

Show all your work. A right answer is a correct result together with the correct steps used to obtain it:
Right Answer = Correct Result + Correct Steps

Solve the following problems from the book

Chapter 1

16. Show that the function f defined by $f(x) := x/\sqrt{x^2 + 1}$, $x \in \mathbb{R}$, is a bijection of \mathbb{R} onto $\{y : -1 < y < 1\}$.
17. For $a, b \in \mathbb{R}$ with $a < b$, find an explicit bijection of $A := \{x : a < x < b\}$ onto $B := \{y : 0 < y < 1\}$.
20. (a) Suppose that f is an injection. Show that $f^{-1} \circ f(x) = x$ for all $x \in D(f)$ and that $f \circ f^{-1}(y) = y$ for all $y \in R(f)$.
(b) If f is a bijection of A onto B , show that f^{-1} is a bijection of B onto A .
21. Prove that if $f : A \rightarrow B$ is bijective and $g : B \rightarrow C$ is bijective, then the composite $g \circ f$ is a bijective map of A onto C .
22. Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be functions.
(a) Show that if $g \circ f$ is injective, then f is injective.
(b) Show that if $g \circ f$ is surjective, then g is surjective.

Also consider the following problem.

Problem 1 Identify the issue with the following misuse of mathematical induction: We would like to “prove” that for any nonnegative integer n , we have that $2n = 0$. For the initial case, $n = 0$, clearly the result is true. Now suppose that it is true for all $n \leq k$ for nonnegative integer k , that is $2n = 0$ for all $n \leq k$ and we “prove” it for all $n = k + 1$. Note that we can write $k + 1 = i + j$ where $0 \leq i, j \leq k$, and then

$$2(k + 1) = 2(i + j) = 2i + 2j = 0 + 0 = 0,$$

and it is “proven”.