Parsing Binomials & Multinomials in Probability

Andrew Jones

Introduction

Note that

i. first point
ii. second point
iii. third point

1 Binomial Theorem

The binomial theorem expresses the expansion of two monomial terms such as $(x+y)^2 = x^2 + 2xy + y^2$. In probability the binomial theorem can express the total probability of two independent events.

Example 1. Let an unfair coin be flipped twice with $P(Tails) = 0.3$ and $P(Heads) = 0.7$
We know the probability must sum to 1. In two flips then, $(T+H)^2 = T^2 + 2TH + H^2$. This aligns with the outcomes of TT, TH, HT, and HH for two flips. Substituting in the probabilities we have $0.3^2 + 2 * 0.3 * 0.7 + 0.7^2 = 1$.

To do this the theorem uses:

Definition 1 (Factorial n!)**.** Count everyway to permute a set of n distinct objects

$$n! = \prod_{i=1}^{n} i$$

with $0! = 1$ and $n \geq 0$.

Building on factorials, the theorem uses the

Definition 2 (Binomial Coefficient)**.** $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

to count combinations of groups of events. The inductive proof of the theorem uses:

Pascal's Identity.

$$\begin{aligned}
\binom{n-1}{k} + \binom{n-1}{k-1} &= \frac{(n-1)!}{k!(n-1-k)!} + \frac{(n-1)!}{(k-1)!(n-k)!} \\
&= (n-1)![\frac{n-k}{k!(n-k)!} + \frac{k}{k(n-k)!}] \\
&= (n-1)!\frac{n}{k!(n-k)!} \\
&= \frac{n!}{k!(n-k)!} \\
&= \binom{n}{k}
\end{aligned}$$

□

Proof. □

2 Multinomial Theorem

Lemma 1. *We have*

$$\int_0^\pi \sin(3x)\,\mathrm{d}x = \frac{2}{3}.$$

Proof. A direct computation yields

$$\begin{aligned}
\int_0^\pi \sin(3x)\,\mathrm{d}x &= \frac{1}{3}\int_0^{3\pi} \sin u\,\mathrm{d}u, \qquad\qquad u = 3x,\\
&= \frac{1}{3}\big[-\cos u\big]_0^{3\pi}\\
&= \frac{1}{3}\big[1-(-1)\big]\\
&= \frac{2}{3}.
\end{aligned}$$

□

Remark 1. This is interesting since...

3 Possible Outcomes to Equations