Did not do any of the examples after this—all fair game!

$$P(C|A) = \frac{P(A|C)P(C)}{P(A|C)P(C) + P(A|C^c)P(C^c)}$$

note $C^c = F$

so

$$P(C|A) = \frac{P(A|C)P(C)}{P(A|C)P(C) + P(A|F)P(F)}$$

$$= \frac{(2/3)(1/2)}{(2/3)(1/2) + (1/2)(1/2)} = \frac{\frac{2}{3}}{\frac{2}{3} + \frac{1}{2}} = \frac{1}{1 + 3/4}$$

$$= \frac{4}{7}$$

probability she took chemistry

Ch. 3 Problem 3.18

46% of voters in a city classify themselves as independents.

30% of " " " " " " " liberals

24% " " " " " conservatives.

In a recent election...

35% of independents voted

62% of liberals voted

58% of conservatives voted.

A voter is chosen at random. Given that this person voted...

What is the probability that he/she is

a) independent? b) liberal? c) conservative?

d) What percentage of voters participated in the election?

a) ans. $P(I|V)$ = probability of I = indep. given that they voted, V.

$$P(I|V) = \frac{P(IV)}{P(V)}$$

$$P(V) = P(IV) + P(LV) + P(CV)$$

(from $V = \underbrace{IV \cup LV \cup CV}_{\text{union of mutually exclusive events}}$)

so
$$P(I|V) = \frac{P(IV)}{P(IV) + P(LV) + P(CV)}$$

$$P(IV) = P(V|I)P(I) = (.35)(.46) = 0.161$$
$$P(LV) = P(V|L) \cdot P(L) = (.62)(.3) = 0.186$$
$$P(CV) = P(V|C)\,P(C) = (.58)(.24) = 0.1392$$
$$0.4862$$

$$P(I|V) = \frac{0.161}{0.161 + 0.186 + 0.1392} = \frac{0.161}{0.4862} \approx .331$$

b) $$P(L|V) = \frac{P(LV)}{P(V)} = \frac{0.186}{0.4862} \approx .383$$

c) $$P(C|V) = \frac{P(CV)}{P(V)} = \frac{0.1392}{0.4862} \approx .286$$

d) $P(V) = 0.4862$

Generalized Version of Bayes's Formula.

Suppose events $F_1, F_2, \ldots, F_n$ are mutually exclusive (i.e. $F_iF_j = \phi$ if $i \neq j$) and together they make up the whole sample space. So

$$\bigcup_{i=1}^{n} F_i = S$$

Then we can write

$$E = EF_1 \cup EF_2 \cup \ldots \cup EF_n$$

and then, since $\underbrace{EF_i}_{\text{and } EF_j}$ are mutually exclusive for $i \neq j$

$$P(E) = P(EF_1) + P(EF_2) + \ldots + P(EF_n)$$

$$= \sum_{i=1}^{n} P(EF_i)$$

$$\boxed{P(E) = \sum_{i=1}^{n} P(E|F_i)P(F_i)}$$

Ross ch. 3.3 p. 69 eq. (3.4)

So

$$P(F_j|E) = \frac{P(EF_j)}{P(E)}$$

Prop. 3.1 Bayes in Ross p. 69

$$= \frac{P(EF_j)}{\sum_{i=1}^{n} P(E|F_i)P(F_i)} = \frac{P(E|F_j)P(F_j)}{\sum_{i=1}^{n} P(E|F_i)P(F_i)}$$

EXAMPLE 3a

Insurance company believes there are two classes of people, accident prone and not accident prone.

Statistics →

- accident prone person has probability 0.4 of having an accident in a 1-year period
- non-accident prone person has probability 0.2 of having an accident in a 1-year period.

⊛ Assume the 30% of the population is accident prone.

Q: What is the probability that a new policy holder will have an accident with a year of purchasing a policy

$$P(A_1) = \underbrace{P(A_1|A)}_{0.4}\,\underbrace{P(A)}_{.3} + \underbrace{P(A_1|A^c)}_{0.2}\,\underbrace{P(A^c)}_{0.7}$$

prob. of new policy holder having accident

$$= 0.12 + 0.14 = \boxed{0.26}$$

Q: A new policy holder has an accident in first year. What is the probability that he/she is accident prone?

$$P(A|A_1) = \frac{P(AA_1)}{P(A_1)} = \frac{P(A_1|A)P(A)}{P(A_1)} = \frac{(0.4)(0.3)}{0.26}$$

$$= \frac{0.12}{0.26} = \frac{12}{26} = \frac{6}{13}$$

Note:

$$P(A^c \mid A_1) = \frac{P(A^c A_1)}{P(A_1)} = \frac{P(A_1 \mid A^c)\,P(A^c)}{P(A_1)}$$

probability of being not accident prone given that the person had an accident in the first year.

$$= \frac{(0.2)(0.7)}{0.26} = \frac{0.14}{0.26}$$

$$= \frac{14}{26} = \frac{7}{13} \quad \text{i.e. } 1 - \frac{6}{13}$$

Q: What if we make a different assumption about the % of the population that is accident prone? (call it p_A)

note: the accident prone proportion of the population is relatively small in this assumption. (30%)

