HW3 - EL2700

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Part 1: LQR Implementation

Q1 - Experimenting with the parameters of Q and R

First, we fixed $R = 0.1$ and changed each element of Q to see their effect. Initially, Q was changed to $[0, 0, 0, 0]$ to generate the plot shown in Fig.1a; this figure shows that the states do not change very much. On the other hand, in Fig.1b we set $Q_1 = 1000$, setting high weight on distance and fix $R = 1000$. This shows that the cart goes to $10m$ is quickly as possible. Similarly, we set $Q_2 = 1000$, leaving the rest to $Q_1 = Q_3 = Q_4 = 1$, this makes the cart not move at all by not providing any input. In the next step, we set $Q_3 = 1000$, observing the pendulum angle change to be minimized in Fig.1c. Similarly, we see angular velocity being minimized in Fig.1d when $Q_4 = 1000$. And finally, in 1e, keeping only $R = 1000$, the cart seldom moves.

Q2. Effect of Disturbance

Next we tuned and Q and R matrix without disturbance to get the desired performance as shown in Fig.2 left. The desired performance was:

1. It moves the cart to the reference point r = 10
2. The cart should move 90% of the reference position within 10 seconds;
3. The pendulum has to be kept within 10o around the vertical position.

We then added the disturbance $w = 0.1$ to see a steady state error at the output, shown in right of Fig.2. The controller could not overcome the unmodelled disturbance.

Part II: Adding Integral Action

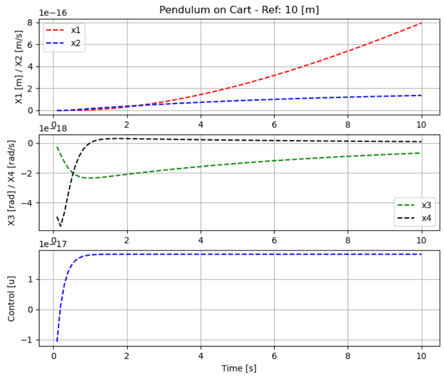
Q3 - Augmented system response

To compensate for the disturbance we added integral action to our controller. We augmented the system matrix to get the problem in the LQR form. Then the weight matrices Q and R were adjusted and the controller was modified. We tuned the Q matrix to $Q = diag(4, 50, 1, 1000, 1)$, with $R = 0.1$. The Q matrix was adjusted such that the weights on angular and linear velocity is higher and the integral weight is 1; the weights on distance and angle was adjusted to get the best performance. The response in Fig3 shows that the performance requirements are met while correcting for the disturbance.

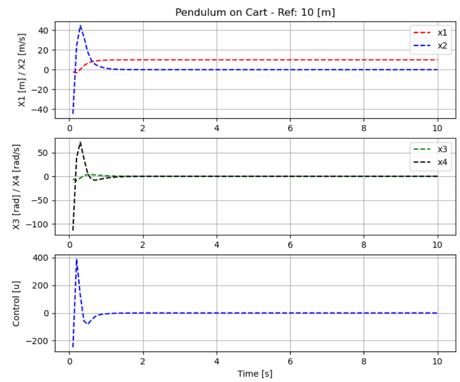
Part III: Output feedback controller

Q4 - Comparing performance

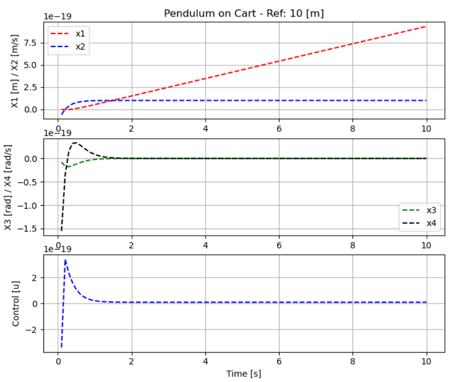
Building on top of the previous part, we assume that we don't have access to all the states, and hence, perform output feedback. We feed the system output with uniform noise between $[-0.005, 0.005]$.



