Math 426.2SY
Calculus II

University of New Hampshire

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Outline

5.1-Area and Estimating with Finite Sums

- Suppose we want to find the area of the region R above the $x-$ axis, below the graph of $y = x^2$, and between the vertical lines $x = 0$ and $x = 1$.

- We don't yet have a method to compute the exact area of R (which $=$ $1/3$), but we can estimate.

Area and Estimating with Finite Sums

- Idea: estimate the area of R by breaking the region into rectangles.
- Example: upper sum with 2 rectangles of equal width.

Area and Estimating with Finite Sums

- Example: upper sum with 4 for $y = 64 - x^3$ on the interval $[0, 4]$

Area and Estimating with Finite Sums

- Example: Estimate the area under $y = sin(\pi x) + 1$ on the interval $[0, 1]$ using 4 rectangle and the right end points.

Area and Estimating with Finite Sums

- Some things to notice about these estimates with finite sums:
  - Lower sums are always underestimates to the actual area.
  - Upper sums are always overestimates to the actual area.
  - The more rectangles we use, the better our estimate becomes

n	L_n	U_n
10	0.2850000	0.3850000
20	0.3087500	0.3587500
30	0.3168519	0.3501852
50	0.3234000	0.3434000
100	0.3283500	0.3383500
1000	0.3328355	0.3338335

5.2-Sigma Notation and Limits of Finite Sum

- Sigma notation: convenient notation for writing sums with lots of terms.

Sigma Notation

- Examples: evaluate the following sums:

- $\displaystyle\sum_{k=1}^{5} k$

- $\displaystyle\sum_{k=0}^{3} (-1)^k k$

- $\displaystyle\sum_{k=4}^{6} \frac{k^2}{k-1}$

Sigma Notation

Example

Express the sum $1 + 3 + 5 + 7 + 9$ in sigma notation.

Sigma Notation

Algebra rules for finite sums:

1. Sum Rule:

2. Difference Rule:

3. Constant Multiple Rule:

4. Constant Value Rule:

Sigma Notation

Example

Suppose that $\sum_{k=1}^{n} a_n = 0$ and $\sum_{k=1}^{n} b_n = 1$. Find the values of:

- $\sum_{k=1}^{n} 8a_k$
- $\sum_{k=1}^{n} a_k + 1$
- $\sum_{k=1}^{n} b_k - \frac{1}{n}$

Sigma Notation

Helpful sum formulas:

- $\displaystyle\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$
- $\displaystyle\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

Riemann Sums

- Let's return to our estimates for the area of the region R
- How do we find the exact area?
  - We compute a lower (or upper) sum approximation using n rectangles of equal width
  - Then we take the limit as $n \to \infty$

Riemann Sums

- The finite sum approximations we've computed are examples of a more general notion called a Riemann sum
- The Riemann sum for a function f on the interval $[a, b]$ is computed as follows:

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