GEORGE MASON UNIVERSITY, MATHEMATICAL SCIENCES DEPARTMENT

Advanced Calculus - Math 315

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Summer 2025, Homework 2:

Due date: **June 9** - Real paper submissions



Show all your work. A right answer is a correct result together with the correct steps used to obtain it: Right Answer = Correct Result + Correct Steps

Solve the following problems from the book

Chapter 1

- 9. Prove in detail that if S and T are denumerable, then $S \cup T$ is denumerable.
- 12. Use Mathematical Induction to prove that if the set S has n elements, then $\mathcal{P}(S)$ has 2^n elements.

Chapter 2

- 12. Find all $x \in \mathbb{R}$ that satisfy the inequality 4 < |x+2| + |x-1| < 5.
- 4. Let S be a nonempty bounded set in \mathbb{R} .
 - (a) Let a > 0, and let $aS := \{as : s \in S\}$. Prove that

$$\inf(aS) = a \inf S$$
, $\sup(aS) = a \sup S$.

(b) Let b < 0 and let $bS = \{bs : s \in S\}$. Prove that

$$\inf(bS) = b \sup S$$
, $\sup(bS) = b \inf S$.

- 5. Find the infimum and supremum, if they exist, of each of the following sets.
 - (a) $A := \{x \in \mathbb{R} : 2x + 5 > 0\},$
- (c) $C := \{x \in \mathbb{R} : x < 1/x\},$
- (b) $B := \{x \in \mathbb{R} : x + 2 \ge x^2\},$ (d) $D := \{x \in \mathbb{R} : x^2 2x 5 < 0\}.$