

## Worksheet 4

Name: \_\_\_\_\_

Due by midnight of **Monday**, Mar. 3, on Gradescope.

Let  $R$  be a relation from  $A$  to  $B$ , let  $S$  be a relation from  $B$  to  $C$ , and let  $T$  be a relation from  $C$  to  $D$ . Prove the following statements.

1.  $I_A \circ R = R$
2.  $R \circ I_A = R$
3.  $(R^{-1})^{-1} = R$
4.  $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$
5.  $(T \circ S) \circ R = T \circ (S \circ R)$
6.  $\text{Dom } R = \text{Rng } R^{-1}$
7.  $\text{Rng } R = \text{Dom } R^{-1}$

For Question 8–10, suppose that  $A = B = C$ .

8. If  $R$  and  $S$  are equivalence relations, then  $S \circ R$  is an equivalence relation.
9. If  $R$  is a partial order, then  $R \circ R$  is a partial order.
10. If  $R$  and  $S$  are partial orders, then it is not generally true that  $S \circ R$  is a partial order.

### Bonus questions.

Give an example of two relations  $R$  and  $S$  on a set  $A$  such that

11.  $R \circ S \neq S \circ R$ .
12.  $S \circ R$  is an equivalence relation, but neither  $R$  nor  $S$  is an equivalence relation.