BAN-PT AKREDITASI INSTITUSI UNGGUL

Kampus Merdeka INDONESIA JAYA

INFORMATICS

Week 4
A Survey of Probability Concepts

IMT01303404 Statistics and Probability

4 Credits

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Learning Objectives

- LO5-1 Define the terms probability, experiment, event, and outcome
- LO5-2 Assign probabilities using a classical, empirical, or subjective approach
- LO5-3 Calculate probabilities using the rules of addition
- LO5-4 Calculate probabilities using the rules of multiplication
- LO5-5 Calculate probabilities using Bayes' theorem

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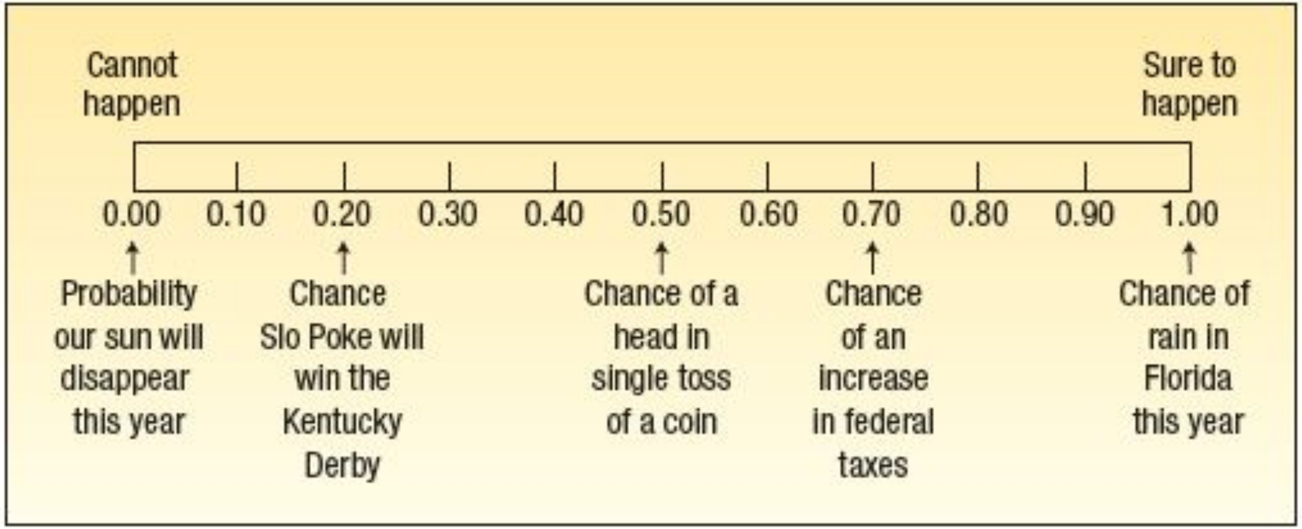
Probability

PROBABILITY A value between 0 and 1 inclusive that represents the likelihood a particular event happens.



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Probability (cont’d)

		
Experiment	Roll a die	Count the number of members of the board of directors for Fortune 500 companies who are over 60 years of age
All possible outcomes	Observe a 1 Observe a 2 Observe a 3 Observe a 4 Observe a 5 Observe a 6	None is over 60 One is over 60 Two are over 60 ... 29 are over 60 ... ... 48 are over 60 ...
Some possible events	Observe an even number Observe a number greater than 4 Observe a number 3 or less	More than 13 are over 60 Fewer than 20 are over 60

EXPERIMENT A process that leads to the occurrence of one and only one of several possible results.

OUTCOME A particular result of an experiment.

EVENT A collection of one or more outcomes of an experiment.

