Homework 1 in EL2450 Hybrid and Embedded Control Systems

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Task 1

The gain "Tap" models the amount of water that is flowing back into the reservoir from tank 1. Setting that gain to zero means all the water flows to the second tank before it reaches the reservoir.

Task 2

```
uppertank = tf(k_tank,[Tau 1]); % Transfer function for upper tank
lowertank = tf(gamma_tank,[Tau*gamma_tank 1]); % Transfer function ...
    for upper tank
G = uppertank*lowertank; % Transfer function from input to lower ...
    tank level
```

Task 3

The reference $r(t)$ value is a step that has the following characteristic:

$$r(t) = \begin{cases} 0, & t < 25 \\ 10 & t \geq 25 \end{cases}$$

The **uss** block resemble the input steady-state and the **yss** block resemble the out put steady-state. They are needed in order to ensure that the system operates around the desired steady-state values.

Task 4

```
% Calculate PID parameters
[K_pid,Ti,Td,N]=polePlacePID(chi,omega0,zeta,Tau,Gamma,K)
F = K_pid*tf([N*Td*Ti+Ti N*Ti+1 N], [Ti N*Ti 0])
```

Task 5

χ	ζ	ω_0	$T_r(s)$	M	$T_{set}(s)$
0.5	0.7	0.1	8.1	14.49	44.49
0.5	0.7	0.2	4.86	34.66	25.29
0.5	0.8	0.2	5.02	31.71	26.58

By thus we deduce that the parameters yielding best performance according to the given criteria is $\chi = 0.5$, $\zeta = 0.8$ and $w_0 = 0.2$.

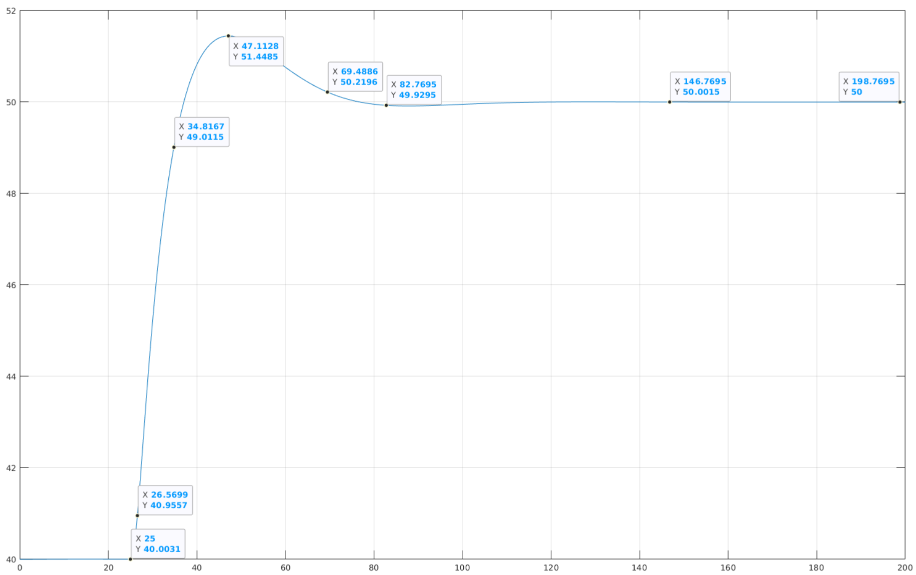


Figure 1: The step response with the labels used to calculate T_r, M, T_{set} for the system with $\chi = 0.5$, $\zeta = 0.7$ and $w_0 = 0.1$

