Math 426.2SY
Calculus II

University of New Hampshire

July 5, 2017

Outline

Introduction

Example

Compute $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$

Infinite Series

An infinite series is a sum of the form

$$a_1 + a_2 + a_3 + \dots$$

Shorthand notation:

$$\sum_{k=1}^{\infty} a_k$$

In this course, a_k s are real numbers.

Infinite Series

We'd like to presicely define what $\sum_{k=1}^{\infty} a_k$ means.

Keep in mind that $\sum_{k=1}^{\infty} a_k$ is a convenient way of writing $a_1 + a_2 + a_3 + \dots$

- First for every integrer n define S_n, the n^{th} **Partial Sum**:

Infinite Series

Terminology

If $\lim_{n\to\infty} S_n = L$ exists, then we say the infinit series $\sum_{k=1}^{\infty} a_k$ converges to L.

If $\lim_{n\to\infty} S_n$ does not exist, we say the infinit series $\sum_{k=1}^{\infty} a_k$ diverges.

Geometric Series

Definition

Geometric series are series of the form

$$a + ar + ar^2 + ar^3 + \dots$$

In sigma notation:

$$\sum_{n=0}^{\infty} ar^n$$

a and r are fixed real numbers and $a \neq 0$.

Example

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$

Geometric Series

Convergence or Divergence of the geometric series

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Convergence or Divergence of the geometric series

- If $|r| < 1$ then the geometric series $\sum_{n=0}^{\infty} ar^n$ converges to $\frac{a}{1-r}$, that is
  $$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$
- If $|r| \geq 1$, then $\sum_{n=0}^{\infty} ar^n$ diverges.

Geometric Series

Example

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$

Geometric Series

Example

$1 + 2 + 4 + 8 + 16 + \dots$

Geometric Series

Not all series are geometric series!

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

Convergence Tests

- The method of looking at the limit of the partial sums $\lim_{n\to\infty} S_n$ to determine the convergence or divergence of series is not usually convenient.
- We need to develop methods to test series for convergence or divergence.

Convergence/Divergence Tests

The n^{th} term Divergence test

If $\displaystyle\sum_{n=0}^{\infty} a_n$ converges, then $a_n \to 0$.

In other words, if $a_n \not\to 0$ or $\displaystyle\lim_{n\to\infty} a_n$ fails to exist, then $\displaystyle\sum_{n=0}^{\infty} a_n$ diverges.

Convergence/Divergence Tests

Example

$$\sum_{n=1}^{\infty} \frac{n^2}{2n^2+1}$$

Convergence/Divergence Tests

Warning!

If $a_n \to 0$, the n^{th} term test does NOT imply that $\sum_{n=1}^{\infty} a_n$ converges. The test is inconclusive in this case.

Example. We'll prove later that:

So if $a_n \to 0$ the n^{th} term test is inconclusive; the series may converge or it may diverge.

Combining Series

Assume $\sum_{n=1}^{\infty} a_n = A$ and $\sum_{n=1}^{\infty} b_n = B$ are convergent series. Then:

- $\sum_{n=1}^{\infty} a_n + b_n =$
- $\sum_{n=1}^{\infty} a_n - b_n =$
- $\sum_{n=1}^{\infty} ka_n =$

Combining Series

Assume $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are divergent series. Then:

- If $k \neq 0$ then $\sum_{n=1}^{\infty} ka_n$ is also divergent.
- $\sum_{n=1}^{\infty} a_n + b_n$ and $\sum_{n=1}^{\infty} a_n - b_n$ may be convergent or divergent.

Infinite Series

Example

Determine if the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{4n^2 - n^3}{10 + 2n^3}$$

Infinite Series

Example

Determine if the series converges or diverges.

$$\sum_{n=1}^{\infty} (-1)^n$$

Infinite Series

Example

Determine if the series converges or diverges.

$$\sum_{n=0}^{\infty} \cos(n\pi)$$

Infinite Series

Example

Determine if the series converges or diverges.

$$\sum_{n=2}^{\infty} e^{-2n}$$

Infinite Series

Example

Determine if the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{2^n + 3^n}{4^n}$$

Infinite Series

Example

Determine if the series converges or diverges.

$$\sum_{n=1}^{\infty} n$$

Infinite Series

Example

Determine if the series converges or diverges.

$$\sum_{n=1}^{\infty} (-1)^n n$$

Infinite Series

Example

Determine if the series converges or diverges.

$$\sum_{n=1}^{\infty} \ln\left(\frac{1}{3^n}\right)$$