Math 426.2SY
Calculus II

University of New Hampshire

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Outline

5.3-The Definite Integral

- Last time we discussed the notion of a Riemann sum for a function f on the interval $[a, b]$:

The Definite Integral

- The definite integral of f over $[a, b]$, denoted by $\int_a^b f(x)dx$ is defined as the limit of these Riemann sums as n approaches ∞ :

The Definite Integral

- Important note: the value of the definite integral of a function over a particular interval depends on the function, not the letter we choose to represent its independent variable.
- So the following expressions all mean the same thing:

The Definite Integral

- We can interpret the definite integral of f over $[a, b]$ as the "signed" area between the graph of f and the x-axis:
- Area above the x-axis is considered positive
- Area below the x-axis is considered negative

Properties of definite integrals

1. Order of Integration:

2. Zero Width Interval:

3. Constant Multiple:

4. Sum and Difference:

5. Additivity:

Properties of definite integrals

- Max-Min Inequality: suppose $min(f)$ is the smallest value that f attains on $[a, b]$, and $max(f)$ is the largest value that f attains on $[a, b]$. Then:

Properties of definite integrals

- Domination: if $g(x) \leq f(x)$ on $[a, b]$ then:

- Special case: if $f(x) \geq 0$ on $[a, b]$ then:

Properties of definite integrals

Example

Suppose that $\int_{-1}^{1} f(x)dx = -5, \int_{-1}^{4} f(x)dx = \pi - 5, \int_{-1}^{1} g(x)dx = 7$. Compute the following:

1. $\int_{4}^{1} f(x)dx$
2. $\int_{2}^{2} 2f(x) - \int_{1}^{-1} 3g(x)dx$
3. $\int_{-1}^{1} f(x) - \frac{g(x)}{2}dx$

Properties of definite integrals

Example

Show that $0 \leq \int_0^1 \sqrt{1+\cos x}\, dx \leq \sqrt{2}$

Definite integrals

- Since we can interpret $\int_a^b f(x)\,dx$ as the "signed" area between the graph of f and the x-axis, we can use geometry to compute some definite integrals.

Definite integrals

Example

$$\int_{-2}^{1} |x| \, dx.$$

Definite integrals

Example

$$\int_{-4}^{4} \sqrt{16 - x^2}\, dx.$$

Definite integrals

- Unfortunately, this approach using areas is only useful if the graph of the function we're integrating consists of simple shapes like circles or triangles
- For instance, it's not so helpful if we are trying to compute

$$\int_0^{\pi} \sin x \, dx$$

The Fundamental Theorem of Calculus, Part2(FTC2)

Theorem

If f is continuous on $[a, b]$ then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where F is any antiderivative of f.

Basic Antiderivative Formulas

TABLE 4.2 Antiderivative formulas, k a nonzero constant

Function	General antiderivative
1. x^n	$\frac{1}{n+1}x^{n+1} + C, \quad n \neq -1$
2. $\sin kx$	$-\frac{1}{k}\cos kx + C$
3. $\cos kx$	$\frac{1}{k}\sin kx + C$
4. $\sec^2 kx$	$\frac{1}{k}\tan kx + C$
5. $\csc^2 kx$	$-\frac{1}{k}\cot kx + C$
6. $\sec kx \tan kx$	$\frac{1}{k}\sec kx + C$
7. $\csc kx \cot kx$	$-\frac{1}{k}\csc kx + C$
8. e^{kx}	$\frac{1}{k}e^{kx} + C$
9. $\frac{1}{x}$	$\ln\lvert x\rvert + C, \quad x \neq 0$
10. $\frac{1}{\sqrt{1-k^2x^2}}$	$\frac{1}{k}\sin^{-1} kx + C$
11. $\frac{1}{1+k^2x^2}$	$\frac{1}{k}\tan^{-1} kx + C$
12. $\frac{1}{x\sqrt{k^2x^2-1}}$	$\sec^{-1} kx + C, kx > 1$
13. a^{kx}	$\left(\frac{1}{k\ln a}\right)a^{kx} + C, a > 0, a \neq 1$

The rules in Table 4.2 are easily verified by differentiating the general antiderivative formula to obtain the function to its left. For example, the derivative of $(\tan kx)/k + C$ is $\sec^2 kx$, whatever the value of the constants C or $k \neq 0$, and this establishes Formula 4 for the most general antiderivative of $\sec^2 kx$.

FTC2

Example

$$\int_0^{\pi} \sin x \, dx$$

FTC2

Example

$$\int_{-\pi/4}^{0} \sec x \tan x \, dx$$

FTC2

Example

$$\int_1^{\sqrt{2}} \frac{x^2+\sqrt{x}}{x^2}\,dx$$

FTC2

Example

$$\int_0^1 \frac{3}{4}\sqrt{x} + \frac{4}{3}\frac{1}{\sqrt{x}}\,dx$$

FTC2

Example

$$\int_{1/2}^{1/4} \frac{4}{4x+1}\,dx$$

Application: Computing Total Area

To find the total area (not "signed" area) between the graph of $y = f(x)$ and the x-axis over the interval [a, b]:

1. Subdivide $[a, b]$ at the zeros of f.
2. Integrate f over each subinterval.
3. Add the absolute values of the integrals.

FTC2

Example

Find the total area (not “signed” area) of the region between the x-axis and the graph of $f(x) = x^3 - x^2 - 2x$ on the interval $[-1, 2]$.