Task 6

As seen in figure 7 the cross over frequence is $w_c = 0.362\,\text{rad}/\sec$

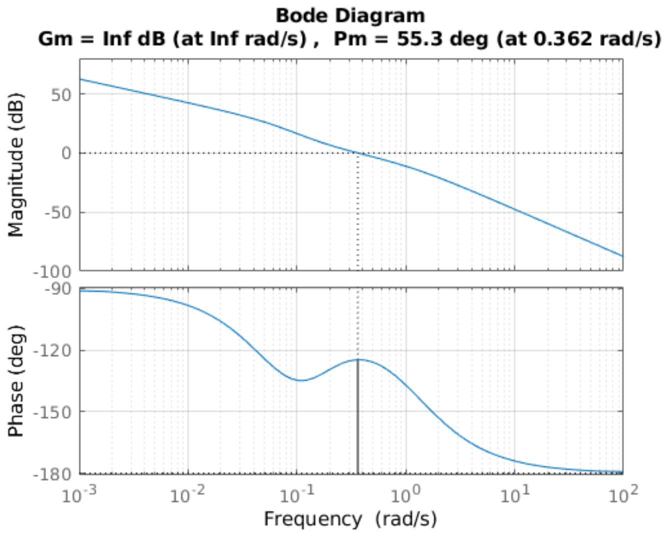


Figure 2: Bode plot of the system using G*F.

Task 7

The overshoot increases, settling time is smaller when sampling time is less than 1s, rise time is almost the same. For settling time above $T = 1$sec the requirements are not fulfilled any more. Solution to the task

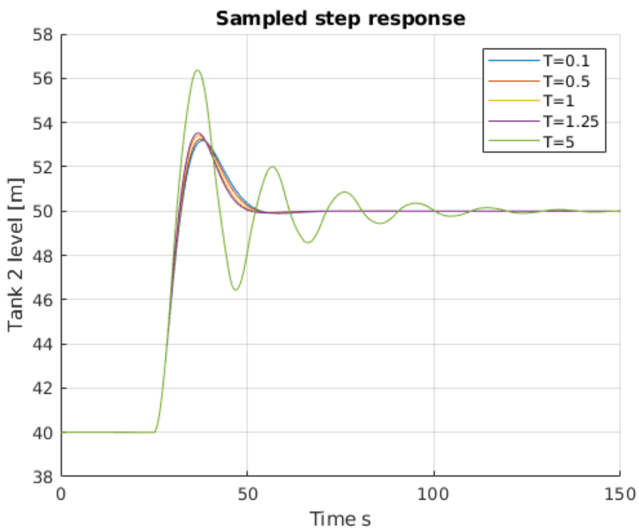


Figure 3: The step responses of the system sampled using ZOH for different sampling times T.

Task 8

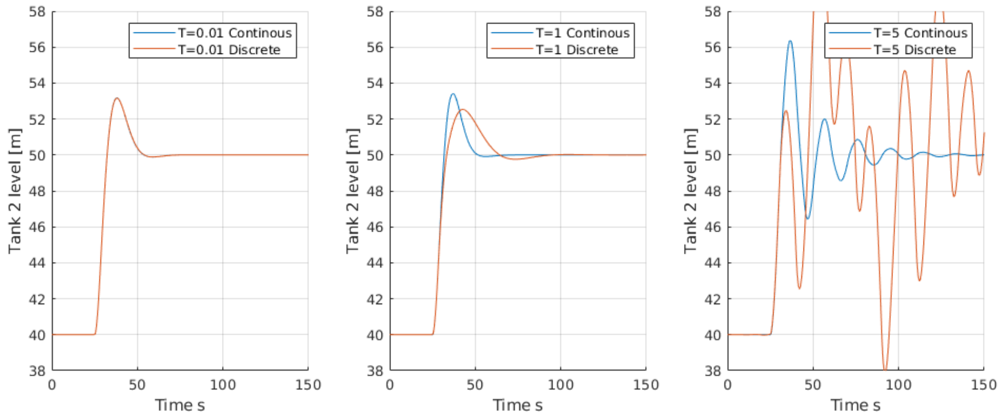


Figure 4: The step responses for the different setups compared using different sampling times T.

