Practice Midterm 2b

Name: _____

Short Answer (*4 pts. each, 20 pts. total*)

Write each of the following quantities in the form $a + bi$.

1. $(3 + 5i) - (7 + 2i)$

   _____

2. $\dfrac{3 + 5i}{-i}$

   _____

3. $(5 - 6i)^2$

   _____

4. $\left(\cos \dfrac{\pi}{18} + i \sin \dfrac{\pi}{18}\right)^9$

   _____

5. $\dfrac{1}{e^{-3\pi i}}$

   _____

Proofs (*5 pts. each, 10 pts. total*)

Prove or dispove the following statements.

6. The relation $\sim$ on $\{(p, q) \in \mathbb{Z}^2 \mid q \neq 0\} \subseteq \mathbb{Z}^2$ given by

$$(p, q) \sim (p', q') \iff pq' = p'q.$$

   is an equivalence relation.

7. The relation R on $\mathbb{N}$ given by

$$mRn \iff \exists k \in \mathbb{N} : m^k = n$$

   is a partial order.

Web of dependencies (*20 pts.*)[1]

8. Draw a web of dependencies exhibiting the interrelations among the definitions and results introduced in this course.

   - Each definition or result should be represented as a node, along with a brief description.
   - If two definitions or results are related directly, insofar as the relation is not mediated by another definition or result, then this relation should be depicted by an arrow between the two along with a brief description.

[1] 1 point per labeled node, 1 per description, $+\frac{1}{k}$ extra credit for the kth additional node or description