## Worksheet 1 Answer Key

We will prove 1., 2., 4., and 6., and disprove 3. and 5.

1.	Put $n = 4$ and observe that $n + 1 = 5$ .
2.	Let $n = 0$ and note that $0 < 7$ .
3.	Fix $x \in \mathbb{R}$ and let $y = x + 1$ . It follows that $x < y$ .
4.	Let $x = 1$ and choose $k \in \mathbb{N}$ . We have $x^k = x$ .
5.	Put $x = 0$ and let $y \in \mathbb{R}$ . Observe that $xy = 0 \neq 1$ .
6.	Let $x = 1$ and fix $y \in \mathbb{R}$ . It follows that $xy = y$ .
7.	Let $P(m, n)$ be the property that $m = n$ .
8.	This is impossible.
	Proof of impossibility. Let $n_0 \in \mathbb{Z}$ be chosen so that $Q(k, n_0)$ is true for all $k \in \mathbb{Z}$ . Fix $m \in \mathbb{Z}$ and put $n = n_0$ . It follows by the condition on $n = n_0$ that $Q(m, n)$ is true.
9.	Yes, it is true.
	<i>Proof.</i> Let $b \in B$ and fix $a \in A$ . We are guaranteed that for every value $a' \in A$ and every value $b' \in B$ the condition $P(a', b')$ is true. In particular, it is true for $a' = a$ and $b' = b$ .
10.	Yes, true.
	<i>Proof.</i> Let $a_0 \in A$ be chosen so that there is a $b_0 \in B$ such that $P(a_0, b_0)$ is true. Put $b = b_0$ and $a = a_0$ and note that $P(a, b)$ is true.