From the graph above we see that a sampling time of $T_s = 0.01$ gives next to unnoticeable differences between the continuous and discrete systems. However, as we increase the sampling time, the discrepancy becomes noticeable and at $T_s = 5$ we see that the method using the discretization of the system becomes unstable. It can be, concluded that for a worst case study, performing a discretization is better than just putting a ZOH after a continous controller.

Task 9

According the given hint, one rule of thumb when implementing a digitized continuous controller is to have the following fulfilled

$$h\omega_c \approx 0.05 \text{ to } 0.14$$

We know from a previous task that the cross-over frequency is $w_c = 0.362\,\text{rad}/\sec$, thus we have that

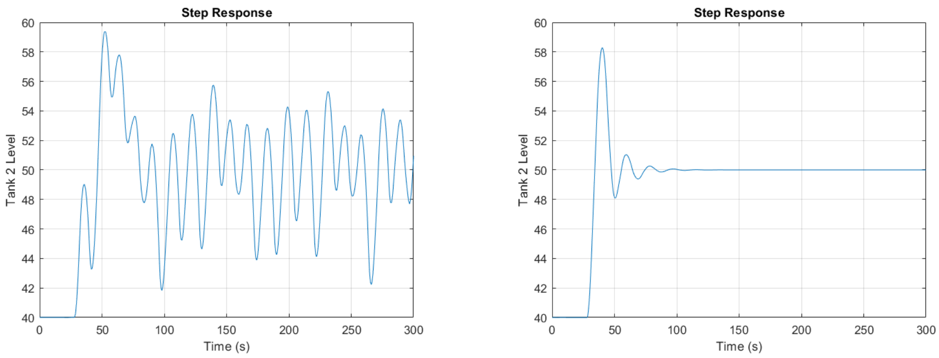
$$h \approx 0.1381 \text{ to } 0.3867$$

Task 10

T_s	$T_r(s)$	M	$T_{set}(s)$
0.3	5.1	28.8	29.5
0.34	5.2	8.4	30.0
0.35	5.1	28.3	30.1
0.38	5.2	28.1	30.4
0.4	5.2	27.9	30.7

From the table above we notice that the maximum possible sampling time without affecting the control performance for a discrete controller is $T_s = 0.34$, which as confirmed in the previous task lies within the range of the suggested rule of thumb.

Task 11



(a) Discrete (b) Continuous with ZOH

Figure 5: Closed loop response when $T_s = 4s$

For both models the performance requirements are not fulfilled.

Task 12

By rewriting equation 1 in the homework description we obtain the state space model with the following A,B,C and matrices

$$A = \frac{1}{\tau}\begin{bmatrix} -1 & 0 \\ 1 & -\frac{1}{\gamma} \end{bmatrix} \quad B = \frac{k}{\tau}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 \end{bmatrix}$$

$$\Phi = \begin{bmatrix} 0.7249 & 0 \\ 0.2332 & 0.7249 \end{bmatrix} \quad \Gamma = \begin{bmatrix} 0.6017 \\ 0.0916 \end{bmatrix}$$

Task 13

The observability matrix is given by

$$W_c = \begin{bmatrix} \Gamma & \Phi\Gamma \end{bmatrix} = \begin{bmatrix} 0.6017 & 0.4362 \\ 0.0916 & 0.2068 \end{bmatrix}$$

$$W_o = \begin{bmatrix} C \\ C\Phi \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0.2332 & 0.7249 \end{bmatrix}$$

Where $det(W_c) = 0.0844$ and $det(W_o) = -0.2332$. Thus, the plant is observable and reachable.

Task 14

l_r is required to satisfy servo properties.

