Introduction to Quantum Computing – Advanced Linear Algebra - Homework 2 (20 points)

1. What is the inner product of the two vectors below? (5 points)

$$\begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix} \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix} = 6$$

See attachment.

2. Find the length of the vector (3, 4) and show your work for partial credit. (5 points)

$$\|\vec{a}\| = \|(3,4)\| \to 5$$

See attachment

3. Find all of the eigenvalues for the matrix below (6 points).

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 5 \\ 0 & 4 & 3 \end{bmatrix} = -1, 2, 8$$

See attachment

4. Find the corresponding eigenvector for the eigenvalue which is negative (4 points)

$$E_{\lambda=-1} = \text{Span}\left(\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}\right)$$

See attachment

Alexander Lizco Homework 2 9/25/20

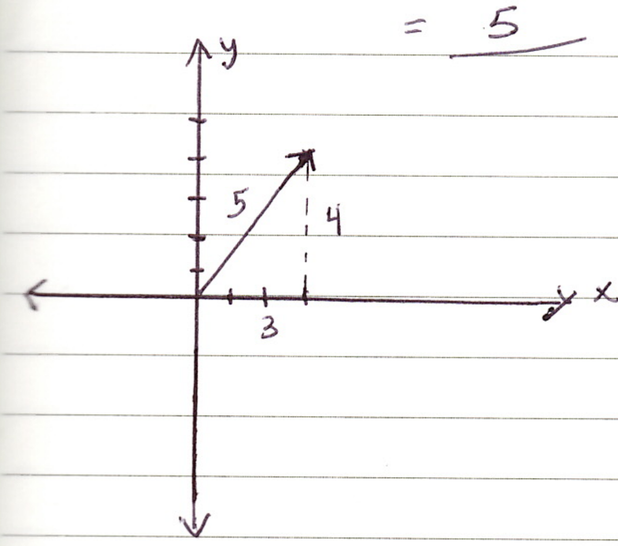
1) What is the inner product of the two vectors below?

$$\begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix} \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix} \Rightarrow \begin{aligned} &= (-1 \cdot 5) + (4 \cdot 2) + (3 \cdot 1) \\ &= -5 + 8 + 3 \\ &= \underline{6} \end{aligned}$$

2) Find the length of the vector (3,4) and show your work for partial credit

$$\text{Length} = \|\vec{a}\| = \sqrt{a_1^2 + a_2^2 + a_3^2 + \ldots + a_n^2}$$

$$b = (3,4) \quad \|\vec{b}\| = \sqrt{3^2 + 4^2}$$
$$= \sqrt{9 + 16}$$
$$= \sqrt{25}$$
$$= \underline{5}$$



3) Find all of the eigenvalues for the matrix below

$$\det(\lambda I_n - A) = 0 = \underline{\det(A - \lambda I_n)}$$

$$\Updownarrow$$

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 5 \\ 0 & 4 & 3 \end{bmatrix} \qquad \overbrace{(\lambda I_n - A)}\vec{v} = \vec{0}$$

$$\lambda \cdot I_n = \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\lambda \cdot I_n = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$(A - \lambda I_n) = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 5 \\ 0 & 4 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = \begin{bmatrix} (2-\lambda) & 0 & 0 \\ 0 & (4-\lambda) & 5 \\ 0 & 4 & (3-\lambda) \end{bmatrix}$$

continued...

Rule of Sarrus

$$\begin{bmatrix} (2-\lambda) & 0 & 0 \\ 0 & (4-\lambda) & 5 \\ 0 & 4 & (3-\lambda) \end{bmatrix} \begin{matrix} (2-\lambda) & 0 \\ 0 & (4-\lambda) \\ 0 & 4 \end{matrix}$$

$$= (2-\lambda)\cdot(4-\lambda)\cdot(3-\lambda) + (0\cdot\cancel{5}\cdot 0) + (0\cdot\cancel{0}\cdot 4)$$
$$- (0\cdot\cancel{(4-\lambda)}\cdot 0) - 4\cdot 5\cdot(2-\lambda) - \cancel{(3-\lambda)\cdot 0\cdot 0}$$

$$(-\lambda+2)\cdot(-\lambda+4)\cdot(-\lambda+3) - 20(-\lambda+2)$$
$$(\lambda^2 - 6\lambda + 8)\cdot(-\lambda+3) + 20\lambda - 40$$
$$-\lambda^3 + 6\lambda^2 - 8\lambda + 3\lambda^2 - 18\lambda + 24 + 20\lambda - 40$$
$$-\lambda^3 + 6\lambda^2 - 8\lambda + 3\lambda^2 - 18\lambda + 20\lambda - 16$$
$$-\lambda^3 + 9\lambda^2 - 6\lambda - \underline{\underline{16}}$$

Roots are factors of this term

potential roots: 1, 2, 4, 8, 16

1: $-1 + 9 - 6 - 16 \neq 0$ ✗
2: $-8 + (9\cdot 4) - 12 - 16 = 0$ ✓
4: $-64 + 9\cdot 16 - 24 - 16 \neq 0$ ✗
8: $-512 + 576 - 48 - 16 = 0$ ✓
16: $-4096 + 2304 - 96 - 16 \neq 0$ ✗
-1: $1 + 9 + 6 - 16 = 0$ ✓
-2: $8 + 36 + 12 - 16 \neq 0$
-4: $64 + 144 + 24 - 16 \neq 0$
-8: $512 + 576 + 48 - 16 \neq 0$
-16: $4096 + 2304 + 96 - 16 \neq 0$

Eigen values are : $-1, 2, 8$

4.) Find the corresponding eigen vector for the eigen value which is negative.

eigen vector / eigen space

$\lambda = -1$

$$\lambda I_n = \begin{bmatrix} (2-\lambda) & 0 & 0 \\ 0 & (4-\lambda) & 5 \\ 0 & 4 & (3-\lambda) \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 5 \\ 0 & 4 & 4 \end{bmatrix} \vec{V} = \vec{0}$$

Row echelon form

$$R_1 \cdot \tfrac{1}{3} \quad R_2 \cdot \tfrac{1}{5} \quad \begin{pmatrix} 3 & 0 & 0 \\ 0 & 5 & 5 \\ 0 & 4 & 4 \end{pmatrix} \cdot \tfrac{1}{3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 5 \\ 0 & 4 & 4 \end{bmatrix} \cdot \tfrac{1}{5} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 4 & 4 \end{bmatrix} \cdot -4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Continued..

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$v_1 = 0$

$v_2 + v_3 = 0$

$x_2 = -v_3 \rightarrow v_2 = -t$

$v_3 = t$

$$E_{\lambda=-1} = \left\{ \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = t \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \text{ for } t \in \mathbb{R} \right\}$$

$$E_{\lambda=-1} = \text{span}\left(\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \right)$$