

# List of Symbols

$a$	distance front axle to c.g.; half of contact length
$a_x$	longitudinal acceleration
$a_y$	lateral acceleration
$a_\mu$	slip velocity dependency coefficient for friction
$A_r$	rolling resistance coefficient
$b$	distance rear axle to c.g.; half contact width
$B$	stiffness factor in ' <i>Magic Formula</i> '
$B_1$	brake force of rolling wheel
$c$	stiffness; factor
$c_c$	lateral carcass stiffness per unit length
$c_{gyr}$	non-dimensional gyroscopic coefficient
$c_{px,y}$	tread element stiffness per unit length of circumference
$c'_{px}$	tread element longitudinal stiffness per unit area
$C$	cornering stiffness ; sum front and rear
$C_i$	cornering stiffness, sum left and right
$C$	contact centre (point of intersection)
$C$	shape factor in ' <i>Magic Formula</i> '
$C_{dA}$	air drag coefficient
$C_{Fx}$	longitudinal stiffness of standing tyre
$C_{Fy}$	lateral stiffness of standing tyre
$C_{Fz}$	stiffness of tyre normal to the road
$C_{Fa}$	cornering stiffness
$C_{Fk}$	longitudinal slip stiffness
$C_{F\gamma}$	camber stiffness for side force
$C_{F\phi}$	spin stiffness for side force
$C_{gyr}$	tyre gyroscopic coefficient
$C_{Ma}$	aligning torque stiffness
$C_{M\gamma}$	camber stiffness for aligning torque
$C_{M\phi}$	spin stiffness for aligning torque
$C_{M\psi}$	torsional yaw stiffness of standing tyre
$C_{Mx\gamma}$	overturning couple stiffness against camber
$C_{cx,y}$	carcass horizontal stiffness of standing tyre
$C_{gyr}$	gyroscopic coefficient
$df_z$	normalised change in normal load, Eq.(4.E2)
$d_t$	tread depth
$D$	peak factor in ' <i>Magic Formula</i> '; dissipation function

$E$	curvature factor in 'Magic Formula'
$e$	caster length; tread element deflection
$f$	trail of c.g.
$f_r$	rolling resistance coefficient
$F_{ax}$	force for forward acceleration
$F_d$	air drag force
$F_{x,tot}$	sum of longitudinal tyre forces
$F_x$	longitudinal tyre force
$F_y$	lateral tyre force
$F_z$	vertical (normal) tyre force (load) ( $>0$ ), in Chap.9: $F_z < 0$
$F_r$	rolling resistance force ( $>0$ )
$F_N$	tyre normal force ( $>0$ )
$F_{No}$	reference vertical load, nominal load ( $=  F_{zo} $ )
$F_V$	tyre vertical force
$F_H$	tyre longitudinal horizontal force
$g$	acceleration due to gravity; feedback rider control gain
$G$	weighting factor
$h$	height
$H$	height; sharpness factor in 'Magic Formula'
$H$	transform; Hurwitz determinant
$i$	$\sqrt{-1}$
$i_z$	radius of inertia
$I$	moment of inertia
$I_w$	wheel polar moment of inertia
$I_p$	wheel polar moment of inertia
$j$	$\sqrt{-1}$
$k$	radius of inertia; viscous damping coefficient
$K$	centrifugal force; force acting in belt, wheel centre
$l$	wheel base
$l_s$	shift; two-point follower length
$l_b$	length of basic curve
$l_f$	offset
$\mathbf{l}$	unit vector along line of intersection
$m$	mass; fraction of $2a$ where adhesion occurs
$m_c$	contact patch mass (dummy)
$m_t$	tyre mass
$m_m$	mass of mainframe (including lower part of rider)
$m_{mr}$	mass of mainframe plus rider
$m_r$	mass of upper torso
$M_{B,D}$	brake, drive torque