Task 15

With an observer and controller L the input signal Δu becomes:

$$\Delta u = -L\Delta\hat{x}(k) + l_r r(k)$$

State space equation for the dynamic observer is

$$\Delta\hat{x}(k+1) = \Phi\Delta\hat{x}(k) + \Gamma\Delta u(k) + K\left[y(k) - C\Delta\hat{x}(k)\right]$$

$$\Delta y(k) = C\Delta x(k)$$

and the stata space equation for the system is given by

$$\Delta x(k+1) = \Phi\Delta x(k) + \Gamma\Delta u(k)$$

Putting all of the above together yields

$$\Delta\hat{x}(k+1) = \Phi\Delta\hat{x}(k) + \Gamma\left[-L\Delta\hat{x}(k) + l_r r(k)\right] + K\left[C\Delta x(k) - C\Delta\hat{x}(k)\right]$$

$$\Delta x(k+1) = \Phi\Delta x(k) + \Gamma\left[-L\Delta\hat{x}(k) + l_r r(k)\right]$$

Thus, with $x_a(k) = [\Delta x(k)\ \Delta\hat{x}(k)]^T$, we get

$$x_a(k+1) = \begin{bmatrix} \Phi & -\Gamma L \\ KC & \Phi - \Gamma L - C \end{bmatrix} x_a(k) + \Gamma l_r \begin{bmatrix} 1 \\ 1 \end{bmatrix} r(k)$$

Task 16

By reusing a few of the expressions in the previous task we have that

$$\Delta x(k+1) = \Phi\Delta x(k) + \Gamma\left[-L\Delta\hat{x}(k) + l_r r(k)\right] =$$

$$= \Phi\Delta x(k) - \Gamma L\Delta x(k) + \Gamma L\Delta x(k) - \Gamma L\Delta\hat{x}(k) + \Gamma l_r r(k) =$$
$$= (\Phi - \Gamma L)\Delta x(k) + \Gamma L\tilde{x}(k) + \Gamma l_r r(k)$$

$$\Delta x(k+1) - \Delta\hat{x}(k+1) = \Phi(\Delta x(k) - \Delta\hat{x}(k)) - KC(\Delta x(k) - \Delta\hat{x}(k)) =$$
$$= (\Phi - KC)(\Delta\tilde{x}(k))$$

Hence we get the desired outcome.

$$\begin{bmatrix} \Delta x(k+1) \\ \Delta\hat{x}(k+1) \end{bmatrix} = \begin{bmatrix} \Phi - \Gamma L & \Gamma L \\ 0 & \Phi - KC \end{bmatrix} \begin{bmatrix} \Delta x(k) \\ \Delta\hat{x}(k) \end{bmatrix} + \Gamma l_r \begin{bmatrix} 1 \\ 0 \end{bmatrix} r(k)$$

Poles of system.

$$|zI - A_z| = 0$$

$$= (zI - \Phi + \Gamma L)(zI - \Phi + KC) = 0$$

$$(zI - \Phi + \Gamma L) = 0, \quad (zI - \Phi + KC) = 0$$

Hence, the separation principle holds.

Task 17

$$G_{cl}(s) = \frac{F(s)G(s)}{1 + F(s)G(s)} \tag{1}$$

Continuous time poles

$$p1, p2 = -0.5, \quad p3 = -0.16 + 0.12j, \quad p4 = -0.16 - 0.12j$$

Discrete time poles

$$z1, z2 = 0.1353, \quad z3 = 0.4677 + 0.2435j, \quad z4 = 0.4677 - 0.2435j$$

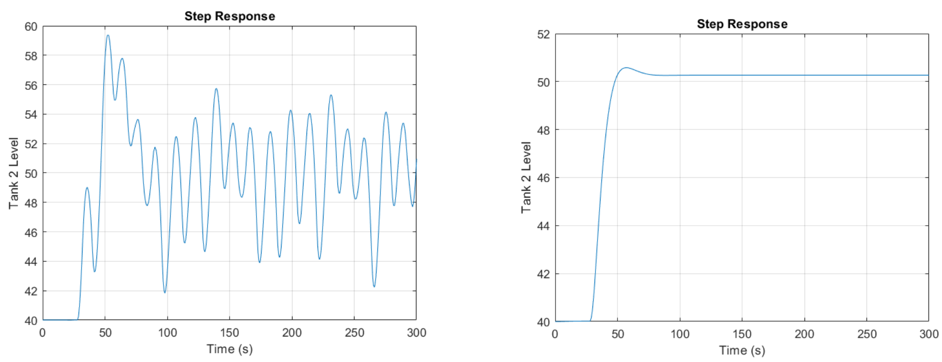
Acker command was used to design and controller and observer. z1 and z2 were used for the observer design because they are closer to zero, hence will have faster dynamics. Moreover, K can be design using the acker formula by first taking the transpose of the determinant.