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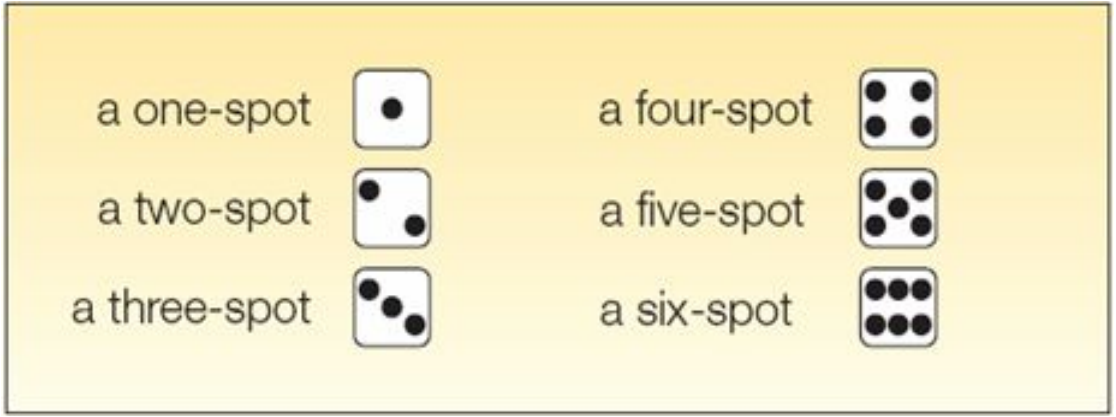
Classical Probability

MUTUALLY EXCLUSIVE The occurrence of one event means that none of the other events can occur at the same time.

COLLECTIVELY EXHAUSTIVE At least one of the events must occur when an experiment is conducted.

CLASSICAL PROBABILITY

$$\text{Probability of an event} = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}} \quad [5\text{–}1]$$



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Empirical Probability

EMPIRICAL PROBABILITY The probability of an event happening is the fraction of the time similar events happened in the past.

LAW OF LARGE NUMBERS Over many trials, the empirical probability of an event will approach its true probability.

The empirical definition occurs when the number of times an event happens is divided by the number of outcomes

$$\text{Probability of a successful flight} = \frac{\text{Number of successful flights}}{\text{Total number of flights}}$$

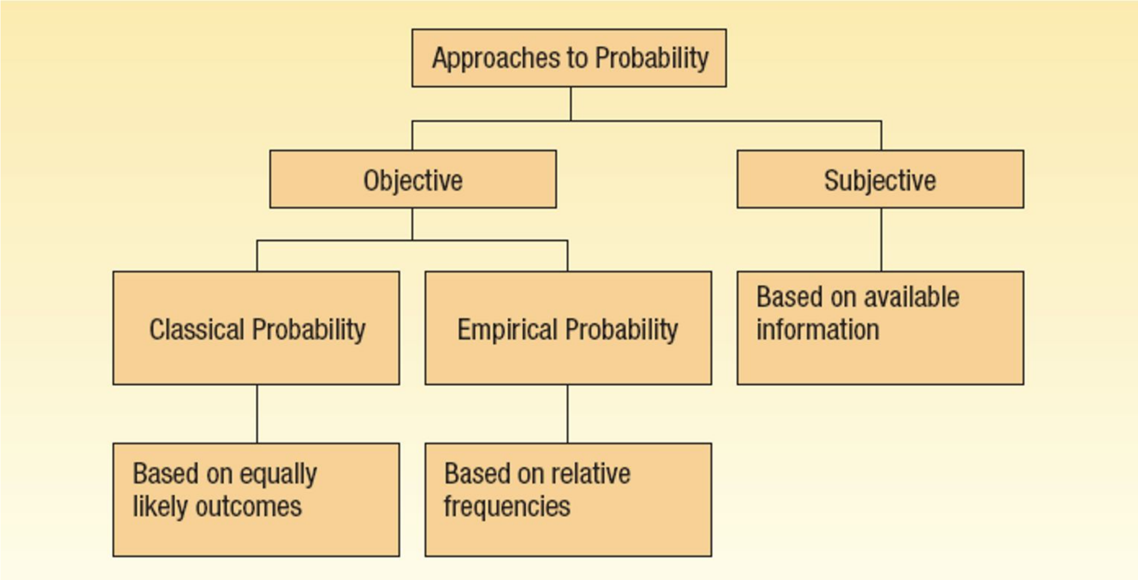
$$= \frac{121}{123} = 0.98$$

Subjective Probability

SUBJECTIVE CONCEPT OF PROBABILTIY The likelihood (probability) of a particular event happening that is assigned by an individual based on whatever information is available.

