An Introduction to Category Theory

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Introduction

One of math's most abstract fields: Category Theory arose from the habit of representing relations as diagrams on blackboards. While it's origins might be in the corporeal world of chalkboards and erasers, Category Theory is a field of mathematics that ephaises the abstract study of mathematics as form and relation over the applied use of mathematics as calculation.

1 Objects and Arrows

Fundamental to Category Theory are categories.

Definition 1. A category consists of

1. A class of Objects ob(c)
2. A class mor(C) of Arrows
3. A source of Objects to map from
4. A target of Objects to map to

Arrows can be and usually are functions. Objects can be and usually are sets. However, "Category theory is extreme in the sense that it actively discourages us from looking inside the objects. An object in category theory is an abstract nebulous entity." - Page 10 - Category Theory for Programmers - Bartosz Milewski.Therefore a category can be something as abstract as a class or a collection of functions.

Theorem 1. *The set of all sets is a category*

Proof. Let Ω be the set of all sets. Fix $s \in \Omega$. As Ω is a set of sets, it is therefore a class which implies s is an Object. $\square$

Example 1. Use the Monoid A(Z, +) here.

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Definition 2. Lorem ipsum dolor sit amet, consectetur adipisci tempor incidunt ut labore et dolore magna aliqua veniam, quis nostrud exercitation ullamcorpor s commodo consequat.

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Theorem 2. *Duis autem vel eum irrure esse molestiae consequat, vel illum dolore eu fugi et iusto odio dignissim qui blandit praesent luptat exceptur sint occaecat cupiditat non provident, deserunt mollit anim id est laborum et dolor fuga distinct.*

2 Functors and Natural Transformations

Applying Theorem 1 to Definition 2, it follows that ipsum. This was previously established in [**?**] and [**?**, **?**].

Theorem 3. *X is a functor*

Proof. Proof that X is a functor □