

# Math 426.2SY

## Calculus II

University of New Hampshire

June 19, 2017

# Outline

## 1 Section 8.4 - Partial Fractions

# Introduction

## Main Idea:

Break up rational functions  $\frac{P(x)}{Q(x)}$  as a sum of simpler fractions which are easily integrated.

## Example

$$\frac{5x - 3}{x^2 - 2x - 3} = \frac{2}{x + 1} + \frac{3}{x - 3}$$

# Partial Fractions

## Case 1

- degree of numerator  $<$  degree of denominator
- Denominator can be factored into **non-repeated** linear factors

# Partial Fractions

## Example of Case 1

$$\frac{5x - 3}{x^2 - 2x - 3}$$

# Partial Fractions

Example of case 1

$$\int \frac{x^2 + 4x + 1}{(x - 1)(x + 1)(x + 3)} dx$$

# Partial Fractions

# Partial Fractions

## Case 2

- degree of numerator  $<$  degree of denominator
- Denominator can be factored into **repeated** linear factors



# Partial Fractions

Example of case 2

$$\int \frac{6x + 7}{(x + 2)^2} dx$$

# Partial Fractions

# Partial Fractions

## Case 3

- degree of numerator  $<$  degree of denominator
- When factored, denominator has an irreducible quadratic

# Partial Fractions

Example of case 3

$$\int \frac{dx}{(x+1)(x^2+1)}$$

# Partial Fractions

# Partial Fractions

## Case 4

- degree of numerator  $\geq$  degree of denominator
- In this case we begin by dividing the denominator into the numerator to get a polynomial plus a proper fraction.

## Example

$$\int \frac{2x^3 - 4x^2 - x - 3}{x^2 - 2x - 3} dx$$

# Dealing with combinations of cases

## Example

$$\int \frac{-2x + 4}{(x^2 + 1)(x - 1)^2} dx$$

# Partial Fractions



# Dealing with combinations of cases

Example

$$\int \frac{dx}{x(x^2 + 1)^2} dx$$

# Partial Fractions