

# Parsing Binomials & Multinomials in Probability

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## Introduction

Note that

- i. first point
- ii. second point
- iii. third point

## 1 Binomial Theorem

The binomial theorem an event most only have 2 outcomes. The binomial theorem accounts for multiple combinations of outcomes.

*Example 1.*

To do this the theorem uses:

**Definition 1** (Factorial  $n!$ ). Count every way to permute a set of  $n$  distinct objects

$$n! = \prod_{i=1}^n i$$

with  $0! = 1$  and  $n \geq 0$ .

Building on factorials, the theorem uses the

**Definition 2** (Binomial Coefficient).  $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

to count combinations of groups of events. The inductive proof of the theorem uses:

*Pascal's Identity.*

$$\begin{aligned} \binom{n-1}{k} + \binom{n-1}{k-1} &= \frac{(n-1)!}{k!(n-1-k)!} + \frac{(n-1)!}{(k-1)!(n-k)!} \\ &= (n-1)! \left[ \frac{n-k}{k!(n-k)!} + \frac{k}{k(n-k)!} \right] \\ &= (n-1)! \frac{n}{k!(n-k)!} \\ &= \frac{n!}{k!(n-k)!} \\ &= \binom{n}{k} \end{aligned}$$

□

*Proof.*

□

## 2 Multinomial Theorem

**Lemma 1.** *We have*

$$\int_0^\pi \sin(3x) \, dx = \frac{2}{3}.$$

*Proof.* A direct computation yields

$$\begin{aligned} \int_0^\pi \sin(3x) \, dx &= \frac{1}{3} \int_0^{3\pi} \sin u \, du, & u = 3x, \\ &= \frac{1}{3} [-\cos u]_0^{3\pi} \\ &= \frac{1}{3} [1 - (-1)] \\ &= \frac{2}{3}. \end{aligned}$$

□

*Remark 1.* This is interesting since...

## 3 Possible Outcomes to Equations