Math 426.2SY
Calculus II

University of New Hampshire

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Outline

Absolute Convergence

Definition

A series $\sum a_n$ **converges Absolutely** (is **absolutely convergent**) if the corresponding series of absolute values, $\sum |a_n|$ converges.

The Absolute Convergence Test

If $\sum |a_n|$ converges, then $\sum a_n$ converges.

Caution

If $\sum a_n$ converges, $\sum |a_n|$ may converge or diverge.

Absolute Convergence

Example

Determine if the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges absolutely.

Absolute Convergence

Example

Determine if the series $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}$ converges.

Absolute Convergence

Example

The absolute convergence test states that if $\sum |a_n|$ converges then $\sum a_n$ also converges. Why?

Introduction to the Ratio Test

- For a Geometric series

$$\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + \dots$$

the ratio of consecutive terms $(= r)$ is constant.

- The series converges if $|r| < 1$ and diverges if $|r| \geq 1$

- The Ratio Test extends this idea to the case where the ratio of consecutive terms of a series is not necessarily constant.

Introduction to the Ratio Test

Example

Determine if the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^{10}}{10^n}$ converges.

The RatioTest

Let $\sum a_n$ be any series and suppose that

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = r.$$

Then:

1. The series converges absolutely if $r < 1$
2. The series diverges if $r > 1$ or $r = \infty$
3. The test is inconclusive if $r = 1$

The Ratio Test

Example

Determine if the series $\sum_{n=1}^{\infty} \frac{2^{n+1}}{n3^{n-1}}$ converges.

The Root Test

Let $\sum a_n$ be any series and suppose that

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = r.$$

Then:

1. The series converges absolutely if $r < 1$
2. The series diverges if $r > 1$ or $r = \infty$
3. The test is inconclusive if $r = 1$

The Root Test

Example

Determine if the series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{1+n}\right)^{n}$ converges.

$n!$ (n Factorial)

Definition

Let n be a positive integer. Define

$$n! = 1 \cdot 2 \cdot 3 \ldots (n-2) \cdot (n-1) \cdot n$$

It's convinient to define

$$0! = 1$$

$n!$

Example

- $5! =$
- $1! =$
- $\dfrac{6!}{4! \cdot 3!} =$
- $\dfrac{(n+3)!}{n!} =$

Examples

Example

Determine if the series $\sum_{n=1}^{\infty} \frac{n+1}{n!}$ converges.

Examples

Example

Determine if the series $\sum_{n=1}^{\infty} \frac{n}{2^n}$ converges.

Examples

Example

Determine if the series $\sum_{n=1}^{\infty}(-1)^n \frac{n^2(n+2)!}{(n+1)!\,3^{2n}}$ converges.

Examples

Example

Determine if the series $\sum_{n=1}^{\infty} \frac{2^n}{n^n}$ converges.

Examples

Example

Determine if the series $\sum_{n=3}^{\infty} \frac{-n}{(\ln(n))^n}$ converges.

Examples

Example

Determine if the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^4}{(1+\frac{1}{n})^{n^2}}$ converges.

Useful Limits

- $\lim_{n\to\infty} \sqrt[n]{n} = 1$
- $\lim_{n\to\infty} \frac{\ln(n)}{n} = 0$
- $\lim_{n\to\infty} x^n = 0 \quad (|x| < 1)$
- $\lim_{n\to\infty} (1 + \frac{1}{n})^n = e$
- $\lim_{n\to\infty} (1 + \frac{k}{n})^n = e^k$
- $\lim_{n\to\infty} x^{\frac{1}{n}} = 1 \quad (x > 0)$

Why are the Ratio and the Root test inconclusive when $r = 1$?