# Math 426.2SY
# Calculus II

University of New Hampshire

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# Outline

# Taylor Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

# The complex number $i$

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# The Euler's identity

# Growth Rate of Functions

$p$ and $r$ are constants. $p > 0$ and $r > 1$.

| | | |
|---|---|---|
| $\ln(n) \to \infty$ | $\dfrac{1}{\ln(n)} \to 0$ | Slowly |
| $n^p \to \infty$ | $\dfrac{1}{n^p} \to 0$ | Moderate |
| $r^n \to \infty$ | $\dfrac{1}{r^n} \to 0$ | Fast |
| $n! \to \infty$ | $\dfrac{1}{n!} \to 0$ | Very Fast |

$$\boxed{n! >> r^n >> n^p >> \ln(n) >> 1}$$

# Review

## Example

$$\sum_{n=1}^{\infty} \frac{\ln(n)}{n^3}$$

# Review

## Example

$$\sum_{n=3}^{\infty} \frac{1}{n^{1/4}\ln(n)}$$

# Review

## Example

$$\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2+1}$$

# Review

## Basic Limits

- $\lim_{n\to\infty} \sqrt[n]{n} = 1$
- $\lim_{n\to\infty} \frac{\ln(n)}{n} = 0$
- $\lim_{n\to\infty} x^n = 0 \quad (|x| < 1)$
- $\lim_{n\to\infty} (1 + \frac{1}{n})^n = e$
- $\lim_{n\to\infty} (1 + \frac{k}{n})^n = e^k \quad (\text{any } k)$
- $\lim_{n\to\infty} x^{\frac{1}{n}} = 1 \quad (x > 0)$
- $\lim_{n\to\infty} \frac{x^n}{n!} = 0 \quad (\text{any } x)$

# Review

## Example

$$\lim_{n\to\infty} \frac{n^2 - 1}{2n^2}$$

# Review

## Example

$$\lim_{n\to\infty} \sqrt[3n]{n}$$

# Review

## Example

$$\lim_{n\to\infty}\left(\frac{n+3}{n}\right)^{3n}$$

# Review

## Example

$$\lim_{n\to\infty} \frac{1+n-n^2}{2n^2-3n+4}$$

# Review

## Example

$$\lim_{n\to\infty} \frac{2^n - n^2}{4^n + n^{10}}$$

# Review

## Example

$$\lim_{n\to\infty} \frac{e^{2n} + 3^n}{e^n - 2^n}$$

# Review

## Example

$$\lim_{n\to\infty} \frac{(-1)^n}{2n+1}$$

# Review

### Example

$$\lim_{n\to\infty}(-1)^n\left(1-\frac{1}{n^2}\right)$$

# Review

## Conclusion

If $\lim_{n\to\infty} f(n) = 0$, then $\lim_{n\to\infty} (-1)^n f(n) = 0$

If $\lim_{n\to\infty} f(n) \neq 0$, then $\lim_{n\to\infty} (-1)^n f(n)$ DNE