	17	0.47075	7.96E-10	10	22	0.85016	8.81E-07
	$\tau_0^6$	12	26	3.4733	7.53E-07	6	13	0.34623	5.61E-11	10	21	0.60567	4.63E-07
	$\tau_0^7$	13	27	4.5101	5.24E-07	8	17	0.47738	1.61E-09	9	19	1.0242	6.71E-13

Table 12: Numerical Results for Problem 12

Problem 12		IHCgPM				ITIA				TDIA			
DIM	SP	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM
1000	$\tau_0^1$	16	34	0.054605	9.14E-07	24	50	0.05459	7.72E-07	25	52	0.023647	5.28E-07
	$\tau_0^2$	16	34	0.05873	9.32E-07	27	56	0.032615	3.88E-07	27	56	0.025802	8.86E-07
	$\tau_0^3$	16	34	0.027977	5.24E-07	25	52	0.028953	5.66E-07	26	54	0.036573	5.67E-07
	$\tau_0^4$	13	28	0.021862	8.76E-07	22	46	0.028179	7.93E-07	23	48	0.027258	7.85E-07
	$\tau_0^5$	19	39	0.047083	9.47E-07	25	52	0.02675	6.88E-07	30	62	0.044716	4.45E-07
	$\tau_0^6$	18	38	0.027832	8.48E-07	24	50	0.024397	4.1E-07	26	54	0.024276	6.68E-07
	$\tau_0^7$	18	37	0.016175	9.4E-07	23	48	0.024318	7.98E-07	29	60	0.026872	4.63E-07
5000	$\tau_0^1$	18	38	0.087395	8.79E-07	23	48	0.16735	9.5E-07	27	56	0.12039	6.56E-07
	$\tau_0^2$	18	38	0.14456	4.19E-07	26	54	0.13016	7.66E-07	28	58	0.3131	4.06E-07
	$\tau_0^3$	17	36	0.077551	7.89E-07	21	44	0.1169	6.16E-07	21	44	0.29163	9.05E-07
	$\tau_0^4$	13	28	0.11603	5.68E-07	21	44	0.090216	8.18E-07	22	46	0.11749	4.71E-07
	$\tau_0^5$	17	36	0.45579	1E-06	28	58	0.15137	4.63E-07	31	64	0.16183	5.54E-07
	$\tau_0^6$	16	34	0.079907	7.13E-07	25	52	0.16227	7.75E-07	24	50	0.10626	5.86E-07
	$\tau_0^7$	17	36	0.13077	3.16E-07	27	56	0.16923	4.6E-07	24	50	0.12919	8.87E-07
10000	$\tau_0^1$	16	34	0.18563	7.85E-07	25	52	0.2793	5.44E-07	25	52	0.26534	9.39E-07
	$\tau_0^2$	17	36	0.95326	7E-07	25	52	0.28872	8.33E-07	30	62	0.33942	9.12E-07
	$\tau_0^3$	18	38	0.28358	7.52E-07	23	48	0.24762	6.87E-07	24	50	0.50074	7.67E-07
	$\tau_0^4$	14	30	0.15631	7.09E-07	21	44	0.5001	5.95E-07	21	44	0.43812	8.83E-07
	$\tau_0^5$	17	36	0.68784	7.23E-07	28	58	0.49122	6.35E-07	30	62	0.36474	7.5E-07
	$\tau_0^6$	18	38	0.24437	5.94E-07	25	52	0.29969	6.58E-07	21	44	0.23379	7.1E-07
	$\tau_0^7$	17	36	0.61045	8.43E-07	26	54	0.31165	8.7E-07	28	58	0.66163	8.39E-07
50000	$\tau_0^1$	17	36	1.6978	5.11E-07	26	54	1.731	5.25E-07	29	60	1.3138	6.49E-07
	$\tau_0^2$	17	36	5.8225	4.73E-07	27	56	1.7387	7.18E-07	29	60	1.828	3.74E-07
	$\tau_0^3$	17	36	1.5032	6.48E-07	26	54	1.0695	7.43E-07	28	58	1.0499	5.9E-07
	$\tau_0^4$	13	28	0.49257	9.55E-07	21	44	1.4399	8.02E-07	20	42	1.3984	8.53E-07
	$\tau_0^5$	17	36	7.2145	5.12E-07	27	56	1.146	4.73E-07	31	64	1.5276	6.1E-07
	$\tau_0^6$	16	34	2.7065	9.13E-07	27	56	1.6402	5.11E-07	28	58	1.0094	9.25E-07
	$\tau_0^7$	16	34	3.3368	9.14E-07	24	50	1.2168	7.13E-07	31	64	2.0694	5.43E-07
100000	$\tau_0^1$	16	34	4.4119	7.87E-07	25	52	2.3084	3.9E-07	30	62	2.6353	5.78E-07
	$\tau_0^2$	16	34	14.9113	8.96E-07	27	56	2.6854	5.89E-07	29	60	2.572	3.78E-07
	$\tau_0^3$	16	34	3.2659	8.57E-07	23	48	2.148	4.18E-07	27	56	2.0997	8.22E-07
	$\tau_0^4$	14	30	0.678	8.45E-07	21	44	2.192	6.52E-07	22	46	2.5336	5.97E-07
	$\tau_0^5$	16	34	20.2816	9.82E-07	27	56	3.0931	9.16E-07	31	64	3.046	6.97E-07
	$\tau_0^6$	16	34	6.5815	7.66E-07	26	54	2.7764	8.94E-07	28	58	3.0556	9.48E-07
	$\tau_0^7$	16	34	9.2442	7.87E-07	24	50	2.5522	5.31E-07	31	64	2.9053	5.42E-07

