



**Disperzija** (**varianca**)

D
(
X
)
=
E
(
(
X
−
E
(
X
)

)

2


)
=
E
(

X

2


)
−
(
E
(
X
)

)

2




{\displaystyle D(X)=E((X-E(X))^{2})=E(X^{2})-(E(X))^{2}}

Lastnosti:

- D*(*X*) ≥ 0
- D*(*X*) = 0 ⇐⇒ *P*(*X* = *E*(*X*)) = 1
- D*(*aX*) = *a*<sup>2</sup>*D*(*X*)

Standardna diviacija/odklon:

σ
(
X
)
=


D
(
X
)




{\displaystyle \sigma (X)={\sqrt {D(X)}}}

zanjo velja *σ*(*aX*) = |*a*|*σ*(*X*).

**Nekoreliranost**

Sl. sprem. *X* in *Y* sta nekorelirani, če velja:

E
(
X
Y
)
=
E
(
X
)
E
(
Y
)


{\displaystyle E(XY)=E(X)E(Y)}

*X*, *Y* neodvisni  ⇒  *X*, *Y* nekorelirani

Če imata *X* in *Y*, je nekoreliranost ekvivalentna zvezi:

D
(
X
+
Y
)
=
D
(
X
)
+
D
(
Y
)


{\displaystyle D(X+Y)=D(X)+D(Y)}

**Kovarianca**

K
(
X
,
Y
)
=
E
(
(
X
−
E
(
X
)
)
(
Y
−
E
(
Y
)
)
)
=
E
(
X
Y
)
−
E
(
X
)
E
(
Y
)


{\displaystyle K(X,Y)=E((X-E(X))(Y-E(Y)))=E(XY)-E(X)E(Y)}

- K*(*X*, *X*) = *D*(*X*)
- K*(*X*, *Y*) = 0 ⇐⇒ *X*, *Y* nekorelirani
- K*(*aX*, *bY*, *Z*) = *aK*(*X*, *Z*) + *bK*(*Y*, *Z*)

- K*(*X*, *Y*) = *K*(*Y*, *X*)
- K*(*aX* + *b*, *cY* + *d*) = *acK*(*X*, *Y*)
- |*K*(*X*, *Y*)| ≤ √*D*(*X*)*D*(*Y*)

- D*(*X* + *Y*) = *D*(*X*) + *D*(*Y*) + 2*K*(*X*, *Y*)

- D*(*X*<sub>1</sub> + ... + *X*<sub>*n*</sub>) = *D*(*X*<sub>1</sub>) + ... + *D*(*X*<sub>*n*</sub>) + 2∑<sub>*i*=1</sub><sup>*n*-1</sup> ∑<sub>*j*=*i*+1</sub><sup>*n*</sup> *K*(*X*<sub>*i*</sub>, *X*<sub>*j*</sub>)

**Standardizacija**

X

S


=


X
−
E
(
X
)


σ
(
X
)




{\displaystyle X\_{S}={\frac {X-E(X)}{\sigma (X)}}}

**Korelacijski koeficient**

r
(
X
,
Y
)
=


K
(
X
,
Y
)


σ
(
X
)
σ
(
Y
)


=
E
(

X

S


,

Y

S


)


{\displaystyle r(X,Y)={\frac {K(X,Y)}{\sigma (X)\sigma (Y)}}=E(X\_{S},Y\_{S})}

Lastnosti:

- r*(*X*, *Y*) = 0 ⇐⇒ *X*, *Y* nekorelirani
- −1 ≤ *r*(*X*, *Y*) ≤ 1
- r*(*X*, *Y*) = 1 ⇐⇒ *P*(*X*<sub>*S*</sub> = *Y*<sub>*S*</sub>) = 1
- r*(*X*, *Y*) = −1 ⇐⇒ *P*(*X*<sub>*S*</sub> = −*Y*<sub>*S*</sub>) = 1
- r*(*aX* + *b*, *cY* + *d*) = *r*(*X*, *Y*)

**Pogojne porazdelitve**

Pogojna porazdelitev sl. sprem. *X* glede na dogodek *B*:

X

|
B


∼


⎡


a

1




P
(
X
=

a

1


|
B
)



a

2




P
(
X
=

a

2


|
B
)



⋯



⋮



⎣


{\displaystyle X|B\sim {\begin{bmatrix}a\_{1}P(X=a\_{1}|B)&a\_{2}P(X=a\_{2}|B)&\cdots \\ \vdots \end{bmatrix}}}