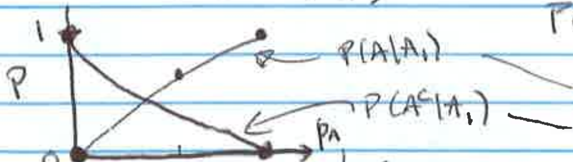
- $P(A_1) = P(A_1 \mid A)P(A) + P(A_1 \mid A^c)\,P(A^c)$

$$= (0.4)\,p_A + 0.2\,(1-p_A)$$

$$= 0.2 + 0.2\,p_A = 0.2(1+p_A)$$

- $P(A \mid A_1) = \dfrac{P(AA_1)}{P(A_1)} = \dfrac{P(A_1 \mid A)\,P(A)}{0.2(1+p_A)} = \dfrac{0.4\,p_A}{0.2(1+p_A)} = \dfrac{2p_A}{1+p_A}$

- $P(A^c \mid A_1) = \dfrac{P(A^cA_1)}{P(A_1)} = \dfrac{P(A_1 \mid A^c)\,P(A^c)}{0.2(1+p_A)} = \dfrac{0.2\,(1-p_A)}{0.2\,(1+p_A)} = \dfrac{1-p_A}{1+p_A}$



$P(A \mid A_1)$ = probability they are accident prone given an accident

$P(A^c \mid A_1)$ = probability they are not accident prone given an accident

$p_A = \frac{\%}{100}$ of population that is accident prone

EXAMPLE 3d p. 64

- A lab blood test is 95% effective at identifying a disease when it is actually present.
- "False positive" tests occur for 1% of healthy people tested.
- Suppose 0.5% of the population has the disease.

Q: What is the probability that a person has the disease given that the test result is positive?

let D = event that person tested has the disease

let tpos = event that the test is positive

$$P(D|tpos) = \frac{P(D\, tpos)}{P(tpos)}$$

D^c = event they do not have disease

$$= \frac{P(tpos|D)P(D)}{P(tpos|D)P(D) + P(tpos|D^c)P(D^c)}$$

$$= \frac{(0.95)(0.005)}{(0.95)(0.005) + (0.01)(.995)}$$

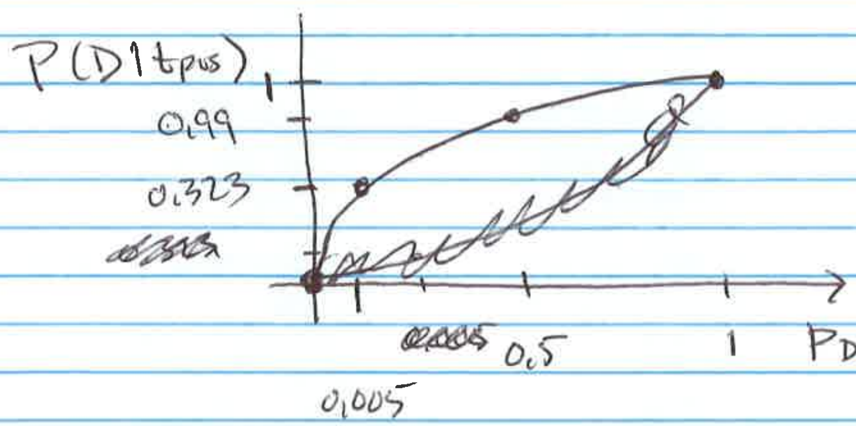
$$= \frac{0.00475}{0.00475 + 0.00995} = \frac{0.00475}{0.0147} \approx 0.323$$

relatively low it would seem..

What if p_D (x100%) is the ~~[illegible]~~ percentage of the population that has the disease?

$$P(D|tpos) = \frac{(0.95)\,p_D}{0.95\,p_D + 0.01\,(1-p_D)}$$

$$= \frac{95\,p_D}{95\,p_D + 1 - p_D} = \frac{95\,p_D}{1 + 94\,p_D}$$



- if $p_D = 0.005$

  $P(D|tpos) = 0.323$

- if $p_D = 0.5$

  $P(D|tpos) = 0.99$

EX

Suppose you have two coins in your pocket (one fair coin and one two-headed coin). Suppose you pick a coin out at random and flip it.

If the coin flip shows H (heads) what is the probability that the coin was 2-headed?

Let 2 = event coin was two headed
F = " " " fair
H = " coin flip was heads
T = " " " " tails

- $$P(2|H) = \frac{P(2H)}{P(H)} = \frac{P(H|2)P(2)}{P(H|2)P(2) + P(H|F)P(F)}$$

$$= \frac{(1)(1/2)}{(1)(1/2) + (1/2)(1/2)} = \frac{1/2}{3/4} = \frac{2}{3}$$

- What if the coin taken out of the pocket was flipped twice and both times showed Heads. What is the probability it is the two-headed coin?

$$P(2|HH) = \frac{P(2HH)}{P(HH)} = \frac{P(HH|2)P(2)}{P(HH|2)P(2) + P(HH|F)P(F)}$$

$$= \frac{(1)(1/2)}{(1)(1/2) + (1/4)(1/2)} = \frac{4}{5}$$

- What if the coin was flipped k times and showed heads k times. What is the probability it is the 2-headed coin?

$$P(2 \mid kH) = \frac{P(kH \mid 2)P(2)}{\underbrace{P(kH \mid 2)P(2) + P(kH \mid F)P(F)}_{= P(kH)}}$$

$$= \frac{(1)(1/2)}{(1)(1/2) + (\frac{1}{2})^k \cdot (1/2)} = \frac{1}{1 + (1/2)^k}$$

as $k \to \infty$ $P(2 \mid kH) \to 1$.

- What if the coin was flipped k+1 times and showed heads k times and tails 1 time. What is the probability it is the two headed coin?

$$P(2 \mid kH1T) = 0$$ ← two headed coin can never be tails.