

# Math 426.2SY

## Calculus II

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# Outline

## 1 5.6 - Substitution and Area Between Curves

## 5.6 - Substitution and Definite Integrals

There are two methods to evaluate definite integrals using the substitution method.

### Example of Method 1

$$\int_{-1}^1 3x^2 \sqrt[3]{x^3 + 1} dx$$

# Substitution and Definite Integrals

## Example of Method 2

$$\int_{-1}^1 3x^2 \sqrt[3]{x^3 + 1} dx$$

# Substitution and Definite Integrals

## More Examples of Method 2

$$\int_0^{\pi/12} \tan(3x) \sec^2(3x) dx$$

# Substitution and Definite Integrals

## More Examples of Method 2

$$\int_{-\pi/4}^{\pi/4} \tan x \, dx$$

# Definite Integrals of Symmetric Functions

## Definition

- A function  $f(x)$  is called **even** if  $f(-x) = f(x)$  for all  $x$ .
- A function  $f(x)$  is called **odd** if  $f(-x) = -f(x)$  for all  $x$ .
- Examples:

# Definite Integrals of Symmetric Functions

## Theorem

Let  $f$  be a function defined on the interval  $[-a, a]$ .

- ① If  $f$  is odd, then  $\int_{-a}^a f(x) dx = 0$
- ② If  $f$  is even, then  $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$

- Proof:



# Definite Integrals of Symmetric Functions

## Examples

❶  $\int_{-\pi/3}^{\pi/3} e^{x^2} \sin x \, dx$

❷  $\int_{-2}^2 x^4 - 4x^2 + 6 \, dx$

# Areas Between Curves

- Suppose we want to find the area of a region that is bounded above by the curve  $y = f(x)$ , below by the curve  $y = g(x)$ , and on the left and right by the lines  $x = a$  and  $x = b$ .

# Areas Between Curves

If  $f(x) \geq g(x)$  throughout  $[a, b]$ , then the area of the region between the curves  $y = f(x)$  and  $y = g(x)$  from  $a$  to  $b$  is given by:

$$A = \int_a^b f(x) - g(x) dx$$

# Areas Between Curves

## Example

Find the area of the region enclosed by the parabola  $y = 2 - x^2$  and the line  $y = -x$ .

# Areas Between Curves

## Note

If the formula for a bounding curve changes at one or more points, we subdivide the region into subregions that correspond to the formula changes and apply the formula for the area between curves to each subregion.

# Areas Between Curves

## Example

Find the area of the region in the first quadrant that is bounded above by  $y = \sqrt{x}$  and below by the  $x$ -axis and the line  $y = x-2$ .

# Areas Between Curves

## Example

Find the area of the region enclosed by  $y = 4x + 3$  and  $y = 3 - x^2$ .

# Areas Between Curves

## Example

Find the area of the region enclosed by  $x + y^2 = 0$  and  $x + 3y^2 = 2$ .