

Math 351 - Spring 2025: Homework 4

Due: Friday, March 21, 2025 - NOTE: Friday due date - drop off at my office or math office if possible ...

Instructions: Be sure to give explanations to your answers. I'm interested not only in whether you get the correct answer but also how you obtained it and your thought process along the way. Don't just write down a number even if the answer seems obvious.

1. An urn has 1 red ball and 10 green balls. Select 5 balls for the two cases described below and for each case obtain the probability mass function, the expected value $E[X]$ and the variance $\text{Var}(X)$. Here X is a random variable representing the number of red balls selected.
 - (a) The 5 balls are selected from the urn without replacement.
 - (b) The 5 balls are selected from the urn with replacement. That is, select a ball, note its color, then put it back in the urn, and mix up the balls. Repeat this process until you have selected 5 balls.
2. A fair die is rolled until a 6 appears for the 5th time. Let X denote the number of times any number other than 6 appears on the die.
 - (a) Compute the probability mass function of X .
 - (b) Recompute the probability mass function of X if the die is 'loaded' so that the number 6 appears with probability $1/3$.
 - (c) Plot (or make a table of numerical values for) the probability mass function for these two cases. Compare $E[X]$ in both cases.
3. You are at the county fair and you are given the offer to play the following game. For the cost of \$1 you may draw three balls (randomly and without replacement) from an urn containing 3 red balls, 3 green balls, and 5 blue balls. The red balls are worth \$1 each, the green balls are worth -1 \$ each (i.e. the green balls are bad – they have negative value), and the blue balls are worth 0\$ each. Let X be a random variable representing your winnings (only count the value on the balls - the dollar you gave to the person running the game is long gone).

- (a) Compute the probability mass function and compute $E[X]$. Would you play this game?
- (b) If the rules stayed the same with the exception that if you drew three red balls you would get a bonus \$100 (in addition to the \$3 for the three red balls). Would you play now? Compute $E[X]$ to help make an informed decision.