Math 426.2SY
Calculus II

University of New Hampshire

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Outline

Introduction

Improper Integrals of Type I

Domain of integration is not finite:

$$\int_0^\infty e^{-x}\,dx, \quad \int_{-\infty}^{-1} \frac{1}{x}\,dx, \quad \int_{-\infty}^{\infty} \frac{x}{x^2+1}\,dx, \quad \ldots$$

Improper Integrals of Type II

The function we're integrating is not bounded in the domain of integration (Vertical Assymptote).

$$\int_0^1 \frac{1}{x^2}\,dx, \quad \int_0^1 \frac{1}{x}\,dx, \quad \int_0^1 \frac{1}{\sqrt{x}}\,dx, \quad \int_{-2}^{2} \frac{x}{x-1} \ldots$$

Improper Integrals, Type II

Integrands with vertical asymptotes

Consider the area of the region above $[0, 4]$ and under the curve $f(x) = \dfrac{1}{\sqrt{x}}$.

Improper Integrals, Type II

Formal Definitions

If $f(x)$ is continuous on $(a, b]$ and discontinuous at a, then

$$\int_a^b f(x)\,dx = \lim_{c \to a^+} \int_c^b f(x)\,dx$$

If $f(x)$ is continuous on $[a, b)$ and discontinuous at b, then

$$\int_a^b f(x)\,dx = \lim_{c \to b^-} \int_a^c f(x)\,dx$$

If $f(x)$ is continuous on $[a, c) \cup (c, b]$ and discontinuous at c, then

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

In each case if the limit is finite, we say that the improper integral **converges**. If the limit fails to exist, we say it **diverges**.

Improper Integrals, Type II

Example

$$\int_0^1 \frac{1}{x}\,dx$$

Improper Integrals, Type II

Example

$$\int_{-1}^{1} \frac{1}{x^2}\,dx$$

Improper Integrals, Type II

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$$\int_{-1}^{1} \frac{1}{x^2}\, dx$$

Improper Integrals, Type II

Example

$$\int_0^4 \frac{1}{\sqrt{4-x}}\,dx$$

Last time we showed that

$$\int_1^\infty \frac{1}{x^p}\,dx = \begin{cases} \frac{1}{p-1} & ; \text{ if } p > 1 \\ \text{diverges} & ; \text{ if } p \leq 1 \end{cases}$$

Similarly, if $a > 0$

$$\int_a^\infty \frac{1}{x^p}\,dx = \begin{cases} \text{converges} & ; \text{ if } p > 1 \\ \text{diverges} & ; \text{ if } p \leq 1 \end{cases}$$

Important Picture to keep in mind