Math 351 Topics for Final Exam
(BIG PICTURE)

R.1

(1) Basic Principles / Combinatorial Analysis
Counting, Permutations, Combinations, ...

(2) Axioms of Probability
Events, Sample Space, $A \cup B$, $A \cap B = $ "AB"

(3) Conditional Probabilities
Bayes' Formula — $P(F_j|E) = \dfrac{P(E|F_j)P(F_j)}{\sum_{i=1}^{n} P(E|F_i)P(F_i)}$
Notion of Independence — $P(A|B) = \dfrac{P(AB)}{P(B)} = \dfrac{P(A)P(B)}{P(B)} = P(A)$

(4) Discrete Random Variable(s)
- types
- probability mass function
- cumulative distribution function

(5) Continuous Random Variables
- types
- probability density function
- cumulative distribution function

DeMoivre-Laplace Limit Thm

(6) Jointly Dist. Random Variables (X, Y) Calc III integrals!

(7) More on Expected Values, Variance, Covariance

(8) Limit Theorems (Central Limit Thm.)

Even with this given information, you should be able to compute

- the expected value of anything
- the variance of anything
- the prob. mass function of anything
- the prob. density function " "
- the marginal pmf or pdf
- the joint pmf or pdf
- the ~~p~~ conditional pmf or pdf
- the cumulative distribution function ("regular" or "joint")
- use Bayes' Formula
- be familiar with limit theorems...

Math 351 - Spring 2025: Exam Materials

Instructions: This material will be provided on the exam. So, you do not need to memorize this stuff but you should certainly be familiar with it and know how to make use of it.

1. I'll provide the Table of numerical values of the function $\Phi(x)$ for the Standard Normal Curve to the left of $X = x$.

2. Some Standard Discrete Random Variables:

- Bernoulli Random Variable [$X = 0$ (failure), $X = 1$ (success)]

$$p(n) = P\{X = n\} = \begin{cases} 1-p & n = 0, \\ p & n = 1 \end{cases}$$

$E[X] = p$, $Var(X) = p(1-p)$.

- Binomial Random Variable [X = number of successes in n trials]

$$p(i) = P\{X = i\} = \binom{n}{i} p^i (1-p)^{n-i}.$$

$E[X] = np$, $Var(X) = np(1-p)$.

- Poisson Random Variable [$X = 0, 1, 2, \ldots$] with parameter $\lambda > 0$

$$p(i) = P\{X = i\} = e^{-\lambda} \frac{\lambda^i}{i!}.$$

$E[X] = \lambda$, $Var(X) = \lambda$.

- Geometric Random Variable [X = number of trials required until success]

$$p(n) = P\{X = n\} = (1-p)^{n-1} p.$$

$E[X] = 1/p$, $Var(X) = (1-p)/p^2$.

- Negative Binomial Random Variable [X = number of trials required until r successes]

$$p(n) = P\{X = n\} = \binom{n-1}{r-1} p^r (1-p)^{n-r}.$$

$E[X] = r/p$, $Var(X) = r(1-p)/p^2$.

- Hypergeometric Random Variable $[X = 0, 1, \ldots, n$ where $n, m \leq N]$

$$p(i) = P\{X = i\} \;=\; \frac{\binom{m}{i}\binom{N-m}{n-i}}{\binom{N}{n}}.$$

$E[X] = nm/N$, $Var(X) = np(1-p)(1-(n-1)/(N-1))$ where $p = m/N$.

3. Some Standard Continuous Random Variables:

- Uniform Random Variable X probability density function

$$f(x) = 1/(b-a) \qquad \text{for } a < x < b$$

and $f(x) = 0$ otherwise. $E[X] = \frac{1}{2}(a+b)$, $Var(X) = \frac{1}{12}(b-a)^2$.

- Exponential Random Variable X with parameter $\lambda > 0$ probability density function

$$f(x) = \lambda e^{-\lambda x} \qquad \text{for } x \geq 0$$

and $f(x) = 0$ otherwise. $E[X] = 1/\lambda$, $Var(X) = 1/\lambda^2$.

- Normal Random Variable X with parameters μ and σ probability density function

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/(2\sigma^2)} \qquad \text{for } -\infty < x < \infty$$

$E[X] = \mu$, $Var(X) = \sigma^2$.

- Standard Normal Random Variable X probability density function

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \qquad \text{for } -\infty < x < \infty$$

$E[X] = 0$, $Var(X) = 1$.

4. Other Results:

- Markov's Inequality (for any random variable X with $X \geq 0$): For any $a > 0$

$$P\{X \geq a\} \leq \frac{E[X]}{a}$$

- Chebyshev's Inequality (for any random variable X finite mean μ and variance σ^2): For any $k > 0$

$$P\{|X - \mu| \geq k\} \leq \frac{\sigma^2}{k^2}$$