**Pogojna porazdelitvena funkcija**

sl. sprem. *X* glede na dogodek *B*:

F

X
|
B


(
x
)
=

F

X


(
x
|
B
)
=
P
(
X
≤
x
|
B
)
=



P
(
(
X
≤
x
)
∩
B
)


P
(
B
)





{\displaystyle F\_{X|B}(x)=F\_{X}(x|B)=P(X\leq x|B)={\frac {P((X\leq x)\cap B)}{P(B)}}}

Če je pogojna porazdelitev zvezna, obstajaja tudi **pogojna porazdelitvena gostota**:

p

X
|
B


(
x
)
=

F

′

X
|
B


(
x
)


{\displaystyle p\_{X|B}(x)=F'\_{X|B}(x)}

**Pogojna gostota**

p

X


(
x
|
Y
=
y
)
≡

p

X


(
x
|
y
)
=



p
(
X
,
Y
)
(
x
,
y
)


p

Y


(
y
)





{\displaystyle p\_{X}(x|Y=y)\equiv p\_{X}(x|y)={\frac {p\_{X,Y}(x,y)}{p\_{Y}(y)}}}

**Pogojno matematično upanje**

E
(
h
(
X
)
|
B
)
=

∑

x



h
(
x
)
P
(
X
=
x
|
B
)


{\displaystyle E(h(X)|B)=\sum \_{x}h(x)P(X=x|B)}

E
(
X
|
Y
=
y
)
=

∫

−
∞


∞



x

p

(
X
|
Y
)
(
x
|
y
)
d
x


=



1


p

Y


(
y
)


∫

−
∞


∞



x

p

(
X
,
Y
)
(
x
,
y
)
d
x





{\displaystyle E(X|Y=y)=\int \_{-\infty }^{\infty }xp\_{(X|Y)(x|y)}dx={\frac {1}{p\_{Y}(y)}}\int \_{-\infty }^{\infty }xp\_{(X,Y)(x,y)}dx}

E
(
h
(
X
,
Y
)
|
Y
=
y
)
=
E
(
h
(
X
,
y
)
|
Y
=
y
)


{\displaystyle E(h(X,Y)|Y=y)=E(h(X,y)|Y=y)}

E
(
h
(
X
,
Y
)
|
Y
)
=

∑

x



h
(
x
,
Y
)
P
(
X
=
x
|
Y
)


{\displaystyle E(h(X,Y)|Y)=\sum \_{x}h(x,Y)P(X=x|Y)}

E
(
h
(
X
,
Y
)
|
Y
)
=

∫

−
∞


∞



h
(
x
,
Y
)

P

X
|
Y


(
x
|
Y
)
d
x


{\displaystyle E(h(X,Y)|Y)=\int \_{-\infty }^{\infty }h(x,Y)P\_{X|Y}(x|Y)dx}

Za vsako slučajno spremenljivko *X* in dogodek *B* veleja:

E
(
X
|
B
)
=



E
(
X
Z
)


P
(
B
)


=



E
(
X
Z
)


E
(
Z
)





{\displaystyle E(X|B)={\frac {E(XZ)}{P(B)}}={\frac {E(XZ)}{E(Z)}}}

kjer je sl. sprem. *Z* indikator dogodka *B*.

Za vsako sl. sprem. *X* z mat. up. in popoln sistem dogodkov *H*<sub>1</sub>, *H*<sub>2</sub>, ... velja **izrek o polni pričakovani vrednosti**

E
(
X
)
=
P
(

H

1


)
E
(
X
|

H

1


)
+
P
(

H

2


)
E
(
X
|

H

2


)
+
.
.
.


{\displaystyle E(X)=P(H\_{1})E(X|H\_{1})+P(H\_{2})E(X|H\_{2})+...}

**Regresijska funkcija**

 
ϕ
(
y
)
=
E
(
X
|
Y
=
y
)


{\displaystyle \ \phi (y)=E(X|Y=y)}

Za vsako sl. sprem. *X* z mat. up. in diskretno sl. sprem. *Y* velja:

E
(
X
g
(
Y
)
|
Y
)
=
E
(
X
|
Y
)
g
(
Y
)


E
(
X
g
(
Y
)
)
=
E
(
E
(
X
|
Y
)
g
(
Y
)
)


E
(
X
)
=
E
(
E
(
X
|
Y
)
)


