Math 426.2SY Calculus II

University of New Hampshire

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1 / 20

Outline

The Euler's Identity

2 Review of chapter 9

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Taylor Series

$$e^{x} = 1 + x + \frac{x^{2}}{2!} + \frac{x^{3}}{3!} + \frac{x^{4}}{4!} + \dots = \sum_{n=0}^{\infty} \frac{x^{n}}{n!}$$
$$\sin(x) = x - \frac{x^{3}}{3!} + \frac{x^{5}}{5!} - \frac{x^{7}}{7!} + \dots = \sum_{n=0}^{\infty} (-1)^{n} \frac{x^{2n+1}}{(2n+1)!}$$
$$\cos(x) = 1 - \frac{x^{2}}{2!} + \frac{x^{4}}{4!} - \frac{x^{6}}{6!} + \dots = \sum_{n=0}^{\infty} (-1)^{n} \frac{x^{2n}}{(2n)!}$$



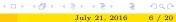
The complex number i

The complex number i

5 / 20

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The Euler's identity



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Growth Rate of Functions

p and r are constants. p > 0 and r > 1.

$$ln(n) \to \infty$$
 $\frac{1}{ln(n)} \to 0$ Slowly

$$n^p \to \infty$$
 $\frac{1}{n^p} \to 0$ Moderate

$$r^n \to \infty$$
 $\frac{1}{r^n} \to 0$ Fast

$$n! \to \infty$$
 $\frac{1}{n!} \to 0$ Very Fast

$$|n!>>r^n>>n^p>>\ln(n)>>1$$



$$\sum_{n=1}^{\infty} \frac{\ln(n)}{n^3}$$



$$\sum_{n=3}^{\infty} \frac{1}{n^{1/4} \ln(n)}$$



$$\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2 + 1}$$



Basic Limits

- $\bullet \lim_{n \to \infty} \sqrt[n]{n} = 1$
- $\bullet \lim_{n \to \infty} \frac{\ln(n)}{n} = 0$
- $\lim x^n = 0 \quad (|x| < 1)$ $n \rightarrow \infty$
- $\bullet \lim_{n \to \infty} (1 + \frac{1}{n})^n = e$
- $\bullet \lim_{n \to \infty} (1 + \frac{k}{n})^n = e^k \quad (\text{any } k)$
- $\bullet \lim_{n \to \infty} x^{\frac{1}{n}} = 1 \quad (x > 0)$
- $\bullet \lim_{n \to \infty} \frac{x^n}{n!} = 0 \quad (\text{any } x)$



$$\lim_{n\to\infty}\frac{n^2-1}{2n^2}$$



Example

 $\lim_{n\to\infty} \sqrt[3n]{n}$



$$\lim_{n \to \infty} \left(\frac{n+3}{n} \right)^{3n}$$



$$\lim_{n\to\infty}\frac{1+n-n^2}{2n^2-3n+4}$$



$$\lim_{n\to\infty}\frac{2^n-n^2}{4^n+n^{10}}$$



Example

$$\lim_{n\to\infty}\frac{e^{2n}+3^n}{e^n-2^n}$$



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$$\lim_{n \to \infty} \frac{(-1)^n}{2n+1}$$



$$\lim_{n\to\infty} (-1)^n \left(1 - \frac{1}{n^2}\right)$$



Conclusion

If
$$\lim_{n \to \infty} f(n) = 0$$
, then $\lim_{n \to \infty} (-1)^n f(n) = 0$

If
$$\lim_{n\to\infty} f(n) \neq 0$$
, then $\lim_{n\to\infty} (-1)^n f(n)$ DNE