$M_x$	overturning couple
$M_y$	rolling resistance moment
$M_z$	(self) aligning torque
$M'_z$	(self) aligning torque due to lateral deflections
$M_z^*$	aligning torque due to longitudinal deflections
$M_{z,gyr}$	gyroscopic couple
$M_\delta$	steer torque
$n$	number of elements; frequency [Hz]
$\mathbf{n}$	unit vector normal to the road $= (0, 0, -1)^T$
$n_{st}$	steer system ratio
$p$	Laplace variable [1/m]
$p_i$	inflation pressure
$q$	average vehicle yaw resistance arm; generalised coordinate
$q$	contact force per unit length of circumference, vector
$Q$	generalised force
$r$	yaw rate; tyre (loaded) radius
$r_c$	radius of carcass (belt), unloaded; cross section crown radius
$r_{yo}$	free tyre radius varying along cross section contour, $r_{yo} = r_{yo}(y_{co})$
$r_e$	effective rolling radius of freely rolling wheel
$r_f$	free unloaded tyre radius
$r_l$	loaded radius
$r_o$	free unloaded tyre radius ( $= R_o$ )
$R$	radius of curvature
$R_o$	free unloaded tyre radius ( $= r_o$ )
$s$	forward position of neutral steer point; half track width
$s$	Laplace variable; travelled distance
$s_{sx}$	$\kappa$ (practical slip component)
$s_{sy}$	$\tan\alpha$ (practical slip component)
$\mathbf{s}$	unit vector along wheel spin axis
$S$	wheel slip point; impulse; string tension force
$S_{v,H}$	vertical, horizontal shift
$t$	pneumatic trail; time
$t_c$	caster length
$t_r$	rise time
$\mathbf{t}$	unit vector in road plane perpendicular to line of intersection $\mathbf{l}$
$T$	kinetic energy; moment acting around belt, wheel centre
$u$	forward velocity of c.g.; longitudinal deflection
$U$	potential energy
$v$	lateral velocity of c.g.; lateral deflection
$V$	speed of travel of c.g. (with $x$ , $y$ components)

$V$	speed of travel of wheel centre (with $x, y$ components)
$V_c$	speed of contact centre $C$ (with $x, y$ components)
$V_g$	speed of sliding (with $x, y$ components)
$V_o$	reference velocity $=\sqrt{gR_o}$
$V_r$	wheel linear speed of rolling $(= V_{cx} - V_{sx})$
$V_s$	wheel slip velocity of slip point $S$ (with $x, y$ components)
$V_x$	longitudinal speed component of wheel centre
$V_s^*$	velocity of contact patch mass (with $x, y$ components)
$w$	vertical road (effective) profile (positive downwards)
$W$	work
$x, y, z$	longitudinal, lateral, vertical displacement
$x, y, z$	coordinates with respect to moving axes system, $z$ axis vertical
$x^o, y^o, z^o$	global coordinates
$\bar{x}, \bar{y}, \bar{z}$	global coordinates
$X$	longitudinal horizontal tyre force
$X, Y, Z$	global coordinates
$y_{co}$	distance from wheel centre plane
$y_{mr}$	lateral offset of $m_{mr}$ c.g.
$\alpha$	wheel (side) slip angle; axle (side) slip angle
$\alpha$	road transverse slope angle
$\alpha'$	transient tyre slip angle
$\alpha_a$	virtual axle slip angle
$\beta$	vehicle side slip angle; tyre yaw torsion angle
$\beta_{x,y}$	road transverse, forward (effective) slope angle
$\beta_{gyr}$	gyroscopic wheel coupling coefficient, Eq.(6.35)
$\gamma$	camber (wheel inclination) angle
$\gamma'$	transient tyre camber angle
$\Gamma$	unit step response function
$\delta$	steer angle of front wheels
$\delta_o$	$\approx 1/R$ , steer angle at $V \rightarrow 0$
$\Delta$	increment
$\varepsilon$	roll steer coefficient; rake angle of steering axis
$\varepsilon$	string length ratio, Eq.(5.153); eff. roll. radius gradient $-\partial r_f / \partial d_i$
$\varepsilon_\gamma$	camber stiffness reduction factor
$\varepsilon_{NL}$	non-lagging part
$\varepsilon$	small quantity to avoid singularity
$\zeta$	damping ratio; spin factor ( $=1$ if spin influence is disregarded)
$\zeta_h$	height ratio, Eq.(6.36)
$\zeta_\alpha$	cornering stiffness load transfer coefficient