Table 13: Numerical Results for Problem 13

Problem 13		IHCGPM				ITIA				TDIA			
DIM	SP	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM
1000	$\tau_0^1$	19	40	0.85858	9.29E-07	14	30	0.47715	9.14E-08	20	42	0.1727	4.6E-07
	$\tau_0^2$	19	40	0.17823	7.52E-07	13	28	0.23152	9.25E-07	17	36	0.13548	2.99E-07
	$\tau_0^3$	19	40	0.1092	3.21E-07	14	29	0.11422	8.16E-07	18	38	0.12587	6.2E-07
	$\tau_0^4$	17	36	0.21487	7.95E-07	11	24	0.092696	1.47E-07	16	34	0.11227	2.65E-07
	$\tau_0^5$	19	40	0.42625	6.87E-07	15	32	0.21003	1.78E-07	21	44	0.1645	6.24E-07
	$\tau_0^6$	22	46	0.12399	6.28E-07	14	30	0.30012	5.07E-07	16	34	0.12942	4.21E-07
	$\tau_0^7$	22	46	0.12171	3.24E-07	14	29	0.45473	4.96E-07	18	38	0.27883	4.99E-07
5000	$\tau_0^1$	23	48	0.37125	3.04E-07	15	32	0.44605	3.22E-07	18	38	0.49523	5.77E-07
	$\tau_0^2$	19	40	0.7418	2.59E-07	14	30	0.47272	3.9E-07	22	46	1.4052	8.16E-07
	$\tau_0^3$	19	40	0.69998	5.71E-07	14	30	1.0637	4.74E-07	19	40	0.97008	5.47E-07
	$\tau_0^4$	17	36	0.25941	7.73E-07	13	28	0.357	1.81E-07	11	24	0.28923	6.1E-07
	$\tau_0^5$	20	42	0.89958	6.28E-07	17	36	0.45277	3.97E-07	19	40	0.67614	4.19E-07
	$\tau_0^6$	21	44	0.86529	6.36E-07	15	31	0.87385	9.27E-07	20	42	0.784	3.52E-07
	$\tau_0^7$	24	50	0.76853	5.42E-07	18	38	0.74581	3.23E-07	23	47	0.47629	8.65E-07
10000	$\tau_0^1$	20	42	0.75885	6.45E-07	16	33	0.99969	4.61E-07	21	44	2.1137	2.77E-07
	$\tau_0^2$	19	40	2.5382	3.45E-07	19	40	1.2943	3.8E-07	22	46	1.4022	5.25E-07
	$\tau_0^3$	19	40	1.0375	4.24E-07	16	34	0.82723	1.1E-07	20	42	0.95923	4.74E-07
	$\tau_0^4$	17	36	0.74641	6.46E-07	11	24	0.57269	1.61E-07	12	26	0.63679	8.93E-07
	$\tau_0^5$	19	39	3.1405	7.26E-07	15	32	1.6615	1.57E-07	23	48	1.4827	3.38E-07
	$\tau_0^6$	21	44	1.1292	4.91E-07	14	29	0.68929	8.52E-07	21	44	1.5374	7.67E-07
	$\tau_0^7$	21	44	1.6853	9.25E-07	17	36	0.93449	9.64E-07	21	44	1.0185	2.9E-07
50000	$\tau_0^1$	19	40	6.9668	5.17E-07	18	38	5.5473	3.3E-07	20	42	5.3971	4.7E-07
	$\tau_0^2$	23	48	21.0452	7.17E-07	18	38	4.7386	5.99E-07	22	46	6.3284	9.98E-07
	$\tau_0^3$	19	40	6.4349	7.64E-07	12	26	3.4733	6.69E-07	17	36	4.3549	3.32E-07
	$\tau_0^4$	16	34	2.838	5.34E-07	12	25	2.9152	6.02E-07	17	36	4.3071	3.02E-07
	$\tau_0^5$	21	44	26.9111	3.68E-07	21	44	6.3289	7.81E-07	24	50	6.2542	4.67E-07
	$\tau_0^6$	22	46	10.981	6.8E-07	18	38	4.7344	3.63E-07	23	47	6.0467	9.13E-07
	$\tau_0^7$	22	46	14.3101	3.39E-07	15	32	4.2906	8.15E-07	22	46	6.0341	4.41E-07
100000	$\tau_0^1$	20	42	17.2817	6.36E-07	19	40	9.3296	2.73E-07	15	32	6.9804	7.32E-07
	$\tau_0^2$	24	50	58.6543	6.71E-07	17	35	8.5958	8.56E-07	24	49	11.8541	9.1E-07
	$\tau_0^3$	19	40	14.8857	6.97E-07	17	35	8.3059	7.28E-07	19	40	8.7448	3.18E-07
	$\tau_0^4$	15	32	4.4018	9.05E-07	12	26	6.1098	5.49E-07	17	35	6.5374	9.08E-07
	$\tau_0^5$	21	44	72.8444	4.29E-07	19	40	9.048	5.44E-07	28	58	14.2284	8.33E-07
	$\tau_0^6$	23	48	28.1974	6.24E-07	19	40	9.9799	7.76E-07	22	46	10.0934	6.17E-07
	$\tau_0^7$	22	46	33.2843	6.73E-07	22	46	11.4746	3.68E-07	21	44	9.735	5.31E-07