Controller: $L = acker(\Phi, \Gamma, [z3; z4])$
Observer: $K^T = acker(\Phi^T, C^T, [z1; z2])$

$$L = \begin{bmatrix} 0.7187 & 0.8937 \end{bmatrix}, \quad K = \begin{bmatrix} 1.4902 \\ 1.1791 \end{bmatrix}, \quad l_r = 2.0697$$

A_a does indeed have the desired poles.

Task 18



(a) Discrete Controller (b) Discrete Designed Controller

Figure 6: Closed loop response when $T_s = 4s$

The output with Discrete Designed Controller is stable as compared to discrete controller. The output has a steady state error of 0.244. However, it was found that by modifying l_r slightly and using instead $l_r^* = 0.985 l_r$, the steady state error was almost eliminated.

Task 19

Quantization level $= \frac{100-0}{2^{10}} = 0.0977$

Task 20

Saturation block is necessary to saturate the output between the limits of the A/D block. This is used to replicate the effect of A/D saturation.

Task 21

Quantization Level $= \frac{100}{2^{bits}}$

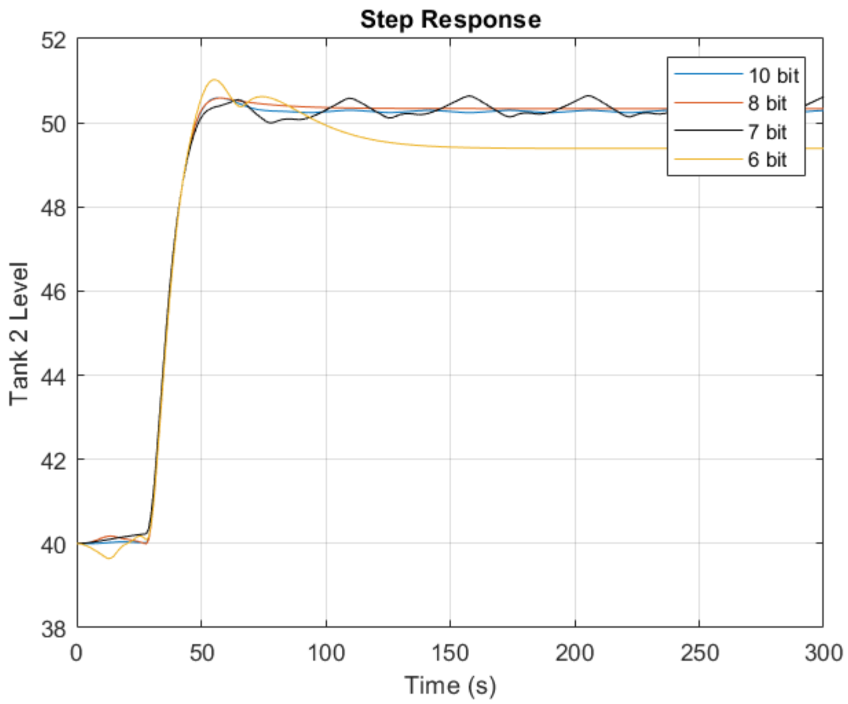


Figure 7: The step responses of the system for different quantization bits.

Performance starts degrading at and after 7 bits, which is equivalent to the quantization of 0.7813.