{\displaystyle E(Xg(Y)|Y)=E(X|Y)g(Y)E(Xg(Y))=E(E(X|Y)g(Y))E(X)=E(E(X|Y))}

Za vsak dododek *A* in vsako sl. sprem *Y* velja:

E
(
P
(
A
|
Y
)
)
=
P
(
A
)


{\displaystyle E(P(A|Y))=P(A)}

**Momenti**

Moment reda *k* glede na točko *a* je

m

k


(
a
)
=
E
(
(
X
−
a

)

k


)
 
 
 
č
e
 
o
b
s
t
a
j
a


{\displaystyle m\_{k}(a)=E((X-a)^{k})\quad \quad \quad {\ce {e obstaja}}}

- Začetni moment** *z*<sub>*k*</sub> := *m*<sub>*k*</sub>(0) = *E*(*X*<sup>*k*</sup>)
- Centralni moment** *m*<sub>*k*</sub> := *m*<sub>*k*</sub>(*E*(*X*)) = *E*((*X* − *E*(*x*)<sup>*k*</sup>)
- Faktorski moment** reda *r*: *E*(*X*(*X* − 1) ... (*X* − *r* + 1)) = *G*<sub>*X*</sub><sup>(*r*)</sup>(1)

z

1


=
E
(
X
)
 
 
 

m

2


=
D
(
X
)


{\displaystyle z\_{1}=E(X)\quad \quad m\_{2}=D(X)}

Če obstaja *m*<sub>*n*</sub>(*a*), obstaja tudi *m*<sub>*k*</sub>(*a*) za ∀*k* < *n*.

Če obstaja *z*<sub>*n*</sub>, obstaja tudi *m*<sub>*n*</sub>(*a*) za ∀*a* ∈ ℝ

Centralne momente lahko izračunamo iz začetnih:

m

n


=

∑

k
=
0


n





n


k





(
−
1

)

n
−
k



z

1


n
−
k



z

k





{\displaystyle m\_{n}=\sum \_{k=0}^{n}{n \choose k}(-1)^{n-k}z\_{1}^{n-k}z\_{k}}

**Asimetrija**

 
A
(
X
)
=
E
(

X

S


3


)
=
E
⎡
⎣
(


X
−
E
(
X
)


σ
(
X
)



)

3




⎦
⎣
=



m

3




m

2




3





{\displaystyle \ A(X)=E(X\_{S}^{3})=E\left({\frac {X-E(X)}{\sigma (X)}})^{3}\right)={\frac {m\_{3}}{m\_{2}^{3}}}}

∀λ > 0 : *A*(λ*X*) = *A*(*X*)

**Sploščenost (kurtozis)**

 
K
(
X
)
=
E
⎡
⎣
⎡


X
−
E
(
X
)


√
D
(
X
)





⎦


⎦
=



m

4




m

2




2





{\displaystyle \ K(X)=E\left[\left[{\frac {X-E(X)}{\sqrt {D(X)}}}\right]^{4}\right]={\frac {m\_{4}}{m\_{2}^{2}}}}

Presežna sploščenost:

 
K
∗
(
X
)
=
K
(
X
)
−
3


{\displaystyle \ K^{\*}(X)=K(X)-3}

**Vrstilne karakteristike**

**Kvantil reda** *p*

je vsaka vrednost *x*<sub>*p*</sub>, za katero velja:

 
P
(
X
≤

x

p


)
≥
p
 
i
n
 
P
(
X
≥

x

p


)
=
1
−
p


{\displaystyle \ P(X\leq x\_{p})\geq p\;{\rm {in}}\;P(X\geq x\_{p})=1-p}

oz. *F*(*x*<sub>*p*</sub>−) ≤ *p* ≤ *F*(*x*<sub>*p*</sub>)

- Mediana: *x*






1
2





{\displaystyle x\_{\frac {1}{2}}}
- Kvartili: *x*






1
4





,

x

2
4


,

x

3
4





{\displaystyle x\_{\frac {1}{4}},x\_{\frac {2}{4}},x\_{\frac {3}{4}}}
- (Per)centili: *x*






1
100


,
.
.
.
,

x

99
100





{\displaystyle x\_{\frac {1}{100}},\ldots ,x\_{\frac {99}{100}}}

**Semi interkvartilni razmik**

 
s
=



1
2



⎡


x

3
4


−

x

1
4




⎦


{\displaystyle \ s={\frac {1}{2}}\left[x\_{\frac {3}{4}}-x\_{\frac {1}{4}}\right]}

**Rodovne funkcije**

Naj bo *X* sl. sprem. z zalogo vrednosti ℕ ∪ {0}:

p

k


=
P
(
X
=
k
)
 
 
 
k
=
0
,
1
,
2
,
.
.
.


