Math 426.2SY
Calculus II

University of New Hampshire

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Outline

1. 9.8- Taylor Series

Introduction

Definition

We have seen how to find power series that relate to the function

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots \quad (\text{for } |x| < 1)$$

.

Some Examples

$$\frac{1}{1+x} =$$

$$\frac{1}{(1-x)^2} =$$

But what can we do about other kind of functions?

Taylor Series

Example

Let f be a function with derivatives of all orders on some interval containing a. Suppose also that $f(x)$ can be represented by a power series on that interval:

$$f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n$$

How can we go about finding the coefficients $c_0, c_1, c_2, \ldots$?

Taylor Series

$$f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + c_4(x-a)^4 + c_5(x-a)^5$$

$$+ \dots$$

Taylor Series

Definition

Let f be a function with derivatives of all orders on some interval containing a. Then the **Taylor series generated by f at $x = a$** is

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

The Maclaurin series of f is the Taylor series generated by f at $x = 0$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}x^n$$

Taylor Series

example

Find the Maclurin series of $f(x) = \sin(x)$.

Taylor Series

example

Find the Maclurin series of $f(x) = \cos(x)$.

Taylor Series

example

Find the Maclurin series of $f(x) = e^x$.

Taylor Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

Taylor Series

Example

Find the Taylor series generated by $f(x) = 1/\sqrt{x}$ at $x = 1$.

Taylor Series

Example

Find the Taylor series generated by $f(x) = \ln(1 + x)$ at $x = 0$.

Taylor Series

Example

Find the Taylor series generated by $f(x) = 1/x$ at $x = 1$.

Taylor Series

Example

Find the Taylor series generated by $f(x) = x^2 e^x$ at $x = 0$.

Taylor Series

Example

Find the Taylor series generated by $f(x) = sin(x^3)$ at $x = 0$.

Taylor Series

Example

Find the Taylor series generated by $f(x) = x^3 - 2x + 4$ at $x = 0$.