Appendix 3

SWIFT parameters

App. 3.1. Parameter values of *Magic Formula* and *SWIFT* model

Table A3.1. (205/60R15 91V, 2.2 bar. ISO sign definition. Also, cf. App.3.2)

$R_o\,(=r_o) = 0.313$m , $F_{zo}\,(=F_{No}) = 4000$N , $m_o = 9.3$kg , $V_o = 16.67$m/s

$p_{Cx1} = 1.685$	$p_{Dx1} = 1.210$	$p_{Dx2} = -0.037$	$p_{Ex1} = 0.344$	$p_{Ex2} = 0.095$
$p_{Ex3} = -0.020$	$p_{Ex4} = 0$	$p_{Kx1} = 21.51$	$p_{Kx2} = -0.163$	$p_{Kx3} = 0.245$
$p_{Hx1} = -0.002$	$p_{Hx2} = 0.002$	$p_{Vx1} = 0$	$p_{Vx2} = 0$	
$r_{Bx1} = 12.35$	$r_{Bx2} = -10.77$	$r_{Bx3} = 0$	$r_{Cx1} = 1.092$	$r_{Hx1} = 0.007$
$q_{sx1} = 0$	$q_{sx2} = 0$	$q_{sx3} = 0$		
$p_{Cy1} = 1.193$	$p_{Dy1} = -0.990$	$p_{Dy2} = 0.145$	$p_{Dy3} = -11.23$	$p_{Ey1} = -1.003$
$p_{Ey2} = -0.537$	$p_{Ey3} = -0.083$	$p_{Ey4} = -4.787$	$p_{Ky1} = -14.95$	$p_{Ky2} = 2.130$
$p_{Ky3} = -0.028$	$p_{Ky4} = 2$	$p_{Ky5} = 0$	$p_{Ky6} = -0.92$	$p_{Ky7} = -0.24$
$p_{Hy1} = 0.003$	$p_{Hy2} = -0.001$	$p_{Hy3} = 0$		
$p_{Vy1} = 0.045$	$p_{Vy2} = -0.024$	$p_{Vy3} = -0.532$	$p_{Vy4} = 0.039$	
$r_{By1} = 6.461$	$r_{By2} = 4.196$	$r_{By3} = -0.015$	$r_{By4} = 0$	$r_{Cy1} = 1.081$
$r_{Hy1} = 0.009$	$r_{Vy1} = 0.053$	$r_{Vy2} = -0.073$	$r_{Vy3} = 0.517$	$r_{Vy4} = 35.44$
$r_{Vy5} = 1.9$	$r_{Vy6} = -10.71$			
$q_{Bz1} = 8.964$	$q_{Bz2} = -1.106$	$q_{Bz3} = -0.842$	$q_{Bz5} = -0.227$	$q_{Bz6} = 0$
$q_{Bz9} = 18.47$	$q_{Bz10} = 0$	$q_{Cz1} = 1.180$	$q_{Dz1} = 0.100$	$q_{Dz2} = -0.001$
$q_{Dz3} = 0.007$	$q_{Dz4} = 13.05$	$q_{Dz6} = -0.008$	$q_{Dz7} = 0.000$	$q_{Dz8} = -0.296$
$q_{Dz9} = -0.009$	$q_{Dz10} = 0$	$q_{Dz11} = 0$		
$q_{Ez1} = -1.609$	$q_{Ez2} = -0.359$	$q_{Ez3} = 0$	$q_{Ez4} = 0.174$	
$q_{Ez5} = -0.896$	$q_{Hz1} = 0.007$	$q_{Hz2} = -0.002$	$q_{Hz3} = 0.147$	$q_{Hz4} = 0.004$
$s_{sz1} = 0.043$	$s_{sz2} = 0.001$	$s_{sz3} = 0.731$	$s_{sz4} = -0.238$	
$q_{Iay} = 0.109$	$q_{ma} = 0.237$	$q_{cbx0,z} = 121.4$	$q_{kbx,z} = 0.228$	$q_{cb\theta0} = 61.96$
$q_{Iaxz} = 0.071$	$q_{mb} = 0.763$	$q_{cby} = 40.05$	$q_{kby} = 0.284$	$q_{cb\gamma,\psi} = 20.33$
$q_{Iby} = 0.696$	$q_{mc} = 0.108$	$q_{ccx} = 391.9$	$q_{kcx} = 0.910$	$q_{cc\psi} = 55.82$
$q_{Ibxz} = 0.357$		$q_{ccy} = 62.7$	$q_{kcy} = 0.910$	$q_{kb\theta} = 0.080$
$q_{Ic} = 0.055$				$q_{kb\gamma,\psi} = 0.038$
				$q_{kc\psi} = 0.834$
$q_{V1} = 7.1\times10^{-5}$	$q_{Fz3} = 0$	$q_{a1} = 0.135$	$B_{reff} = 9$	$q_{Fcx1} = 0.1$
$q_{V2} = 2.489$	$q_{sy1} = 0.01$	$q_{a2} = 0.035$	$D_{reff} = 0.23$	$q_{Fcy1} = 0.3$
$q_{Fz1} = 13.37$	$q_{sy3} = 0$	$q_{bvx,z} = 3.957$	$F_{reff} = 0.01$	$q_{Fcx2} = 0$
$q_{Fz2} = 14.35$	$q_{sy4} = 0$	$q_{bv\theta} = 3.957$		$q_{Fcy2} = 0$

