1 Homework

Prove or Disprove the following statements:

1. $\exists n \in \mathbb{Z} : n + 1 = 5$
   - There exists a number n in the integers such that n + 1 = 5
   - Set n equal to 4
   - Observe that $4 + 1 = 5 \in \mathbb{Z}$
2. $\forall n \in \mathbb{Z} : n > 7$
   - For all numbers n in the integers, n is greater than 7
   - Set n equal to 5
   - Observe that $5 \in \mathbb{Z}$
3. $\exists x \in \mathbb{R} : \forall y \in \mathbb{R} : x \geq y$
   - There exists a real number x such that for all real numbers $y : n \geq y$
   - Fix x to 5.
   - Fix y to 7.
   - Observe that 5, $7 \in \mathbb{R}$
   - Observe that $y \geq n$ where y = 7 and x = 5
4. $\exists x \in \mathbb{R} : \forall k \in \mathbb{N} : x^k = x$
   - There exists a real number x such that for all integers y: $x^k = x$
   - Fix x to 1.
   - Observe that $1^y = 1$
5. ∀ x ∈ ℝ: ∃ y ∈ ℝ: xy = 1
   - For all x in the reals there exists a real number y such that xy = 1.
   - Put $y = \frac{1}{x}$
   - Observe that $x * \frac{1}{x} = 1$
   - $\frac{1}{x} \in \mathbb{R}$
6. ∃ x ∈ ℝ: ∀ y ∈ ℝ: xy = y

- There exists a real number x such that for all real numbers xy = y.
- Put x = 1
- Observe that 1*y=y
- $1 \in \mathbb{R}$

7. Give an example of a proposition P for which:

   $\forall m \in \mathbb{Z} : \exists n \in \mathbb{Z} : P(m,n)$ is true and $\exists n \in \mathbb{Z} : \forall m \in \mathbb{Z} : P(m,n)$ is false

   - For all integers m there exists an integer n such that P(m,n) is true
   - There exists an integer n such that for all integers m P(m,n) is false
   - Let P = m = n
   - Put n = m
   - Observe that n, m $\in \mathbb{Z}$
   - Observe that for all integers m there exists an integer n such that m = n
   - Observe that there is not a single integer n such that for all integers m m = n

8. Find a Proposition Q for which:

   $\forall m \in \mathbb{Z} : \exists n \in \mathbb{Z} : Q(m,n)$ is false and $\exists n \in \mathbb{Z} : \forall m \in \mathbb{Z} : Q(m,n)$ is true

   - For all integers m there exists an integer n such that Q(m,n) is false
   - There exists an integer n such that for all integers m Q(m,n) is true
   - Put Q equal to m ¡ n
   - Put n = (m + 1)
   - Observe that m, (m + 1) $in\mathbb{Z}$
   - Observe that for all integers m there is an integer m - 1 such that m ¡ (m + 1)
   - Put m = 5 and n = 4
   - Observe that 5 ¡ 4 is false
   - Observe that there does not exist a single integer n such that all integers m are greater than it.

9. Is the statement $\forall a \in \mathbb{A} : \forall b \in \mathbb{B} : P(a,b)$ communative and there for $\forall b \in \mathbb{B} : \forall a \in \mathbb{A} : P(a,b)$ is also true?

- For all numbers a in A such that for all numbers b in B satisfy P(a,b)
- For all numbers b in B such that for all numbers a in A satisfy P(a,b)
- As the order of the arguments to the proposition does not change, I would assume that switching the order of the for all statements should not effect the value of the proposition.

10. $\exists a \in \mathbb{A} : \exists b \in \mathbb{B} : P(a, b)$ does it follow that $\exists b \in \mathbb{B} : \exists a \in \mathbb{A} : P(a, b)$?

    - There exists a number a in Set A such that there exists a number B in set B that satisifies P(a,b)
    - There exists a number b in Set B such that there exists a number A in Set A that satisfies P(a,b)
    - I would also assume here that exists is communative and what matters is switching the order of the parameters to the Property.