Introduction to Quantum Computing – Linear Algebra Fundamentals - Homework 1 (25 points)

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1. Compute the tensor product of the zero and one basis vectors (5 points)

$$|0\rangle|1\rangle = \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} = \begin{matrix}(1)(0)\\(1)(1)\\(0)(0)\\(0)(1)\end{matrix} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

2. Compute the tensor product shown below (5 points).

$$\begin{bmatrix}1\\0\end{bmatrix} \otimes \frac{1}{\sqrt{2}} \begin{bmatrix}1\\1\end{bmatrix}$$

$$\begin{bmatrix}1\\0\end{bmatrix} \otimes \begin{bmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{bmatrix} = \begin{bmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\\0\\0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\\0\\0\end{bmatrix}$$

3. Compute the transpose of the matrix shown below (5 points).

$$\begin{pmatrix}5 & 4 & 3\\4 & 0 & 4\\7 & 10 & 3\end{pmatrix}^T = \begin{pmatrix}5 & 4 & 7\\4 & 0 & 10\\3 & 4 & 3\end{pmatrix}$$

4. What is the result of applying the matrix below to |1> ? (5 points)

$$\begin{pmatrix}1 & 1\\0 & 0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{matrix}(1)(0) + (1)(1)\\(0)(0) + (0)(1)\end{matrix} = \begin{matrix}1\\0\end{matrix}$$

Set to constant Zero

5. Is it possible to factor this 1 x 4 column vector into a tensor product of two other vectors? If so, provide the values to be substituted for the question marks in the equation below; if not, explain why not. (5 points)

$$\begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}?\\?\end{pmatrix} = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}$$

yes, it is possible to solve the equation given. Although, there are infinite many solutions to solving the Kronecker product because any product of [0, a] and [1/a, 0],

where a ≠ 0 would provide the answer [0,0,1,0]. For example, $\begin{matrix}0\\3\end{matrix} \otimes \begin{matrix}1/3\\0\end{matrix}$ = [0,0,1,0]. However, since a is given the output would be $\begin{matrix}0\\1\end{matrix} \otimes \mathbf{\begin{matrix}1\\0\end{matrix}}$ = [0,0,1,0]. = 1107

$$\begin{pmatrix} ? \\ ? \end{pmatrix} = \begin{matrix}1\\0\end{matrix}$$