Math 426.2SY
Calculus II

University of New Hampshire

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Outline

Introduction

Main idea:

Use trig identities to simplify integrals involving trig functions.

Important Identities:

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin^2(x) = \frac{1 - \cos(2x)}{2}$$

$$\cos^2(x) = \frac{1 + \cos(2x)}{2}$$

$$\tan^2(x) = \sec^2(x) - 1$$

$$\sec^2(x) = 1 + \tan^2(x)$$

Integrals of powers of $\sin(x)$ and $\cos(x)$

We'll start with integrals of the form:

$$\int \sin^m(x)\cos^n(x)\,dx$$

where m and n are nonnegative integers.

Case 1: m is odd

Integrals of powers of $\sin(x)$ and $\cos(x)$

Example

$$\int \sin^3(x)\cos^6(x)\,dx$$

Integrals of powers of $\sin(x)$ and $\cos(x)$

Case 2: n is odd

Integrals of powers of $\sin(x)$ and $\cos(x)$

Example

$$\int \sin^2(x)\cos^5(x)\,dx$$

Integrals of powers of $\sin(x)$ and $\cos(x)$

Note:

If m and n are both odd, either method may be used.

Example

$$\int \sin^3(x)\cos^3(x)\,dx$$

Integrals of powers of $\sin(x)$ and $\cos(x)$

Case 3: Both m and n are even

In this case we use

$$\sin^2(x) = \frac{1-\cos(2x)}{2}$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$

to reduce the integral to one in lower powers of $\cos(2x)$

Integrals of powers of $\sin(x)$ and $\cos(x)$

Example

$$\int \sin^2(x)\cos^4(x)\,dx$$

Eliminating Square Roots

We can also use the identities

$$\sin^2(x) = \frac{1 - \cos(2x)}{2}$$

$$\cos^2(x) = \frac{1 + \cos(2x)}{2}$$

to eliminate a square root.

Eliminating Square Roots

Example

$$\int_0^{\pi} \sqrt{1-\cos(2x)}\, dx$$

Integrals of Even Powers of $\tan(x)$ and $\sec(x)$

To integrate higher powers of tangent and secant, we can use the identities

$$\tan^2(x) = \sec^2(x) - 1$$

$$\sec^2(x) = 1 + \tan^2(x)$$

as follows:

Integrals of Even Powers of $\tan(x)$ and $\sec(x)$

For even integeers n and m

$$\int \tan^n(x)\,dx$$

$$\int \sec^n(x)\,dx$$

$$\int \tan^n(x)\sec^m(x)\,dx$$

Integrals of even powers of $\tan(x)$ and $\sec(x)$

Example

$$\int \tan^4(x)\,dx$$

Integrals of evem powers of $\tan(x)$ and $\sec(x)$

Example

$$\int \sec^4(x)\,dx$$

Integrals of even powers of $\tan(x)$ and $\sec(x)$

Example

$$\int \tan^4(x)\sec^4(x)\,dx$$

Products of Sines and Cosines

Finally, we will consider integrals of the forms:

$$\int \sin(mx)\sin(nx)\,dx$$

$$\int \sin(mx)\cos(nx)\,dx$$

$$\int \cos(mx)\cos(nx)\,dx$$

Products of Sines and Cosines

To simplify these integrals, we can use the identities:

$$\sin(mx)\sin(nx) = \frac{1}{2}[\cos(m-n)x - \cos(m+n)x]$$

$$\sin(mx)\cos(nx) = \frac{1}{2}[\sin(m-n)x + \sin(m+n)x]$$

$$\cos(mx)\cos(nx) = \frac{1}{2}[\cos(m-n)x + \cos(m+n)x]$$

Products of Sines and Cosines

Example

$$\int \sin(3x)\cos(5x)\,dx$$