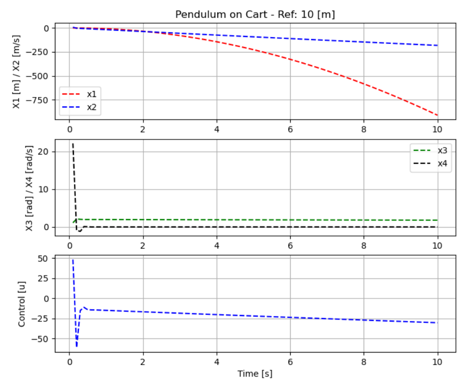
(a) $Q = diag(0, 0, 0, 0), R = 0.1$



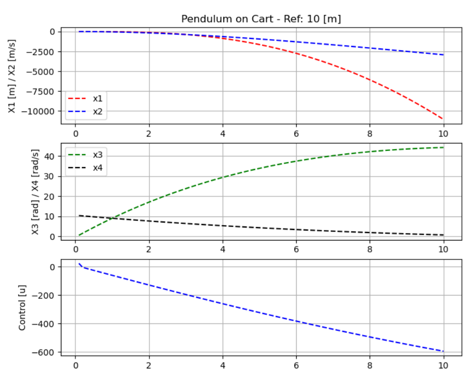
(b) $Q = diag(1000, 0, 0, 0), R = 1000$



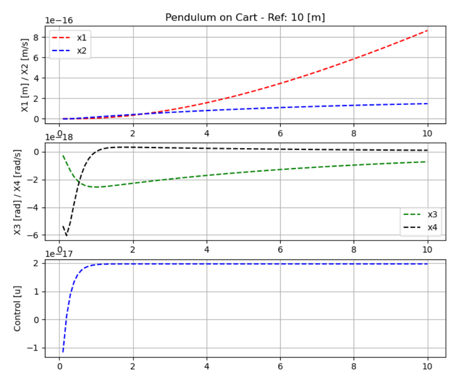
(c) $Q = diag(0, 1000, 0, 0), R = 1000$



(d) $Q = diag(0, 0, 1000, 0), R = 1000$

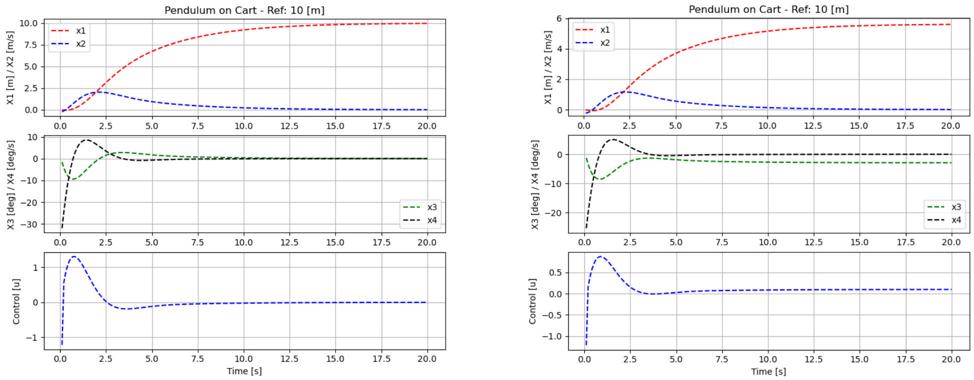


(e) $Q = diag(0, 0, 0, 1000), R = 1000$



(f) $Q = diag(0, 0, 0, 0), R = 1000$

Figure 1: The simulation results for different values of Q and R.



(a) $Q = diag(4, 50, 1, 1000), R = 0.01, w = 0$ (b) $Q = diag(4, 50, 1, 1000), R = 0.01, w = 0.1$

Figure 2: The parameters sets that fulfil the requirements with (left) and without (right) disturbance.

