Math 426.2SY
Calculus II

University of New Hampshire

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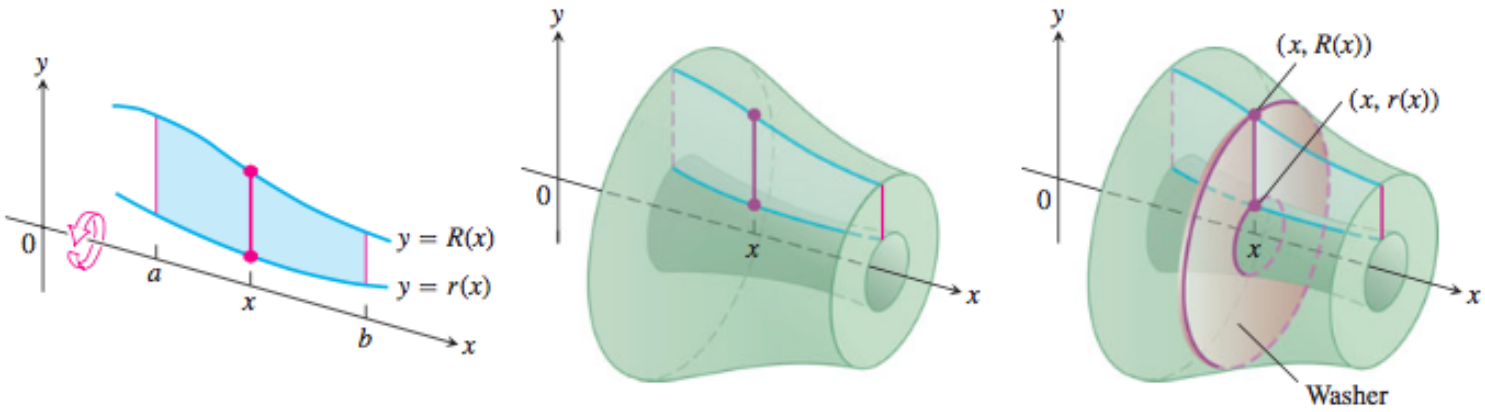
Outline

Introduction

- Solid of revolution: a 3D solid generated by rotating (or revolving) a 2D region about an axis.



- We will study two methods for computing the volume of such a solid:
  - Method of Disks/Washers (section 6.1)
  - Method of Cylindrical Shells (section 6.2)

Method of Disks

Example

The region between the curve $y = \sqrt{x}, 0 \leq x \leq 4$ and the x-axis is revolved about the x-axis to generate a solid. Find its volume.

Method of Disks

Example

Find the volume of the solid generated by revolving the region above the x-axis and below the curve $x^2 + y^2 = a^2$ about the x-axis.

Method of Disks

Example

Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1, x = 4$ about the line $y = 1$.

Method of Disks

Example

Find the volume of the solid generated by revolving the region between the y-axis and the curve $x = 2/y, 1 \leq y \leq 4$ around the $y-$axis.

Method of Disks

Example

Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.

Method of Washers

- What if the 2D region we are revolving does not border on or cross the axis of revolution?

Method of Washers

Example

The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the x-axis to generate a solid. Find the volume of the solid.

Method of Washers

Example

The region bounded by the parabola $y = x^2$ and the line $y = 2x$ in the first quadrant is revolved about the y-axis to generate a solid. Find the volume of the solid.

Procedure

Finding the volume using the Method of Disks/Washers

- Look at a slice of the 2D region **perpendicular** to the axis of revolution.
- Find the volume of this slice after rotation. (Call it ΔV_k)
- Approximate the total volume by $V \approx \sum_{k=1}^{n} \Delta V_k$
- Replace the $\sum$ by $\int$
- Find the limits of integration by looking at the region.
- Evaluate the definite integral.