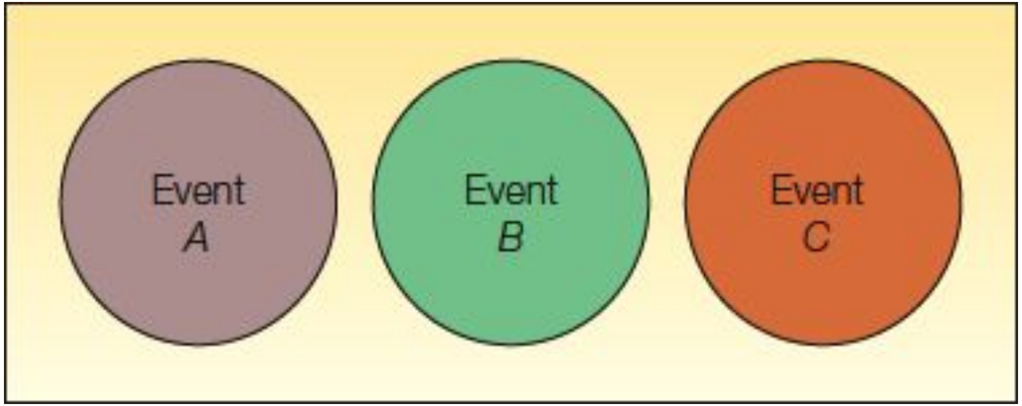
- Examples of subjective probability are:
  - Estimating the likelihood, the Indonesian National Football team will be in the next World Cup
  - Estimating the likelihood, the Republic of Indonesia budget deficit will be reduced by half in the next 10 years

Summary of Approaches to Probability



Which approach of probability is this?

The Rules of Addition



SPECIAL RULE OF ADDITION P(A or B) = P(A) + P(B)

- The rules of addition refer to the probability that any two or more events can occur
- The special rule of addition is used when the events are **mutually exclusive**

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Special Rule of Addition

Special Rule of Addition Example

Weight	Event	Number of Packages	Probability of Occurrence	
Underweight	*A*	100	.025	← $\frac{100}{4,000}$
Satisfactory	*B*	3,600	.900	
Overweight	*C*	300	.075	
		4,000	1.000	

Note: This example is Mutually Exclusive

A machine fills plastic bags with a mixture of beans, broccoli, and other vegetables. Most of the bags contain the correct weight, but because of the variation in the size of the beans and other vegetables, a package might be underweighted or overweight. A check of 4,000 packages filled in the past month revealed.

What is the probability that a particular package will be either underweight or overweight?

$$P(A \text{ or } C) = P(A) + P(C) = 0.025 + 0.075 = 0.10$$

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Complement Rule

Weight	Event	Number of Packages	Probability of Occurrence	
Underweight	*A*	100	.025	← $\frac{100}{4,000}$
Satisfactory	*B*	3,600	.900	
Overweight	*C*	300	.075	
		4,000	1.000	

Note: This example is Mutually Exclusive

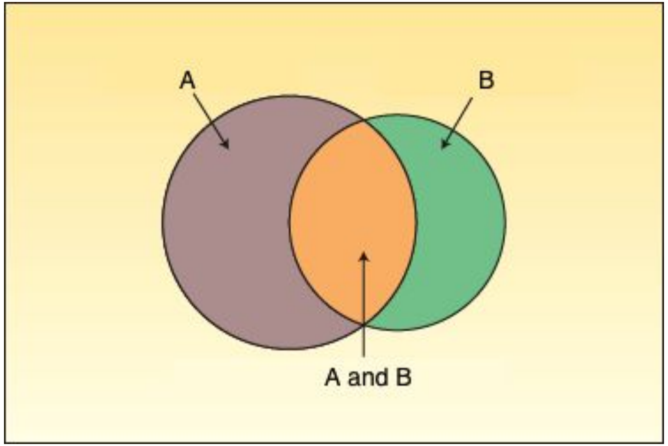
COMPLEMENT RULE P(A) = 1 – P(~A)

The complement rule is used to determine the probability of an event happening by subtracting the probability of an event not happening.

What is the probability that a particular package will be **neither** underweight **nor** overweight?

You can also use the complement rule:

$$P(A \text{ or } C) = P(\sim B) = 1 - P(B) = 1 - 0.900 = 0.10$$



The general rule of addition is used when the events are **NOT** mutually exclusive.