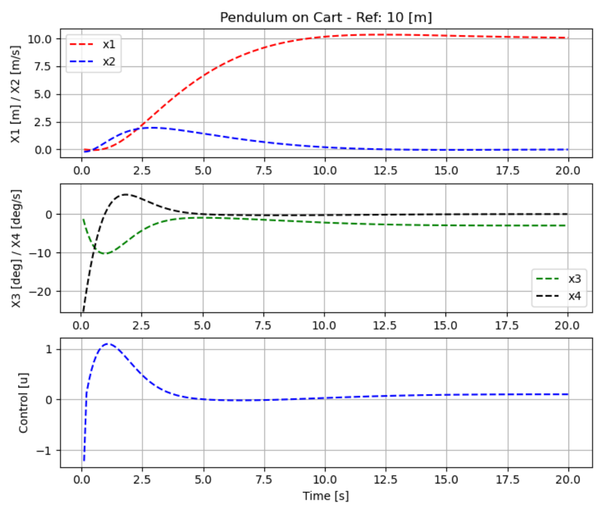
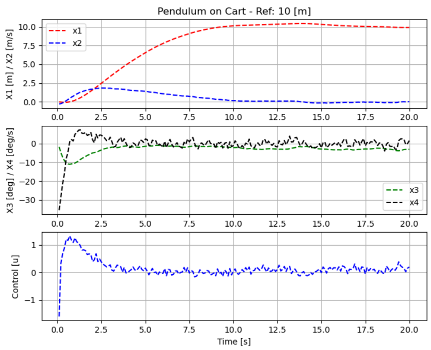


Figure 3: $Q = diag(4, 40, 1, 1000, 1), R = 0.1, w = 0.1$

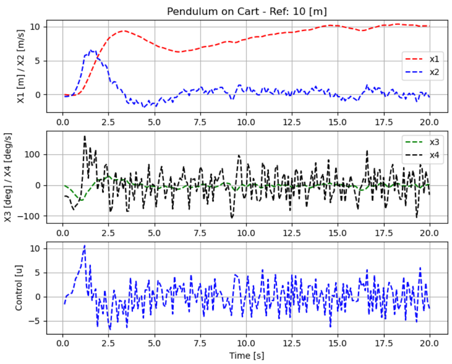
This presents the need for state estimation using a Kalman filter, provided that our pendulum-cart system is observable.

We begin by comparing the effect of using $C_p\,[1, 0, 0, 0]$ and $C_p \begin{bmatrix} 1, 0, 0, 0 \\ 1, 0, 1, 0 \end{bmatrix}$. In other words, either measuring the position of the cart only or measuring both the position and the angle simultaneously. Studying the results of Fig. 4 it is clear that measuring two states yields a much better performance, which is an intuitive results. Measuring only the position of the cart is a bit like balancing a stick on one's hand while blindfolded.

In Fig. 5 we compare the effect that the choice of covariance matrices has on the performance. This is further discussed in the next subsection. Lastly, we compare the performance of the output feedback controller to the state feedback controller in Fig. 6. We see that with proper tuning of the covariance matrices, the position of the cart has very similar curves in both cases. However, we note that the control input and the angular velocities have noisy profiles. More on that in the next subsections.

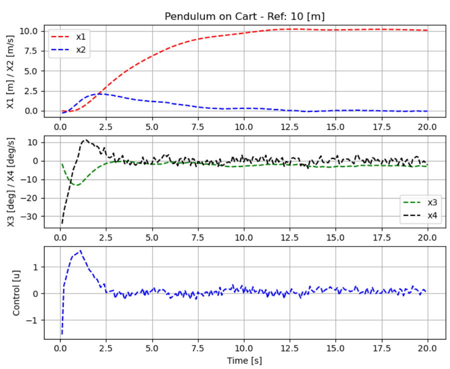


(a) $C = \begin{bmatrix} 1,0,0,0 \\ 0,0,1,0 \end{bmatrix}$, $Q_p = diag(1,1,1,1)$, $R_n = diag(1,1)$

