Non-Parametric Statistics

Sign Test on Proportions

When Do We Use the Sign Test?

- When no assumption about the population distribution can be made
- To obtain a quick and easy answer to the question: "Is there a difference between the two parameters in the data?"
- When comparing populations are not independent

Example of Sign Test: Aspiring Freshmen

Students with low SAT scores are taking a summer remedial math course to be able to enroll as freshmen in the fall.

To evaluate the accuracy of this test they are given a pretest, before the course is started, and an after-test at the conclusion of the course.

If the course is effective, then the students' scores on the after-test should be higher than their score on the pretest.

Let's Pretend...

That the director of the program claims that this program will be a success:

In average, students' grades improve after participating in the summer program.

$\mu_1 > \mu_2$

Some party pooper disagrees and claims that this summer program is a waste of time, energy and money: no improvement at the end of the summer.

$\mu_1 = \mu_2$

Hypothesis

Party-Pooper's Claim:

This will be our H0: $\mu_1 = \mu_2$

Program Director's Claim:

This will be our H1: $\mu_2 > \mu_1$

μ_2: population mean of the students pre-test scores

μ_1: population mean of the students after-test scores

Assuming a confidence level of 95%, then the level of significance $\alpha = 0.05$

If H0: $\mu_2 = \mu_1$

This means that the ratio of "success" is $p \approx 0.5 \Rightarrow q = (1 - p) \approx 0.5$

(i.e. if there are 9 "+", there are also 9 "-". Thus, $p \approx \frac{9}{18} = 0.5$)

Since H1: $\mu_2 > \mu_1$

This is a one-tail test and since $\alpha = 0.05$, the critical value is 1.645

Thus the sample test-statistic will be:

$$Z = \frac{r - p}{Standard\ Deviation} = \frac{r - p}{\sqrt{\left[\frac{(p)(1-p)}{n}\right]}}$$

To perform the sign test, we want to:

- Compare each student pretest with his or her after-test:
- After – Pre = X
- If X > 0, then mark + in the sign column
- If X < 0, then mark – in the sign column
- If X = 0, then mark 0 and remove the student's data from the count
- Note how many:
  - "+" signs
  - "-" signs
  - zeros

Students	Pretest	After-Test	Sign
1	82	95	+
2	76	88	+
3	84	90	+
4	65	77	+
5	54	74	+
6	68	66	-
7	77	86	+
8	48	73	+
9	82	82	0
10	78	77	-
11	81	85	+
12	57	75	+
13	78	78	0
14	76	73	-
15	88	89	+
16	90	98	+
17	83	87	+
18	85	88	+

To be able to use this test, the sum of the + and – signs has to be equal or greater than 12.

Observe there are:

- 13 "+" signs
- 3 "–" signs
- 2 zeros

Note that the 2 zero-values are excluded and with regard to the sign test,
n = total of non zero signs = 18 – 2 => n = 16

Knowing there are

- 13 "+" signs,
- 3 "–" signs,
- and 2 "zeros",

We need to compute the ratio of success:

$$\frac{\boldsymbol{Number\ of + Signs}}{\boldsymbol{\sum(+\ and\ - Signs)}}$$

Knowing there are 13 + signs, 3 – signs, and 2 zeros,

The ratio of “success” is:

$$r = \frac{\boldsymbol{Number\ of + Signs}}{\sum(+\ \boldsymbol{and}\ -\boldsymbol{Signs})} = \frac{\mathbf{13}}{\mathbf{16}} = .8125$$

Recall:

Assuming $\alpha = 0.05$

If H0: $\mu_2 = \mu_1$ $p \approx 0.5$ thus $(1-p) \approx 0.5$

Since H1: $\mu_2 > \mu_1$ => one-tail test and $z = 1.645$

But from the sign test on our sample, we obtain $r = 0.8125$

Hence the test-statistic will be:

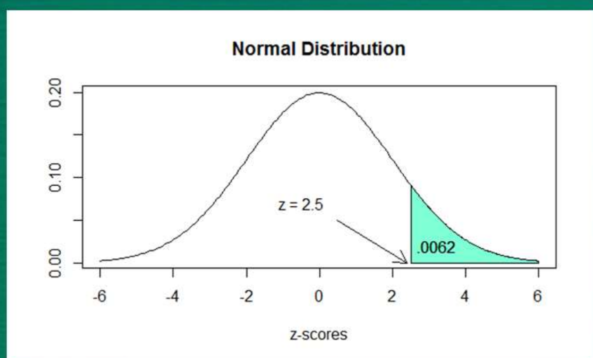
$$\mathbf{Z} = \frac{r - p}{\sqrt{[\frac{(p)(1-p)}{n}]}} = \frac{0.8125 - 0.5}{\sqrt{[\frac{0.25}{16}]}} = 2.5$$

To find the p-value corresponding to $z = 2.5$, we need to look at the Standard Normal Probability table.