GENERAL RULE OF ADDITION P(A or B) = P(A) + P(B) – P(A and B)

JOINT PROBABILITY A probability that measures the likelihood two or more events will happen concurrently.

General Rule of Addition

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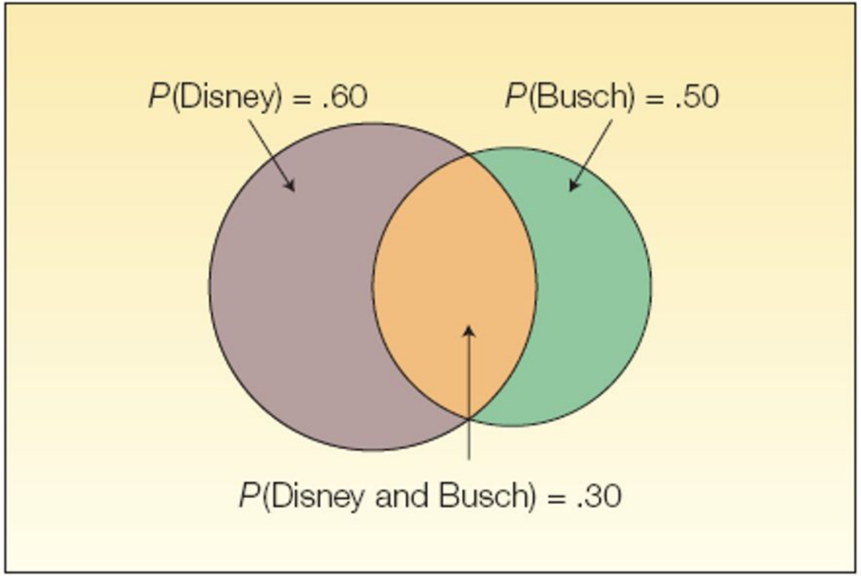
General Rule of Addition Example

A sample of 200 tourists in Florida shows 120 went to Disney, 100 went to Busch Gardens, and 60 visited both during summer holiday.

- P(Disney) =120/200 = 0.60
- P(Busch) =100/200 = 0.50
- P(Disney and Busch) = 60/200 = 0.30



P(Disney or Busch)
= P(Disney) + P(Busch) – P (Disney and Busch)
= 0.60 + 0.50 - 0.30 = 0.80

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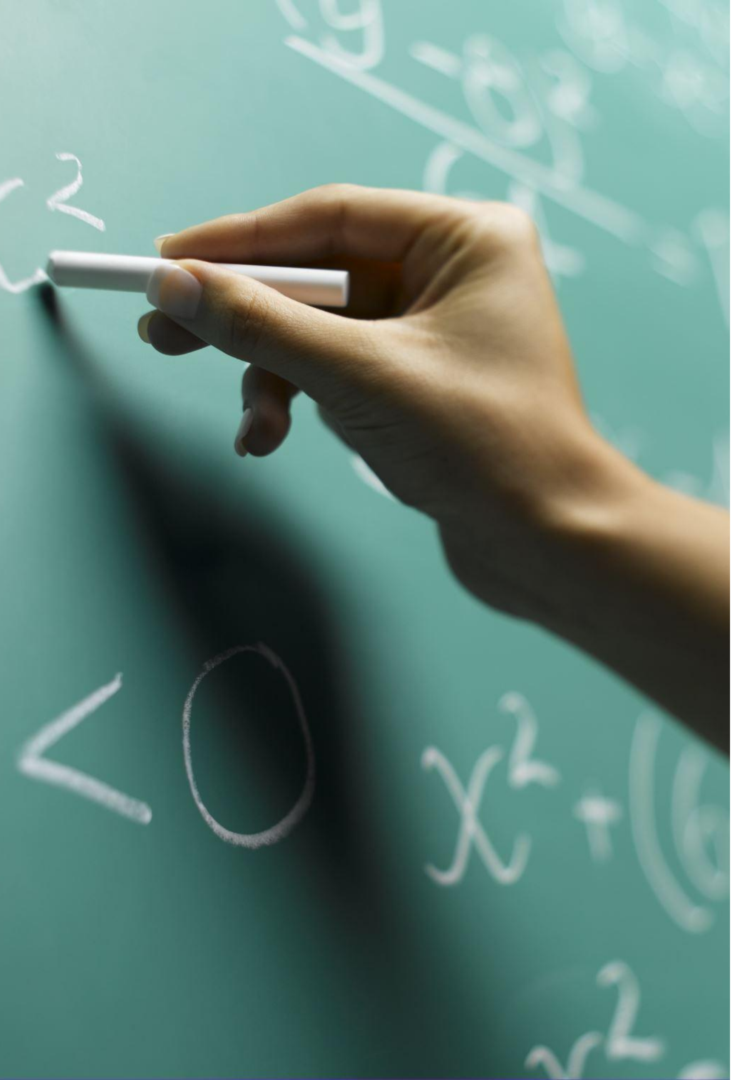
The Rules of Multiplication