App. 3.2. Non-Dimensionalisation (with some remarks)

The quantities listed in Table A3.1 are non-dimensional. They are used in the following equations:

Magic Formula: Eqs.(4.E1-4.E78), except (4.E68, 4.E70)

SWIFT: Eqs.(9.131-236), except (9.177-182).

The inertia quantities have been made non-dimensional by dividing by the reference mass m_o or the reference moment of inertia I_o. For m_o the total tyre mass has been taken. The quantity I_o is defined as:

$$I_o = m_o R_o^2 \quad [\mathrm{kg.m^2}] \tag{A3.1}$$

Consequently, we have the non-dimensional parameters shown in the list above:

$$q_{ma} = \frac{m_a}{m_o}\ \text{, etc. and} \quad q_{Iay} = \frac{I_{ay}}{m_o R_o^2}\ \text{, etc.} \tag{A3.2}$$

As has been introduced already in connection with Eqs.(9.213-216) the non-dimensionalisation of the stiffnesses and damping coefficients is realized by dividing the linear (translational) stiffnesses and damping coefficients by:

$$c_{to} = \frac{F_{zo}}{R_o}\ [\mathrm{N/m}] \quad \text{and} \quad k_{to} = \sqrt{\frac{m_o F_{zo}}{R_o}}\ [\mathrm{Ns/m}] \tag{A3.3}$$

and the rotational stiffnesses and damping coefficients by:

$$c_{ro} = F_{zo} R_o\ [\mathrm{Nm/rad}] \quad \text{and} \quad k_{ro} = \sqrt{m_o F_{zo} R_o^3}\ [\mathrm{Nms/rad}] \tag{A3.4}$$

As a consequence, we have for example:

$$q_{cbx} = \frac{c_{bx}}{c_{to}}\ \text{, etc.} \quad \text{and} \quad q_{kbx} = \frac{k_{bx}}{k_{to}}\ \text{, etc.} \tag{A3.5}$$

$$q_{cb\psi} = \frac{c_{b\psi}}{c_{ro}}\ \text{, etc.} \quad \text{and} \quad q_{kb\psi} = \frac{k_{b\psi}}{k_{ro}}\text{, etc.} \tag{A3.6}$$

Remarks. In (9.131-132) the factors $k_{bx,z}\Omega$ are set equal to zero as rolling resistance has been accounted for already by the moment M_y (9.236,9.230-231). The mass m_a is the mass of that part of the tyre that, according to the model, moves with the wheel rim or axle. The quantity I_{ay} is the moment of inertia of that part of the tyre that rotates about the η axis as if it were attached as a rigid body to the wheel rim. The tyre inertia that is allocated to the belt is denoted with subscript b. The inertia denoted with subscript c is assumed to move with the contact patch. The interaction parameters q_{Fcx2} and q_{Fcy2} have been chosen equal to zero. They would have taken values 0.94 and 1.56 if $q_{Fcx1} = q_{Fcy1} = 0$.

App. 3.3. Estimation of *SWIFT* Parameter Values

using a limited set of (semi-static) measurements
(Contributed by I.J.M. Besselink, TNO-Automotive)

In Chapter 9 the *SWIFT* tyre model has been described. In order to assess the parameters of this tyre model a number of different tests are required, e.g. dynamic braking, yaw oscillation, dynamic cleat experiments on the drum. A full measurement programme to determine all parameters of the *SWIFT* model will be extensive and time consuming. Also the dedicated measurement equipment to perform high frequency testing may not be available or not suited to handle very large tyres (e.g. truck or aircraft tyres).

