Worksheet 3

Name: _____

Due by midnight of **Wednesday**, Feb. 12, on Gradescope.

Let A, B, and C be sets. Prove or disprove the following statements.

1. If $A \cap B = \varnothing$ and $B \cap C = \varnothing$, then $A \cap C = \varnothing$.
2. If $A \not\subseteq B$ and $B \not\subseteq C$, then $A \not\subseteq C$.
3. If $A \subseteq \varnothing$, then $A = \varnothing$.
4. If $A \subseteq C$ and $B \subseteq C$, then $A \cap B \subseteq C$.
5. If $f : A \to B$ is injective and $g : B \to C$ is injective, then $g \circ f : A \to C$ is injective.
6. If $f : A \to B$ is surjective and $g : B \to C$ is surjective, then $g \circ f : A \to C$ is surjective.
7. Give an example of a function $f : A \to A$ that is injective but not surjective.
8. Give an example of a function $g : A \to A$ that is surjective but not injective.
9. Let $f : A \to B$ and $g : B \to A$. If $g \circ f = \mathrm{id}_A$, then both f and g are bijections.[1]
10. If $f : A \to A$ is surjective, and if A is a finite set, then f is injective.

Bonus question.

11. If $f : A \to A$ satisfies the property that $f \circ f = \mathrm{id}_A$, then f is a bijection.

[1] Recall that $\mathrm{id}_A : A \to A$ is the *identity function* on A, given by $\mathrm{id}_A(x) = x$ for all $x \in A$.