EX Game show Problem

A game show host shows you three doors and tells you there is a car behind one of the doors and goats behind the other 2 doors.

- You get to choose a door, say door #1, but you don't get to open it yet.

  at this stage you have a 1/3 chance of picking the correct door

- The host then shows you another door, say door #2, that does not have a car behind it.
- Next, the host offers you the chance to change your choice to door #3.

Q: Should you change your choice?

- Here's a little more information

  • the answer depends on the 'strategy' used by the host. Suppose the host uses the following strategy:

Host 'Standard' Strategy

- the host knows the position of the car and will never reveal it in step 2. If there are two doors that could be revealed, the door revealed will be selected randomly from the two options.

~~'Random' Strategy~~

~~- The host picks one of the two remaining doors at random (which could possibly result in revealing the car).~~

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Answer:

The situation is: $\boxed{\text{Door 1 has been chosen by the contestant}}$

Let C_i = event that the car is behind door i

R_2 = event that the host reveals that behind door 2 is a goat.

We would like to compute...

... $P(C_1|R_2)$ = probability that the car is behind door 1 given that the host revealed door 2 (goat)

... $P(C_2|R_2)$ = " " " car " door 2 given ... host ... door 2 (goat)

... $P(C_3|R_2)$ = " " " " " door 3 given ... door 2 (goat)

Note: Bayes's Formula says

$$P(R_2) = P(R_2|C_1)P(C_1) + P(R_2|C_2)P(C_2) + P(R_2|C_3)P(C_3)$$

and our knowledge of conditional probabilities says

$$P(C_1|R_2) = \frac{P(C_1R_2)}{P(R_2)}, \quad P(C_2|R_2) = \frac{P(C_2R_2)}{P(R_2)}$$

$$P(C_3|R_2) = \frac{P(C_3R_2)}{P(R_2)}$$

Bayes's

Again, remember door 1 has been selected by the contestant - the influences the behavior of the host. 70.3

So

$$P(C_1|R_2) = \frac{P(C_1 R_2)}{P(R_2)} = \frac{P(R_2|C_1)P(C_1)}{\left(P(R_2|C_1)P(C_1) + P(R_2|C_2)P(C_2) + P(R_2|C_3)P(C_3)\right)}$$

$$P(C_1|R_2) = \frac{(\frac{1}{2})(\frac{1}{3})}{(\frac{1}{2})(\frac{1}{3}) + (0)(\frac{1}{3}) + (1)(\frac{1}{3})} = \frac{\frac{1}{6}}{\frac{1}{2}} = \boxed{\frac{1}{3}}$$

The host can choose door 2 or 3 since they both would hide goats

original probability that door 1 hides car

Door 2 hides the car so host cannot select it.

original probability that door 2 hides car

The host must reveal door 2 since door 3 hides car and door 1 was selected

original probability that door 3 hides car

$$P(C_2|R_2) = \frac{P(R_2|C_2)P(C_2)}{P(R_2|C_1)P(C_1) + P(R_2|C_2)P(C_2) + P(R_2|C_3)P(C_3)}$$

$$= \frac{0 \cdot (\frac{1}{3})}{\frac{1}{2}} = \boxed{0}$$

$$P(C_3|R_2) = \frac{P(R_2|C_3)P(C_3)}{P(R_2|C_1)P(C_1) + P(R_2|C_2)P(C_2) + P(R_2|C_3)P(C_3)}$$

$$= \frac{(1)(\frac{1}{3})}{\frac{1}{2}} = \boxed{2/3}$$

(#)

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So, assuming the host uses this strategy, you should always change to door #3, since given the information the host provided, there is a 2/3 probability that the car is behind door 3 and only a 1/3 probability it is behind door 1.

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New variation on the Monty Hall Problem

N Doors (not 3)
1 Prize
$N-1$ Duds

- You pick a door
- Host reveals $N-2$ Duds
- You get to KEEP PICK or SWITCH PICK...

- Host - <u>standard strategy</u> (will not reveal the prize)

Suppose you pick Door 1

$C_i =$ event prize is behind door i

$R_K =$ event that host reveals $N-2$ duds but door K remains closed. $(K \neq i)$

Bayes's

$$P(R_K) = \sum_{i=1}^{N} P(R_K|C_i)P(C_i)$$

with $P(C_i|R_K) = \dfrac{P(C_i R_K)}{P(R_K)}$

(*)

Initially $P(C_i) = \frac{1}{N}$

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Suppose the Contestant Selects Door 1

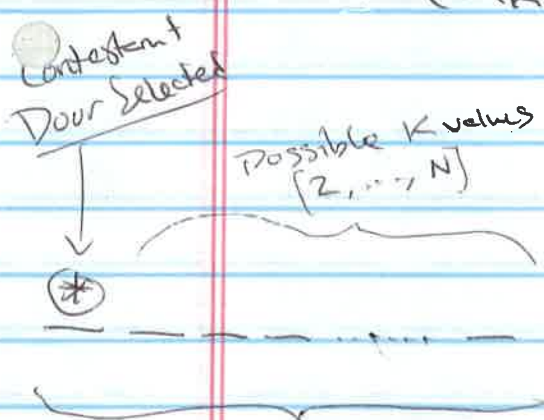
so allowable K values are $k \in [2, N]$

Note:

$$P(C_1|R_K) = \frac{P(C_1 R_K)}{P(R_K)} = \frac{P(R_K|C_1)P(C_1)}{\sum_{i=1}^{N} P(R_K|C_i)P(C_i)}$$

Let's think about

$P(R_K|C_i)$ = probability door K is left closed given prize is behind door i



$$= \begin{cases} \left.\frac{1}{N-1}\right\} & i = 1 \\ \left.\begin{matrix} 0 & K \neq i \\ 1 & K = i \end{matrix}\right\} & i \neq 1 \end{cases}$$

So
- $P(C_1|R_K) = \dfrac{\frac{1}{N-1} \cdot \frac{1}{N}}{\frac{1}{N-1} \cdot \frac{1}{N} + 1 \cdot \frac{1}{N}} = \dfrac{\frac{1}{N-1}}{\frac{1}{N-1} + 1} = \dfrac{1}{1+N-1} = \boxed{\dfrac{1}{N}}$
- $P(C_K|R_K) = \dfrac{1 \cdot \frac{1}{N}}{\frac{1}{N-1} \cdot \frac{1}{N} + 1 \cdot \frac{1}{N}} = \dfrac{1}{\frac{1}{N-1} + 1} = \dfrac{N-1}{1+N-1} = \boxed{\dfrac{N-1}{N}}$

$\Rightarrow$ very much better to switch doors!