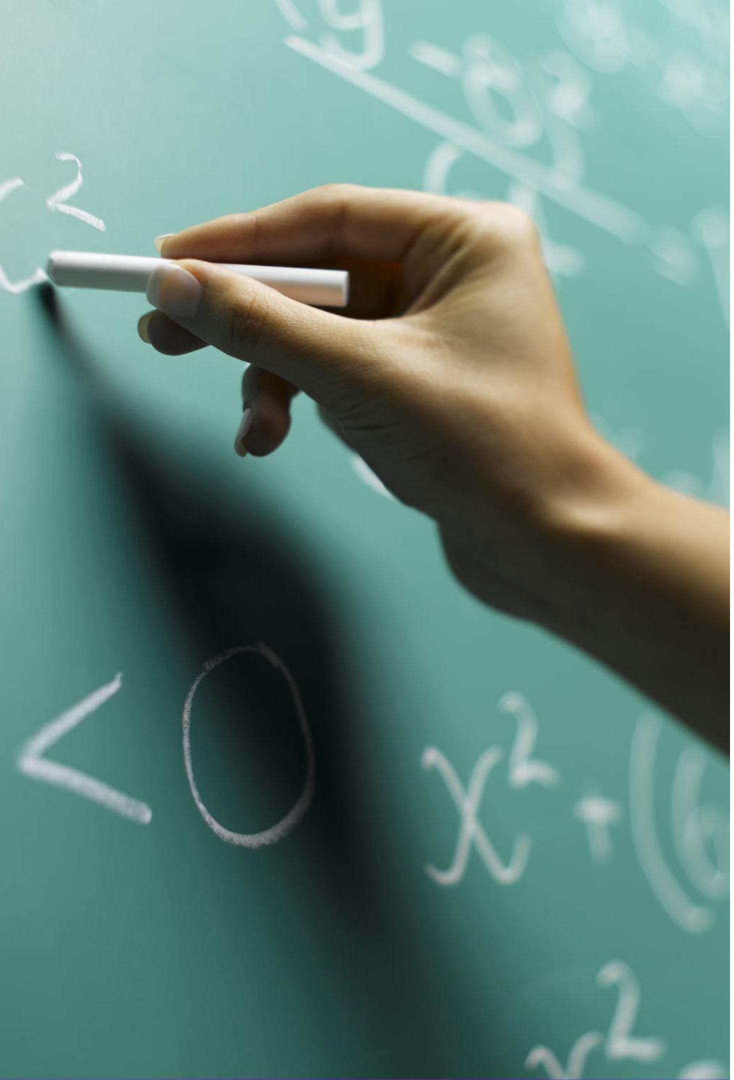


Special Rule of Multiplication

- The rules of multiplication are applied when two or more events occur simultaneously
- The special rule of multiplication refers to events that are independent

INDEPENDENCE The occurrence of one event has no effect on the probability of the occurrence of another event.

