A. Jones Math-300 Feb 14

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Limits

1. The hierarchy of Results] on midterm
2. Proof by Contradiction] on midterm
3. Limits at infinity
4. Limits at points

To dos

- ☐ Midterm 1 is Thursday 02/20
  - will be on Tuesday if snow
- ☐ Practice mid-terms available
- Monday → Practice Mid-Terms
- ☐ WS3 will allow corrections
- ☐ WS3 Graded by Saturday
- ☐ WS3 Corrections due by mid-term 1
- ☐ Solutions on Blackboard WS3

- So far we call everything a "claim"
- Proof by Contradiction sucks
  - doesn't tell you why something is true.

theorem > claim. A theorem is a bigger result than a claim.

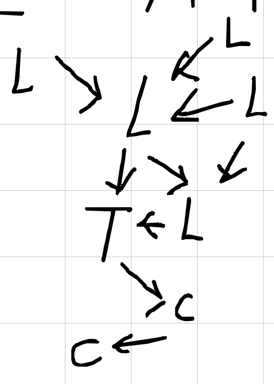
Proposition < theorem. Props aren't main ideas, they support theorems. Propositions are lesser results that can stand alone.

lemma < theorem. Lemmas exist to prove something else. They can chain.

An axiom is not a result, it is an assumption.

Corollary $= \text{lemma}^{-1}$ → the opposite of a lemma. Follows easily from a theorem or propisition. Helper prop.

[thm., prop., lem., cor.]



→ Transitive

[If $a \to b$
If $b \to c$
then $a \to c$]

axiom = there exists an $\emptyset$ set.
axioms are high level.

Proof by Contradiction, Want to prove: P

idea: Assume that $\neg P$ & derive a contradiction.

$$P \equiv P \vee \bot$$ — False

$$\equiv \neg(\neg P) \vee \bot$$

$$\equiv \neg P \Rightarrow \bot$$

$$P \Rightarrow Q$$
$$\neg P \vee Q$$

"Suppose not" $\triangleq$ proof by contradiction
— use "suppose not" or
"suppose for a contradiction"

$2, 3, 5 = 9 + 1 = 10$ 31

$2, 3, 5, 7 = 17 + 1 = 18$ all

for the prime noms ex!

$$m = p_1 \ldots p_n + 1 = \boxed{p_1 \times p_2 \times p_3 \times p_n + 1}$$

by multiplication
not

$$\left(\prod_{i=1}^{n} p_i\right) + 1$$

- End contradiction "proof" w/ "This yields the desired contradiction"

- Euclid wrote a great text book, but very few thms are attributed to him.

Proof By Contradiction #2

0 1 0 1 1 0 1 0 0 ...
1 0 1 0 0 1 1 0 0 ...

Claim: You can not list all the binary sequences.
(This is a proof of multiple infinities)
Some infinites are smaller.
Infinity of naturals < binary seq. ∞.

1	$\boxed{0}$	1	0			
2	1	$\boxed{0}$	1			
3	0	0	$\boxed{0}$	0		
4	1	1	1	$\boxed{0}$	1	1
5	0	0	0	1	$\boxed{0}$	0
6	1	1	1	0	1	$\boxed{1}$

diagnolization arguement

so we take the kth num & invert it

0 0 0 0 0 1

↓

1 1 1 1 1 0

↓

Therefore we can create a new binary sequence not listed before

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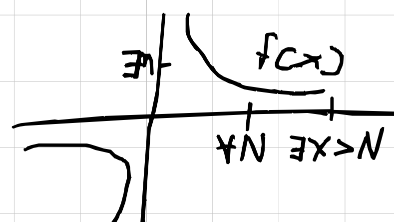
3. Limits at infinity

So we're encoding that for some number m there exista an N such that f(N) > M

<u>Prop.</u> $\lim_{x \to \infty} 2x = \infty$

<u>Prf.</u> Let $M > 0$. Put $N = \frac{M}{2}$.
Let $x > N$. From $x > \frac{M}{2}$,
we can conclude that $2x > M$

<u>Prop.</u> $\lim_{x \to \infty} \frac{1}{x} \neq \infty$



<u>wts.</u> $\exists M > 0 : \forall N > 0 :$
$\exists x > N : f(x) \leq M$

<u>prf.</u> Put $M = 1$. Let $N > 0$.
Put $x = \max(N, 1)$.
If $N > 1$, the $\frac{1}{x} = \frac{1}{N} \leq 1$.
Otherwise, $x = 1$ and so $\frac{1}{x} \leq 1$. □