Midterm 1 Rubric

Questions 1.–5.

i. express the statement in terms of quantifiers, (*1 pt.*)
ii. express the negation in terms of quantifiers, (*1 pt.*)
iii. indicate whether the statement is true or false, (*2 pt.*)
iv. either prove or disprove the statement (*3 pts.* for logical correctness, *3 pts.* for conventional writing.)

Rubric.

i–ii.

points	*conditions*
1	correct
0	incorrect

iii.

points	*conditions*
2	correct
0	incorrect

iv. logical correctness

points	*conditions*
3	entirely correct with no irrelevant data
2	includes superfluous or irrelevant data
1	does not appropriately introduce or assign variables misapplies or misinterprets a mathematical fact includes a logically invalid deduction
0	cites a mathematically false statement attempts to disprove a claim by failing to prove it

writing

points	*conditions*
3	clear and conventional mathematical writing
2	includes unconventional terms or phrases starts a sentence with a mathematical symbol uses quantifiers as shorthand for English phrases
1	grossly unconventional writing
0	not entirely written in complete sentences does not address the question

Example.

$$\boxed{\text{If } f : A \to B \text{ and } g : B \to C \text{ are not injective, then } g \circ f : A \to C \text{ is not injective.}}$$

i. *1 pt.*

- $\forall$ noninjections $f\!:\!A \to B,\ g\!:\!B \to C : g \circ f$ noninjective
- $\forall$ functions $f\!:\!A \to B,\ g\!:\!B \to C : (f$ noninjective $\wedge$ g noninjective$) \implies g \circ f$ noninjective
- $\forall f\!:\!A \to B,\ g\!:\!B \to C : f, g$ noninjective $\implies g \circ f$ noninjective

0 pts.

- $\forall$ noninjections $f\!:\!A \to B\ \wedge\ g\!:\!B \to C : g \circ f$ noninjective
- $(f\!:\!A \to B$ noninjective $\wedge$ $g\!:\!B \to C$ noninjective$) \implies g \circ f$ noninjective

ii. *1 pt.*

- $\exists$ noninjections $f\!:\!A \to B,\ g\!:\!B \to C : g \circ f$ injective
- $\exists$ functions $f\!:\!A \to B,\ g\!:\!B \to C : (f$ noninjective $\wedge$ g noninjective$) \wedge g \circ f$ injective
- $\exists$ functions $f\!:\!A \to B,\ g\!:\!B \to C : f$ noninjective $\wedge$ g noninjective $\wedge$ $g \circ f$ injective
- $\exists f\!:\!A \to B,\ g\!:\!B \to C : f, g$ noninjective $\wedge$ $g \circ f$ injective

0 pts.

- $\neg\forall$ noninjections $f\!:\!A \to B,\ g\!:\!B \to C : g \circ f$ noninjective
- $\forall$ noninjections $f\!:\!A \to B,\ g\!:\!B \to C : g \circ f$ injective
- $\forall f\!:\!A \to B,\ g\!:\!B \to C : f, g$ noninjective $\implies g \circ f$ injective
- $\exists f\!:\!A \to B,\ g\!:\!B \to C : f, g$ injective $\wedge$ $g \circ f$ injective

iii. *2 pts.* true; *0 pts.* false

iv. **logical correctness 3 pts.**, *writing 3 pts.*

- Choose distinct $a, a' \in A$ with $f(a) = f(a')$ and observe that $g(f(a)) = g(f(a'))$.
- Choose distinct $a, a' \in A$ with $f(a) = f(a')$ and observe that $g(f(a)) = g(f(a'))$. It follows that $g \circ f$ is noninjective.
- Let $f : A \to B$ and $g : B \to C$ be functions. Suppose that f and g are not injective. Since f is noninjective, there are $a, a' \in A$ such that $a \neq a'$ and $f(a) = f(a')$. Applying g to the second equality yields $g \circ f(a) = g \circ f(a')$ and we conclude that $g \circ f$ is not injective.

l.c. 3 pts., w. 2 pts.

- Choose distinct $a, a' \in A$ with $f(a) = f(a')$ and observe that $g(f(a)) = g(f(a'))$. So this means the statement is true.
- Since f is injective $\exists a, a' \in A$ such that $a \neq a'$ and $f(a) = f(a')$. Applying g to the second equality yields $g \circ f(a) = g \circ f(a')$ and we conclude that $g \circ f$ is not injective.

l.c. 3 pts., w. 1 pts.

- Assume $a, a' \in A$ such that $a \neq a'$ and $f(a) = f(a')$. Therefore, $g(f(a)) = g(f(a'))$.

l.c. 3 pts., w. 0 pts.

- $a, a' \in A$ s.t. $a \neq a'$ and $f(a) = f(a')$ Therefore $g(f(a)) = g(f(a'))$

l.c. 2 pts., *w. 3 pts.*

- Choose distinct $a, a' \in A$ with $f(a) = f(a')$ and distinct $b, b' \in B$ with $g(b) = g(b')$ and observe that $g(f(a)) = g(f(a'))$.
- Choose distinct $a, a' \in A$ with $f(a) = f(a')$. Since g is noninjective, we have $g(f(a)) = g(f(a'))$.

l.c. 1 pts., *w. 3 pts.*

- Since injections are closed under composition, it follows that if $g \circ f$ were injective then at least one of f and g must be injective as well.

l.c. 0 pts., *w. 3 pts.*

- Since noninjections are closed under composition, it follows that if $g \circ f$ were injective then at least one of f and g must be injective as well.