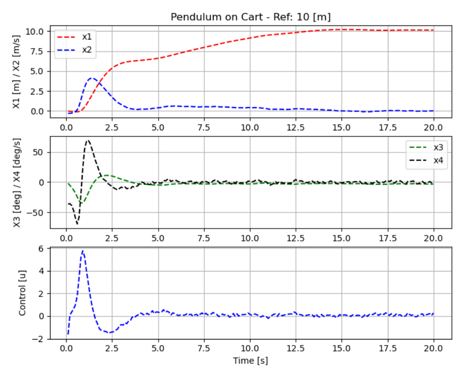


(b) $C = \begin{bmatrix} 1,0,0,0 \end{bmatrix}$, $Q_p = diag(1,1,1,1)$, $R_n = diag(1)$

Figure 4: Comparing the two different choices for the C matrix.

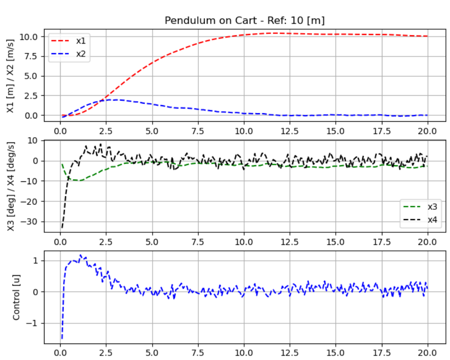


(a) $Q_p = 1000 \times diag(1,1,1,1)$, $R_n = diag(1,1)$

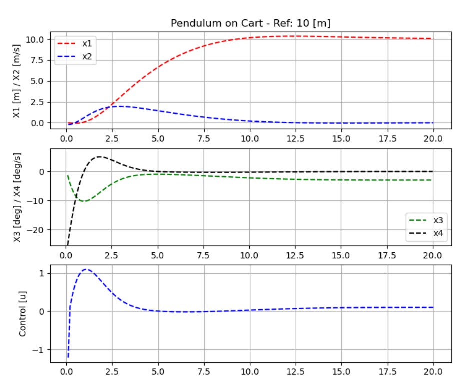


(b) $Q_p = diag(1,1,1,1)$, $R_n = 1000 \times diag(1)$

Figure 5: Comparing the effect of large Q_p vs large R_n with $C = \begin{bmatrix} 1,0,0,0 \\ 0,0,1,0 \end{bmatrix}$



(a) $Q_p = diag(5,100,5,100)$, $R_n = diag(1)$



(b) $Q = diag(4,40,1,1000,1)$, $R = 0.1, w = 0.1$

Figure 6: Performance comparison with the most performant systems with (left) and without (right) measurement noise. $w = 0.1$

Q5 - Robustness of output feedback controller

Diving deeper into the robustness of our observer-infused system, we study the performance given different sets of parameters. Firstly, we study the effect that the choice of the covariance matrices, for the process noise Q_p and the measurement noise R_n. The covariance matrix reflects how the Kalman filter weighs the state transition and the measured output.

When comparing the effects of the covariance matrices we note that with a high Q_p values, the Kalman filters gives higher priority to the information coming from the output feedback, in the estimation of the state. This results in a noisy control signal, since the output feedback itself is corrupted with noise. In contrast, when R_n is large, the output covariance is high. Thus, the Kalman filter gives more precedence to the estimated state and the dynamical system. We see that this cause the system to be less robust and does not fulfil the criteria for the system. On the other hand, the control signal is less noisy.

Lastly, we discuss the tuning of the covariance matrices. Based on the intuition gained so far. We want the Kalman filter to have higher covariance values for states we measure in the process noise versus the measured states. Therefore we give the measured states a covariance of 5 in Q_p and the non-measured states are set to 100. Furthermore, we also learned that we want the covariance of the measurement noise to be lower than the process noise. This ensures the Kalman filters prioritizes the output feedback over the state transition.

Overall we see that the knowledge obtained from the performed experiments was enough to aid the tuning of the Kalman filter parameters. We achieve a controller that fulfils the desired criteria despite measurement noise and a small disturbance, through integral action.