Using the experience gained from a number of full measurement programmes executed on passenger car tyres, a procedure has been developed to make sensible first estimates for the *SWIFT* parameters. This description applies to *MF-Swift* as implemented by TNO-Automotive in a number of simulation codes (e.g. *ADAMS*, *DADS*, *SIMPACK* or *MATLAB/Simulink*). The following steps can be distinguished:

(1) Force and moment testing

Fitting of the *Magic Formula*: F_x, F_y, M_x, M_y, $M_z = f(F_z, \kappa, \alpha, \gamma, V_x)$.

The software program *MF-Tool*, also developed by TNO-Automotive, may be used to fit the *Magic Formula* parameters. This program requires that the measurement data is provided in the form of *TYDEX* files, a standardised format as developed by the international *TYDEX* (*Ty*re *D*ata *Ex*change) workgroup, cf. Oosten et al.(1996).

Note: the contribution of the forward velocity V_x is present in the rolling resistance formula (9.231):

$$f_r = q_{sy1} + q_{sy3}|V_x/V_o| + q_{sy4}(V_x/V_o)^4 \tag{9.231}$$

that is used in connection with Eqs.(9.230,9.236). Of course, in cases such as moving on wet roads, the friction coefficient may be formulated as functions of the speed, cf. Eq.(4.E23).

(2) Loaded radius/effective rolling radius

The tyre normal deflection ρ_z is the difference between the free tyre radius of the rotating tyre r_Ω and the loaded tyre radius r_l: (cf. Eq.(9.218))

$$\rho_z = r_o + \Delta r - r_l = r_\Omega - r_l \tag{A3.7}$$

The following set of equations needs to be fitted:

a) centrifugal growth of the tyre free radius Δr (coefficient q_{V1})

$$\Delta r = q_{V1} r_o (\Omega\, r_o / V_o)^2 \tag{9.219}$$

b) vertical force-deflection equation (coefficients: q_{v2}, q_{Fcx1}, q_{Fcy1}, q_{Fz1} and q_{Fz2})

$$F_z = \left\{1 + q_{V2}|\Omega|\frac{r_o}{V_o} - \left(q_{Fcx1}\frac{F_x}{F_{zo}}\right)^2 - \left(q_{Fcy1}\frac{F_y}{F_{zo}}\right)^2\right\}\left(q_{Fz1}\frac{\rho_z}{r_o} + q_{Fz2}\frac{\rho_z^2}{r_o^2}\right)F_{zo} \tag{A3.8}$$

c) effective rolling radius, similar to (9.232), (coefficients: B, D, F_{reff})

$$r_e = r_o + \Delta r - \frac{F_{zo}}{C_{Fz}}\left\{D_{reff}\arctan\left(B_{reff}\frac{C_{Fz}}{F_{zo}}\rho_z\right) + F_{reff}\frac{C_{Fz}}{F_{zo}}\rho_z\right\} \tag{A3.9}$$

with the vertical stiffness of the standing tyre at nominal load F_{zo} as derived from Eq.(A3.8):

$$C_{Fz} = \frac{F_{zo}}{r_o}\sqrt{q_{Fz1}^2 + 4 q_{Fz2}} \tag{9.232b}$$

Notes: (1) F_{zo}, V_o and $r_o = R_o$ are reference parameters: they are just used to make the coefficients dimensionless. (2) An additional parameter q_{re0} may be introduced that multiplied with r_o in Eqs.(9.218, 9.232) gives an improved value for the free unloaded radius.

The measurement data for assessing the loaded and the effective rolling radius typically consists of values for r_l, r_e, V, F_x, F_y, F_z carried out for a number of different vertical loads, forward velocities and possibly longitudinal and/or lateral forces. Based on the relation $V = \Omega r_e$ it is sufficient to specify two out of three variables: forward velocity V, angular velocity Ω and effective rolling radius r_e. The data for F_x and F_y can be considered optional for passenger car tyres, but generally should be included for racing tyres.

Fitting of the parameters is generally done in two steps. In order to obtain maximum accuracy for the loaded radius the equations (9.219) and (9.217) are fitted first. In a second step the coefficients for the effective rolling radius, as given by equation (9.232,9.232a), are determined.

(3) Contact length

In *SWIFT* the following formula is used for the contact length formula, a represents half of the contact length (coefficients: q_{a1} and q_{a2})

$$a = \left(q_{a1}\sqrt{\frac{F_z}{F_{zo}}} + q_{a2}\frac{F_z}{F_{zo}} \right) r_o \tag{9.207}$$

Consequently, measurement data should provide half the contact length as a function of the vertical load F_z. If no or only very limited data is available the following formula may give a good estimate (where ρ_z is the tyre deflection):

$$a = p_{a1}\left(p_{a2}\sqrt{\frac{\rho_z}{r_o}} + \frac{\rho_z}{r_o} \right) r_o \tag{A3.10}$$

This formula was proposed by Besselink (2000) and better acknowledges the fact that the contact length is a geometrical property and primarily a function of the tyre deflection. The suggested parameter values for a passenger and light truck tyres are respectively:

p_{a1}=0.35 and p_{a2}=2.25 (on a flat surface)

p_{a1}=0.45 and p_{a2}=1.25 (on an external drum with a diameter of 2.5m).