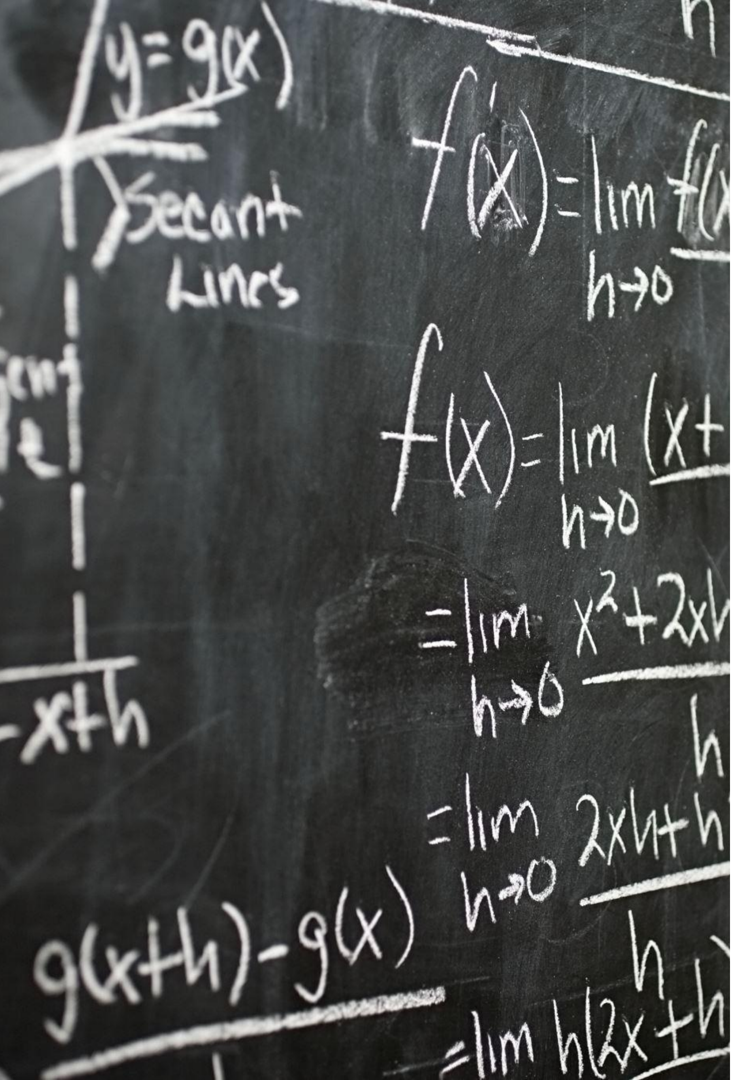
SPECIAL RULE OF MULTIPLICATON P(A and B) = P(A) P(B)



Special Rule of Multiplication Example

A survey by the American Automobile Association (AAA) revealed 60% of its members made airline reservations last year. Two members are selected at random. What is the probability both made airline reservations last year?