$\zeta_\gamma$	camber stiffness load transfer coefficient
$\eta$	understeer coefficient; effective rolling radius gradient $-\partial r_e/\partial \rho_z$
$\eta_y$	c.g. offset steer coefficient
$\theta$	tyre model parameter, Eqs.(3.6,3.24,3.46)
$\theta$	angular displacement about $\eta$ axis; pitch angle
$\theta_c$	string model composite parameter, Eq.(5.160)
$\kappa$	longitudinal wheel slip
$\kappa'$	transient longitudinal tyre slip
$\kappa^*$	damping coefficient due to tread width
$\lambda$	wavelength; root characteristic equation
$\lambda$	fraction of $2a$ where adhesion occurs; user scaling factor
$\mu$	coefficient of friction
$\rho$	tyre radial (vertical) deflection
$\rho_{x,y,z}$	tyre longitudinal, lateral, normal deflection
$\sigma$	relaxation length; load transfer coefficient
$\sigma$	theoretical slip, vector, Eq.(3.34)
$\sigma^*$	intersection length in string model with tread elements
$\sigma_c$	string model length parameter, Eq.(5.153)
$\sigma_c$	contact patch relaxation length
$\tau$	roll camber coefficient
$\varphi$	body roll angle; spin slip
$\varphi'$	transient spin slip
$\varphi_t$	turn slip
$\phi$	phase angle
$\psi$	yaw angle; steer angle
$\psi_{cl}$	compliance steer angle
$\psi_{io}$	toe angle
$\omega$	frequency [rad/s]
$\omega_o$	undamped natural frequency
$\omega_{1,2}$	natural frequencies
$\omega_n$	damped natural frequency
$\omega_s$	path frequency [rad/m]
$\Omega$	wheel speed of revolution
$\xi, \eta, \zeta$	moving axes system, $\eta$ axis along spin axis, $\xi$ horizontal

### ***Subscripts and superscripts***

$a$	axle; from belt to wheel rim centre
$b$	belt; from belt centre to rim
$c$	compliance (steer)

<i>c</i>	contact patch; from contact patch centre to belt; crown; contour
<i>D</i>	drag
<i>e</i>	effective
<i>eff</i>	effective (cornering stiffness)
<i>eq</i>	equivalent
<i>f</i>	free, unloaded; of front frame
<i>g</i>	global
<i>i</i>	1: front, 2: rear
<i>L,R</i>	left, right
<i>m</i>	of mainframe
<i>mr</i>	of mainframe plus rider
<i>NL</i>	non-lagging
<i>o</i>	original; initial; average
<i>o</i>	unloaded; nominal; at vanishing speed; natural
<i>r</i>	roll; rolling; rolling resistance; of residual spring; of rider
<i>s</i>	slip; from road surface to contact patch; of front sub-frame
<i>sl</i>	at verge of total sliding
<i>sf</i>	side force (steer)
<i>ss</i>	steady state
<i>st</i>	static
<i>stw</i>	steering wheel
<i>t</i>	transition from adhesion to sliding
<i>w</i>	wheel
<i>x,y,z</i>	forward (longitudinal), lateral (to the right), downward
<i>zr</i>	residual (torque)
$\xi, \eta, \zeta$	along, around $\xi, \eta, \zeta$ axes
1,2	front, rear; leading, trailing edge