It has been found that this formula is valid for a fairly wide range of tyres and can also be used for different tyre pressures without changing the coefficients, see Fig.A3.1. This figure also clearly illustrates that on an external drum the contact length is shorter compared to the situation on a flat road surface.

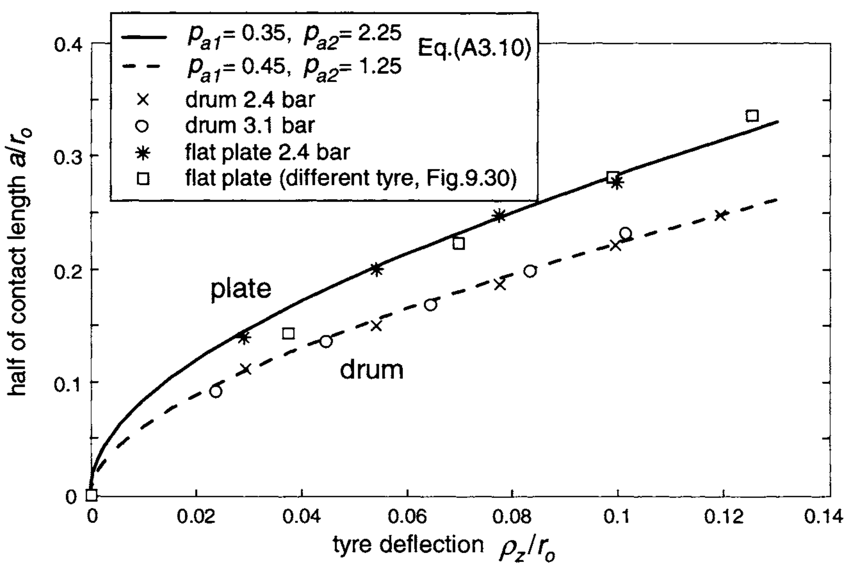

Fig. A3.1. Measurement and empirical formula results for half the contact length as a function of tyre deflection, passenger car tyres.

(4) Relaxation lengths, overall longitudinal and lateral stiffness

Apart from an accurate representation of the high frequency responses, it is important that the low frequency, transient behaviour (i.e. relaxation effects of the tyre) is included correctly. Measurements of the relaxation lengths can be used to assess the resulting longitudinal and lateral stiffness of the tyre carcass (that is excluding the tread) at ground level. For *SWIFT* the following equations hold for the longitudinal and lateral relaxation lengths respectively, cf. Eqs.(9.15) and (9.51) with $m = 1$ (at vanishing slip) and the relatively small effect of the torsional compliance $1/c_\psi$ neglected:

$$\sigma_x = \frac{C_{F\kappa}}{c_x} + a \tag{A3.11}$$

$$\sigma_y = \frac{C_{F\alpha}}{c_y} + a \tag{A3.12}$$

where:

- σ_x longitudinal relaxation length
- $C_{F\kappa}$ longitudinal slip stiffness of the tyre
- c_x longitudinal carcass stiffness at ground level
- σ_y lateral relaxation length
- $C_{F\alpha}$ cornering stiffness of the tyre
- c_y lateral carcass stiffness at ground level
- a half of the contact length

In the lateral direction all parameters may be known (σ_y, $C_{F\alpha}$, c_y, a), but usually the left and right hand side of Eq.(A3.12) turn out not to be the same in magnitude. In general the relaxation length based on the lateral stiffness and contact length is too short compared to the measurements on a rolling tyre. Since the aim of the tyre model is to have an accurate description of the tyre under rolling conditions, Eqs.(A3.11,A3.12) are used to calculate the resulting carcass stiffnesses of the tyre model in lateral and longitudinal direction (c_x and c_y respectively).

In the next step (5), the resulting compliances at ground level will be divided and distributed over the various parts of the tyre model.

(5) Carcass compliances

In *SWIFT* the rigid belt ring is elastically suspended with respect to the rim in all directions. The contact patch is elastically attached to the ring. Normally the various stiffness values are determined from dynamic experimental results. By using these values, it is possible to calculate their respective contribution to the overall carcass deflection at ground level when it is assumed that a longitudinal or lateral force is applied and that the rim is held fixed, cf. Fig.A3.2.

