

Math 426.2SY

Calculus II

University of New Hampshire

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Outline

1 9.6- Alternating Series and Conditional Convergence

Alternating Series

Definition

A series in which the terms are alternately positive and negative is an **alternating series**.

Example

- $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots$
- $\sum_{n=1}^{\infty} \frac{(-1)^n 4}{2^n} = -2 + 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots$
- $\sum_{n=1}^{\infty} (-1)^{n+1} n = 1 - 2 + 3 - 4 + 5 \dots$

The Alternating Series Test

The series

$$\sum_{n=1}^{\infty} (-1)^{n+1} u_n = u_1 - u_2 + u_3 - u_4 + \dots$$

converges if all three of the following conditions are satisfied:

- The u_n 's are all positive
- The positive u_n 's are eventually nonincreasing: $u_n \geq u_{n+1}$ for all $n \geq N$
- $u_n \rightarrow 0$

Caution

This test **cannot** be used to conclude that an alternating series **diverges** unless $u_n \not\rightarrow 0$ (n^{th} term test).

Alternating Series

Example

Determine if the *alternating harmonic series*

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots$$

converges.

Alternating Series

Example

Determine if the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\ln(n)}{n}$ converges.

Conditional Convergence

Definition

A convergent series that is not absolutely convergent is **conditionally convergent**.

Example of a conditionally convergent series

Absolute and Conditional Convergence

Which of the series in the following exercises converge absolutely, which converge and which diverge?

Absolute and Conditional Convergence

Example

$$\sum_{n=1}^{\infty} (-1)^{n+1} (0.1)^n$$

Absolute and Conditional Convergence

Example

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$$

Absolute and Conditional Convergence

Example

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3+n}{5+n}$$

Absolute and Conditional Convergence

Example

$$\sum_{n=2}^{\infty} (-1)^n \frac{4}{(\ln(n))^2}$$

Absolute and Conditional Convergence

Example

$$\sum_{n=1}^{\infty} (-1)^n \ln \left(1 + \frac{1}{n} \right)$$