P(R1 and R2) = P(R1)P(R2) = (0.60)(0.60) = 0.36



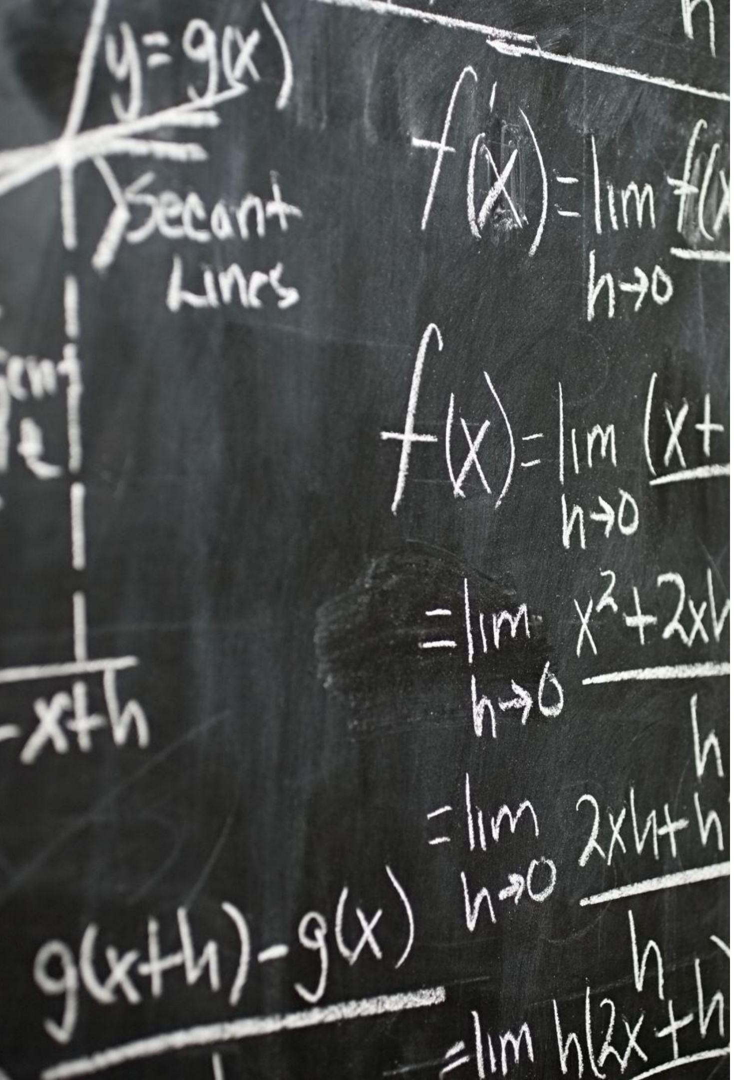
General Rule of Multiplication

- The general rule of multiplication refers to events that are not independent
- A conditional probability is the likelihood an event will happen, given that another event has already happened

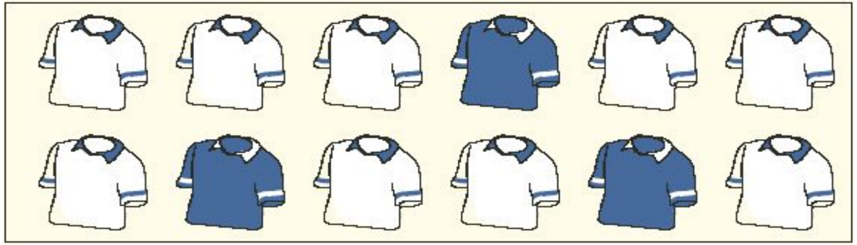
CONDITIONAL PROBABILITY The probability of a particular event occurring, given that another event has occurred.

GENERAL RULE OF MULTIPLICATION $P(A \text{ and } B) = P(A)P(B|A)$ **[5–6]**

The conditional probability is represented a P(B|A) and is read, the probability of B given A



General Rule of Multiplication Example



A golfer has 12 golf shirts in his closet. Suppose 9 of these shirts are white and the others are blue. He gets dressed in the dark , so he just grabs a shirt and puts in on. He plays golf two days in a row and does not return the shirts to the closet.

What is the probability both shirts are white?

$$P(W_1 \text{ and } W_2) = P(W_1)P(W_2|W_1) = (\tfrac{9}{12})(\tfrac{8}{11}) = 0.55$$

Bayes' Theorem

- Bayes' Theorem is a method of revising a probability, given that additional information is obtained
- For two mutually exclusive and collectively exhaustive events

BAYES' THEOREM $$P(A_i|B) = \frac{P(A_i)P(B|A_i)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2)} \quad [5\text{–}7]$$

PRIOR PROBABILITY The initial probability based on the present level of information.

POSTERIOR PROBABILITY A revised probability based on additional information.

Bayes' Theorem Example

BAYES' THEOREM $$P(A_i|B) = \frac{P(A_i)P(B|A_i)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2)} \quad [5\text{–}7]$$

Suppose 5% of the population of Umen have a disease and A_1 represents the part of the population that has the disease and A_2 represents those who do not. There is a new testing method being developed at Umen, Let B denote a test result that shows the disease is present.

$P(A_1) = 0.05$ Individual has the disease
$P(A_2) = 0.95$ Individual does not have the disease
$P(B|A_1) = 0.90$ Test shows the individual has the disease and is correct
$P(B|A_2) = 0.15$ Test incorrectly shows the individual has the disease

Randomly select an individual and perform the test. The test results indicate the disease is present.

