Math 426.2SY
Calculus II

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Outline

5.4-The Fundamental Theorem of Calculus, Part1 (FTC1)

Theorem

Suppose that f is a continuous function. Let $F(x) = \int_a^x f(t)\,dt$, where a is any real number. Then

$$F'(x) = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

FTC1

- Before providing a justification of FTC 1, let's look at a few examples of how to apply the theorem

Example

Use FTC1 to find $\frac{dy}{dx}$ for the following functions:
$y = \int_2^x t^3 + 1\, dt$

FTC1

Example

$$y = \int_x^5 3t \sin t \, dt$$

FTC1

Example

$$y = \int_1^{x^2} \cos t \, dt$$

- Notice here that the upper limit of integration is NOT x, but is some other more complicated function of x .
- Procedure:

FTC1

- This last example gives us a more general version of FTC 1:

Theorem

Suppose that f is a continuous function, and g is some other function of x. Let $F(x) = \displaystyle\int_a^{g(x)} f(t)\,dt$. Then

$$F'(x) = \frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f(g(x)).g'(x)$$

FTC1

Example

$$y = \int_{1+3x^2}^{4} \frac{1}{2+e^t}\,dt$$

Justification for FTC 1

- Let f be a continuous function. Let $F(x) = \int_a^x f(t)\,dt$.

Definition

- Let f be a function with antiderivative F. The indefinite integral of f, denoted $\displaystyle\int f(x)\,dx$ is defined as:

$$\int f(x)\,dx = F(x) + C$$

Important distinction:

- $\displaystyle\int_a^b f(x)\,dx$ is a number.
- $\displaystyle\int f(x)\,dx$ is a function plus an arbitrary constant.

Indefinite Integrals

Example

$$y = \int e^{-4x}\, dx$$

Indefinite Integrals

Example

$$y = \int \sin(\pi x) - \frac{2}{x}\, dx$$

Differentials

Definition

Let $y = f(x)$ be a differentiable function. Let dx an independent variable. Define the differential dy to be

$$dy = f'(x)dx$$

A convenient way to think of differentials is to start with the derivative

$$\frac{dy}{dx} = f'(x)$$

then (viewing this as a ratio) multiply both sides by dx.

Differentials

Example

Find dy in each case:

1. $y = \tan(e^x)$
2. $y = \dfrac{\sin 2x}{x+1}$
3. $y = \sqrt{2x+1}$

The Substitution Method

Example

$$\int (x^2 - 3x)^4 (2x - 3)\, dx$$

The Substitution Method

Example

$$\int \sqrt{4x-2}\,dx$$

The Substitution Method

Example

$$\int e^{2x^3-3x}(2x-1)\,dx$$

The Substitution Method

Example

$$\int x\sqrt{3x+5}\,dx$$

The Substitution Method

Example

$$\int x^2 \sec^2(x^3 + 1)\, dx$$

The Substitution Method

An integrand may require some algebraic manipulation before the substitution method can be applied.

Example

$$\int \frac{1}{e^x + e^{-x}}\,dx$$

The Substitution Method

Sometimes we can use trig identities to transform integrals we do not know how to evaluate into ones we do.

Example

$$\int \sin^2(x)\,dx$$

The Substitution Method

Example

$$\int \sec(x)\,dx$$