Since the p-value is a probability and the probability is an area,

The Probability

$$P(z \geq 2.5) \equiv P(z \leq -2.5)$$



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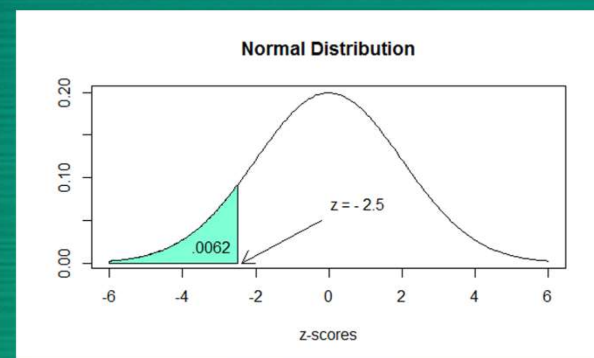


Our Situation

The Standard Normal Probability table.

Finding the P-Value

Standard Normal Probability Table

To find the p-value corresponding to z = 2.5, we need to look at the Standard Normal Probability table.

Since P(z ≥ 2.5) ≡ P(z ≤ -2.5)

Then scroll down to Z = - 2.50

P-value = 0.0062

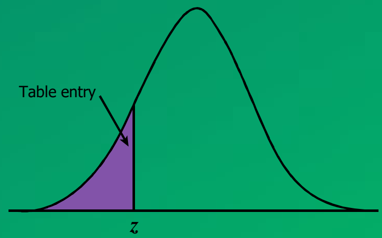
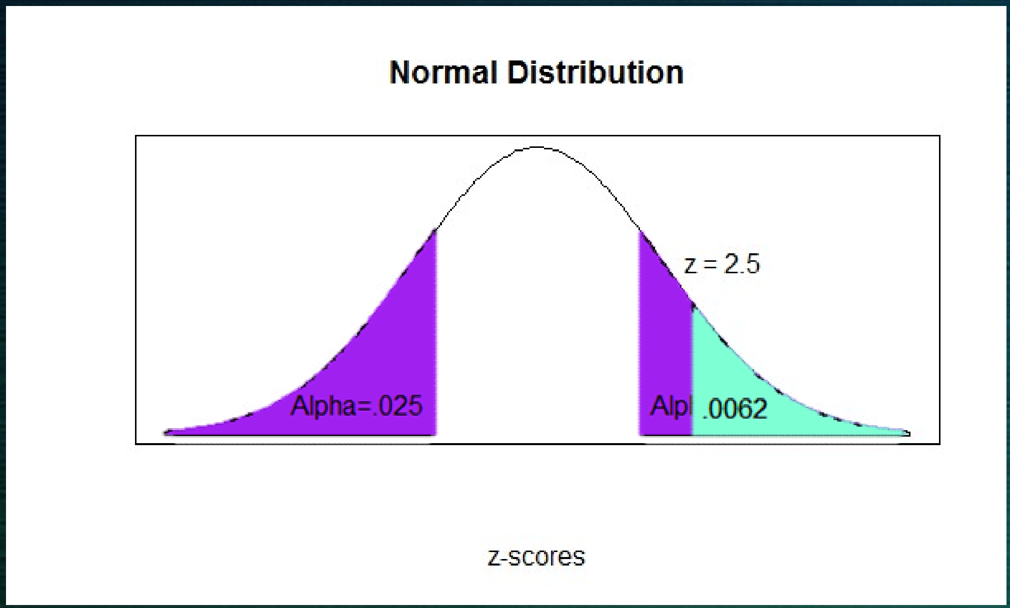


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	[illegible]	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	[illegible]	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	[illegible]	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	[illegible]	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	[illegible]	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	[illegible]	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	[illegible]	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	[illegible]	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	[illegible]	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183

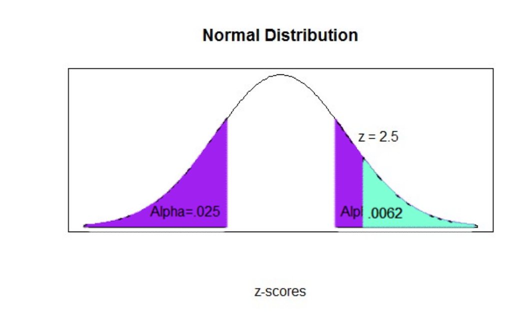

Normal Distribution
z = 2.5
Alpha=.025
Alpl
.0062
z-scores

Conclusion

The null hypothesis for our example states:

H0: $\mu_1 = \mu_2$ that the program did not improve students grades

However, the our sample z score 2.5 is in the critical region



Also, the P-value for the sample test statistic is 0.0062, and 0.0062 < 0.05, the critical area

Thus we reject the null hypothesis H0 that the summer course has no effect on the students performance.

Thus, this evidence is sufficient to claim that the math summer course indeed increases these students scores

Inspired by:

https://www.youtube.com/watch?v=P40BhFAiIKg

“Understandable statistics”, 6th Edition, H. and C. Brase, Houghton Mifflin