Table 14: Numerical Results for Problem 14

Problem 14		IHCGPM				ITIA				TDIA			
SP	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM	
$\tau_0^1$	46	93	0.091535	8.73E-07	44	89	0.011436	6.96E-07	47	95	0.023311	7.78E-07	
$\tau_0^2$	49	99	0.013642	7.9E-07	46	93	0.008471	6.48E-07	49	99	0.013319	6.25E-07	
$\tau_0^3$	46	93	0.006961	7.96E-07	44	89	0.008711	6.28E-07	47	95	0.007692	6.52E-07	
$\tau_0^4$	47	95	0.006367	8.3E-07	42	85	0.007562	7.86E-07	46	93	0.007955	7.12E-07	
$\tau_0^5$	48	97	0.012423	9.3E-07	46	93	0.009685	9.34E-07	49	99	0.014394	7.37E-07	
$\tau_0^6$	47	95	0.007059	7.58E-07	44	89	0.008558	6.45E-07	48	97	0.012968	6.04E-07	
$\tau_0^7$	48	97	0.007524	8.65E-07	44	89	0.008338	9.99E-07	48	97	0.007957	9.1E-07	

Table 15: Numerical Results for Problem 15

Problem 15		IHCGPM				ITIA				TDIA			
SP	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM	ITER	FVAL	TIME	NORM	
$\tau_0^1$	34	70	0.024671	3.21E-07	28	57	0.032968	4.63E-07	53	108	0.04327	7.53E-07	
$\tau_0^2$	40	82	0.015053	7.86E-07	33	68	0.018861	9.15E-07	57	116	0.020521	9.7E-07	
$\tau_0^3$	35	72	0.009519	7.1E-07	29	60	0.010914	9.26E-07	51	104	0.016961	9.33E-07	
$\tau_0^4$	28	58	0.007225	5.43E-07	27	56	0.010948	8.29E-07	41	84	0.012008	8.47E-07	
$\tau_0^5$	45	92	0.016104	8.4E-07	34	70	0.01851	9.46E-07	60	122	0.025511	6.75E-07	
$\tau_0^6$	41	84	0.008847	6.95E-07	34	70	0.013782	3.78E-07	54	110	0.015033	9.76E-07	
$\tau_0^7$	41	84	0.01087	4.98E-07	34	69	0.011241	7.45E-07	55	112	0.01575	9.26E-07	