{\displaystyle p\_{k}=P(X=k)\quad \quad k=0,1,2,\ldots }

Rodovna funkcija sl. sprem. *X*:

G

X


(
s
)
=

p

0


+

p

1


s
+

p

2


s

2


+
.
.
.
=

∑

k
=
0


∞



p

k



s

k




{\displaystyle G\_{X}(s)=p\_{0}+p\_{1}s+p\_{2}s^{2}+\ldots =\sum \_{k=0}^{\infty }p\_{k}s^{k}}

Obstaja za vse |*s*| ≤ 1.

 
P
(
X
=
k
)
=



G

X




(
k
)
(
0
)


k
!





{\displaystyle \ P(X=k)={\frac {G\_{X}^{(k)}(0)}{k!}}}

G

X


(
0
)
=

p

0


 
 
 

G

X


(
1
)
=
1
 
 
 

G

X


(
s
)
=
E
(

s

X


)


{\displaystyle G\_{X}(0)=p\_{0}\quad \quad G\_{X}(1)=1\quad \quad G\_{X}(s)=E(s^{X})}

Izrek o enoličnosti:

 
∀
s
∈
[
−
1
,
1
]
:

G

X


(
s
)
=

G

Y


(
s
)
 
 
 
⇔
 
P
(
X
=
k
)
=
P
(
Y
=
k
)
 
∀
k
=
0
,
1
,
2
,
.
.
.


{\displaystyle \ \forall s\in [-1,1]:G\_{X}(s)=G\_{Y}(s)\quad \quad \quad \Leftrightarrow \;P(X=k)=P(Y=k)\;\forall k=0,1,2,\ldots }

lim

s
↑
1



G

′

X


(
s
)
=
lim

s
↑
1



∑

k
=
1


∞



k

p

k



s

k
−
1


=

∑

k
=
1


∞



lim

s
↑
1



k

p

k



s

k
−
1


E
(
X
)


{\displaystyle \lim \_{s\uparrow 1}G'\_{X}(s)=\lim \_{s\uparrow 1}\sum \_{k=1}^{\infty }kp\_{k}s^{k-1}=\sum \_{k=1}^{\infty }\lim \_{s\uparrow 1}kp\_{k}s^{k-1}E(X)}

Naj bo *X* sl. sprem. z rodovno funkcijo *G*<sub>*X*</sub>, potem je:

G

X




(

n


)


(
1
−
)
=
E
(
X
)
(
X
−
1
)
(
X
−
2
)
.
.
.
(
X
−
n
+
1
)


{\displaystyle G\_{X}^{(n)}(1-)=E(X)(X-1)(X-2)\ldots (X-n+1)}

Naj bosta *X*<sub>1</sub>, ..., *X*<sub>*n*</sub> nedovisne sl. sprem. z rodovnimi funkcijami *G*<sub>*X*1</sub>, ... *G*<sub>*X**n*</sub>:

G

X

1
+
.
.
.
+

X

n




=

G

(

X

1


)
.
.
.
G
(

X

n


)
 
 
 
∀
s
∈
[
−
1
,
1
]


{\displaystyle G\_{X\_{1}+\ldots +X\_{n}}=G(X\_{1})\ldots G(X\_{n})\quad \quad \forall s\in [-1,1]}

Naj bodo ∀*n* ∈ ℕ sl. sprem *N*, *X*<sub>1</sub>, ..., *X*<sub>*n*</sub> neodvisne. Naj ima *N* rodovno funkcijo *G*<sub>*N*</sub> in *X*<sub>*i*</sub> rodovno funkcijo *G*<sub>*X*</sub>(*X*<sub>1</sub>, ..., *X*<sub>*n*</sub> so enako porazdeljene). Naj bo *S* = *X*<sub>1</sub> + ... + *X*<sub>*n*</sub>. Potem je:

G

S


=

G

N


(

G

X


(
s
)
)
 
 
 
∀
s
∈
[
−
1
,
1
]


{\displaystyle G\_{S}=G\_{N}(G\_{X}(s))\quad \quad \forall s\in [-1,1]}

Velja tudi *E*(*S*) = *E*(*N*)*E*(*X*).