Event, A_i	Prior Probability, $P(A_i)$	Conditional Probability, $P(B \mid A_i)$	Joint Probability, $P(A_i \text{ and } B)$	Posterior Probability, $P(A_i \mid B)$
Disease, A_1	.05	.90	.0450	.0450/.1875 = .24
No disease, A_2	.95	.15	.1425	.1425/.1875 = .76
			$P(B) = .1875$	1.00

What is the probability the test is correct? Use Bayes' theorem to solve.

$$P(A_1|B) = \frac{P(A_1)P(B|A_1)}{P(A_1)P(B|A_1)+P(A_2)P(B|A_2)} = \frac{(0.05)(0.90)}{(0.05)(0.90)+(0.95)(0.15)} = \frac{0.0450}{0.1875} = 0.24$$

Exercises

Exercise 1

Card	Probability	Explanation
King	$P(A) = 4/52$	4 kings in a deck of 52 cards
Heart	$P(B) = 13/52$	13 hearts in a deck of 52 cards
King of Hearts	$P(A \text{ and } B) = 1/52$	1 king of hearts in a deck of 52 cards

What is the probability of randomly selected card is King or Heart?

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Exercise 2

A sample of employees of Worldwide Enterprises is to be surveyed about a new health care plan. The employees are classified as follows:

Classification	Event	Number of Employees
Supervisors	*A*	120
Maintenance	*B*	50
Production	*C*	1,460
Management	*D*	302
Secretarial	*E*	68

(a) What is the probability that the first person selected is:
   (i) either in maintenance or a secretary?
   (ii) not in management?
(b) Draw a Venn diagram illustrating your answers to part (a).
(c) Are the events in part (a)(i) complementary or mutually exclusive or both?

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Exercise 3

In each of the following cases, indicate whether classical, empirical, or subjective probability is used.

a. A baseball player gets a hit in 30 out of 100 times at bat. The probability is .3 that he gets a hit in his next at bat.
b. A seven-member committee of students is formed to study environmental issues. What is the likelihood that any one of the seven is chosen as the spokesperson?
c. You purchase one of 5 million tickets sold for Lotto Canada. What is the likelihood you will win the $1 million jackpot?
d. The probability of an earthquake in northern California in the next 10 years above 5.0 on the Richter Scale is 0.8.

Exercise 4

From experience, Dunlop Tire knows the probability is 0.95 that a particular DT-65 tire will last 60,000 miles before it becomes bald or fails. An adjustment is made on any tire that does not last 60,000 miles. You purchase four DT-65s.

What is the probability all four tires will last at least 60,000 miles?

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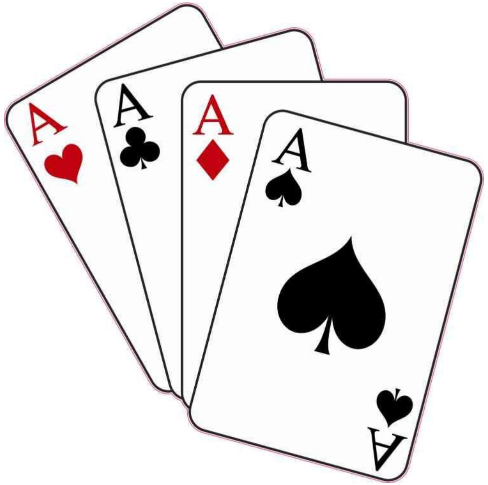
Exercise 5



Two cards are drawn sequentially from a deck of playing cards:

If the first card is an Ace, what is the probability that we get the second card an Ace as well?

Exercise 6

Kehidupan Sekar, seorang dokter sukses dengan kehidupan penuh rasa damai karena memiliki keluarga kecil yang bahagia. Ia tinggal di kawasan elite Jakarta bersama suaminya, Ivan dan anak mereka, Dennis. Pernikahan itu juga telah berjalan selama 15 tahun tanpa menemui masalah besar, sampai dengan suatu hari Sekar mendapati baju suami nya tercium wangi parfum yang umum dipakai wanita.

Namun, situasi pelik terjadi ketika Sekar tanpa sengaja menemukan receipt pembelian rangkaian bunga mawar di kantong baju Ivan. Rasa curiga itu memicu masalah besar yang tidak pernah dibayangkan oleh Sekar sebelumnya.

Dengan menggunakan teorema Bayes, perkirakan berapa probabilitas bahwa Ivan berselingkuh?

Please obtain Basic Python Certificate

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1 More Week!

March 19, 2024

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Next Topic > One Sample Test

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