Table A3.2. Distribution of longitudinal and lateral carcass compliance components

longitudinal at ground level	*tyre 1*	*tyre 2*	*tyre 3*	*rule of thumb*
rigid ring translation (*a*)	27%	28%	31%	30%
rigid ring rotation (*b*)	63%	62%	60%	60%
contact patch translation (*c*)	10%	10%	9%	10%

lateral at ground level	*tyre 1*	*tyre 2*	*tyre 3*	*rule of thumb*
rigid ring translation (*a*)	34%	25%	27%	25%
rigid ring rotation (*b*)	62%	56%	55%	55%
contact patch translation (*c*)	4%	19%	18%	20%

Table A3.2 shows the relative contributions a, b and c that have been assessed for three different tyres. Based on these data it may be concluded that although the compliance values themselves are different, the relative contributions to the overall carcass deflection are fairly constant. Since under

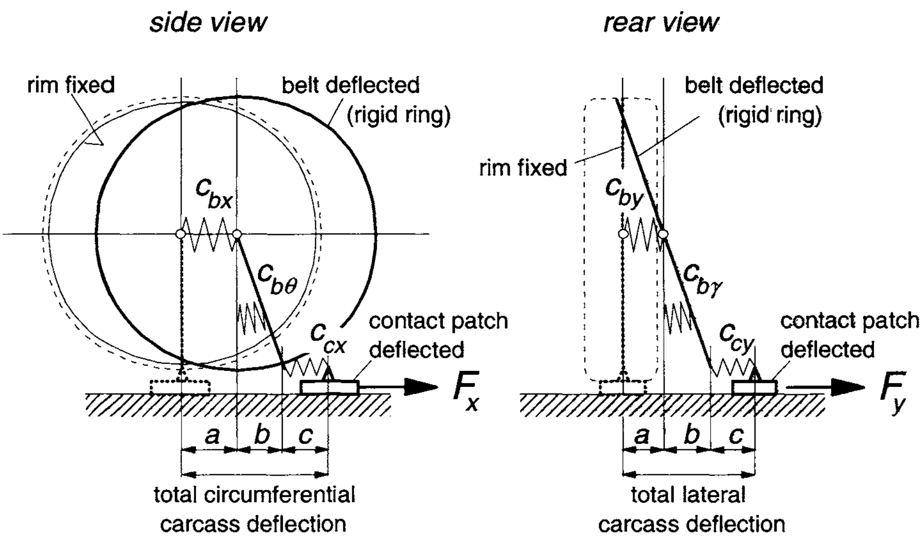

Fig. A3.2. Longitudinal and lateral deflections of the rigid ring and contact patch.

step (4) the overall longitudinal and lateral carcass stiffnesses have been determined, it is now possible to assess approximate individual stiffness values using the suggested 'rule of thumb'.

Additional measurement data (e.g. yaw stiffness or *FEM* model results) may also be used to enhance the estimates or to gain additional confidence in the stiffness/deflection distribution.

(6) Tyre inertia

In the *SWIFT* model the tyre is not considered as a single rigid body. A part of the tyre near the rim is assumed to be rigid and fixed to the rim. The other part that represents the belt, is considered to move as a rigid ring. Consequently, the total tyre mass has to be divided and a distribution has to be made over the inertia of the ring and the inertia of the part rigidly attached to the rim. The latter part is further regarded as a part of the wheel body. An estimate of this division can be made based on past experience or on a detailed weight break-down provided by the tyre manufacturer. The following rough initial estimate is suggested:

75% of the tyre mass is assigned to the tyre belt

85% of the tyre moments of inertia is assigned to the tyre belt

Tentatively, the contact patch body is considered as an additional small mass.

(7) Carcass damping

Generally, the exact amount of damping of the tyre is very difficult to assess experimentally. For instance, it is observed that a large difference in vertical damping exists between a tyre that stands still and a tyre that rolls: under rolling conditions the apparent damping may be a factor 10 smaller compared to the damping of a tyre standing still, also cf. Jianmin et al. (2001). A simple model with finite contact length and provided with radial dry friction dampers (Pacejka 1981a) may explain this phenomenon.

Based on experience, the following guideline may be provided: when the tyre is not in contact with the ground (and the rim is fixed) the damping will be relatively low. Typical values of the damping coefficients lie in the range of 1 to 6% of the corresponding critical damping coefficients. Generally, the modes that contain a large translational component are more heavily damped than the modes with a large rotational component.