Math 426.2SY
Calculus II

University of New Hampshire

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Outline

Introduction

Improper Integrals of Type I

Domain of integration is not finite:

$$\int_0^\infty e^{-x}\,dx, \quad \int_{-\infty}^{-1} \frac{1}{x}\,dx, \quad \int_{-\infty}^{\infty} \frac{x}{x^2+1}\,dx, \quad \ldots$$

Improper Integrals of Type II

The function we're integrating is not bounded in the domain of integration (Vertical Assymptote).

$$\int_0^1 \frac{1}{x}\,dx, \quad \int_{-2}^{2} \frac{1}{x-1}\,dx, \quad \int_{-1}^{3} \frac{x}{\sqrt{9-x^2}}\,dx, \quad \ldots$$

Improper Integrals, Type I

Example

Find the area of the region under the curve $f(x) = e^{-x}$ in the first quadrant.

Improper Integrals, Type I

Formal Definitions

$$\int_a^{\infty} f(x)\,dx = \lim_{b\to\infty} \int_a^b f(x)\,dx$$

$$\int_{-\infty}^{b} f(x)\,dx = \lim_{a\to-\infty} \int_a^b f(x)\,dx$$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_c^{\infty} f(x)\,dx$$

In each case if the limit is finite, we say that the improper integral **converges**. If the limit fails to exist, we say it **diverges**.

Improper Integrals, Type I

Example

$$\int_1^\infty \frac{\ln(x)}{x^2}\,dx$$

Improper Integrals, Type I

Example

$$\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$$

Improper Integrals, Type I

Very Important Example

For what values of p does $\displaystyle\int_1^\infty \frac{1}{x^p}\,dx$ converge? When the integral does converge, what is its value?

Improper Integrals, Type I

Improper Integrals, Type I

$$\int_1^\infty \frac{1}{x^p}\,dx = \begin{cases} \frac{1}{p-1} & ;\text{ if } p > 1 \\ \text{diverges} & ;\text{ if } p \leq 1 \end{cases}$$

Similarly, if $a > 0$

$$\int_a^\infty \frac{1}{x^p}\,dx = \begin{cases} \text{converges} & ;\text{ if } p > 1 \\ \text{diverges} & ;\text{ if } p \leq 1 \end{cases}$$