Worksheet 1

Name: ____

Due by midnight of **Wednesday**, Jan. 29, on Gradescope.

Prove or disprove the following statements.

1. $\exists n \in \mathbb{Z} : n + 1 = 5$
2. $\forall n \in \mathbb{Z} : n > 7$
3. $\exists x \in \mathbb{R} : \forall y \in \mathbb{R} : x \geq y$
4. $\exists x \in \mathbb{R} : \forall k \in \mathbb{N} : x^k = x$
5. $\forall x \in \mathbb{R} : \exists y \in \mathbb{R} : xy = 1$
6. $\exists x \in \mathbb{R} : \forall y \in \mathbb{R} : xy = y$
7. Give an example of a property P for which

$$\forall m \in \mathbb{Z} : \exists n \in \mathbb{Z} : P(m, n)$$

is true while

$$\exists n \in \mathbb{Z} : \forall m \in \mathbb{Z} : P(m, n)$$

is false.[1]

8. Now give an example of a property Q for which

$$\forall m \in \mathbb{Z} : \exists n \in \mathbb{Z} : Q(m, n)$$

is false while

$$\exists n \in \mathbb{Z} : \forall m \in \mathbb{Z} : Q(m, n)$$

is true.

9. Let A and B be sets and suppose that $\forall a \in A : \forall b \in B : P(a, b)$ for some property P. Is it necessarily true that $\forall b \in B : \forall a \in A : P(a, b)$? Justify your reasoning. You do not need to provide a proof, though you are welcome to do so.
10. Now suppose that $\exists a \in A : \exists b \in B : P(a, b)$. Does it necessarily follow that $\exists b \in B : \exists a \in A : P(a, b)$? As above, just your reasoning.

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[1] For example, $P(m, n)$ might be "$m + n$ is even", "m is a multiple of n", "$m > n$", ...