G

2
X


(
s
)
=

G

X


(

s

2


)


{\displaystyle G\_{2X}(s)=G\_{X}(s^{2})}

**Znane rodovne funkcije**

 
∑

n
=
0


∞



q

n


=


1


1
−
q


 
 
 
∑

n
=
0


b



q

n


=


1
−

q

b
+
1




1
−
q





{\displaystyle \ \sum \_{n=0}^{\infty }q^{n}={\frac {1}{1-q}}\quad \quad \sum \_{n=0}^{b}q^{n}={\frac {1-q^{b+1}}{1-q}}}

 
∑

n
=
a


∞



q

n


=


q

a




1
−
q


 
 
 
∑

n
=
a


b



q

n


=


q

a


−

q

b
+
1




1
−
q





{\displaystyle \ \sum \_{n=a}^{\infty }q^{n}={\frac {q^{a}}{1-q}}\quad \quad \sum \_{n=a}^{b}q^{n}={\frac {q^{a}-q^{b+1}}{1-q}}}

 

a

n


−

b

n


=
(
a
−
b
)

(

a

n
−
1


+

a

n
−
2


b
+
...
+

a

b

n
−
2


+

b

n
−
1


)


{\displaystyle \ a^{n}-b^{n}=(a-b)(a^{n-1}+a^{n-2}b+...+ab^{n-2}+b^{n-1})}

a

0


+
.
.
.
+


a

k
−
1



x

k
−
1




1
−

x

k





=

a

0


+
.
.
.
+


a

k
−
1



x

k
−
1


+


a

0


k


+
.
.
.
+


a

k
−
1



x

2
k
−
1


+
.
.
.


{\displaystyle {\frac {a\_{0}+...+a\_{k-1}x^{k-1}}{1-x^{k}}}=a\_{0}+...+a\_{k-1}x^{k-1}+a\_{0}^{k}+...+a\_{k-1}x^{2k-1}+...}

 
(
x
+
y

)

n


=

∑

k
=
0


n





n


k





x

n
−
k



y

k




{\displaystyle \ (x+y)^{n}=\sum \_{k=0}^{n}{n \choose k}x^{n-k}y^{k}}

 


1


(
1
−
x

)

n




=

∑

k
=
0


n





n
+
k
−
1


k





x

k




{\displaystyle \ {\frac {1}{(1-x)^{n}}}=\sum \_{k=0}^{n}{n+k-1 \choose k}x^{k}}

B

λ


(
x
)
=

∑

n





λ


n





x

n


=
(
1
+
x

)

λ


;
 
 
 


λ


n





=



λ

n


n
!





{\displaystyle B\_{\lambda }(x)=\sum \_{n}{\lambda \choose n}x^{n}=(1+x)^{\lambda };~~~{\lambda \choose n}={\frac {\lambda ^{n}}{n!}}}

**Momentno rodovna funkcija**

M

X


(
t
)
=
E
(

e

t
X


)
 
 
 
∀
t
∈

R

 
 
 
č
e
 
o
b
s
t
a
j
a


{\displaystyle M\_{X}(t)=E(e^{tX})\quad \quad \forall t\in R\quad \quad {\ce {e obstaja}}}

 
=
1
+

z

1


t
+


z

2


2
!


t

2


+


z

3


3
!


t

3


+
.
.
.


{\displaystyle \ =1+z\_{1}t+{\frac {z\_{2}}{2!}}t^{2}+{\frac {z\_{3}}{3!}}t^{3}+\ldots }

V primeru, ko ima *X* zalogo vrednosti v ℕ ∪ {0}, je

M

X


(
t
)
=
E
(

e

t
X


)
=

G

X


(

e

t


)


{\displaystyle M\_{X}(t)=E(e^{tX})=G\_{X}(e^{t})}

Za zvezno porazdeljeno sl. sprem. *X* velja:

M

X


(
t
)
=

∫

−
∞


∞



e

t
x



p

X


(
x
)
d
x


{\displaystyle M\_{X}(t)=\int \_{-\infty }^{\infty }e^{tx}p\_{X}(x)dx}

Naj pri nekem δ > 0 *M*<sub>*X*</sub>(*t*) obstaja za vse *t* ∈ (−δ, δ). Potem je porazdelitev za *X* natanko določana z *M*<sub>*X*</sub> in vsi začetni momenti obstajajo: