

Practice Midterm 1c

Name: _____

For each of the following statements,

- i. express the statement in terms of quantifiers, (*1 pt.*)
- ii. express the negation in terms of quantifiers, (*1 pt.*)
- iii. indicate whether the statement is true or false, (*2 pt.*)
- iv. either prove or disprove the statement (*3 pts.* for logical correctness, *3 pts.* for conventional writing.)

1. If $m, n \in \mathbb{Z}$ and $m \leq n$, then $m^2 \leq n^2$.
2. For every $m, n \in \mathbb{Z}$ satisfying $m < n$, there is an $x \in \mathbb{R}$ satisfying $xm > n$.
3. Let A and B be sets and let $C = A \cup B$. If $x \in C$, then $A \neq \emptyset$ or $B \neq \emptyset$.
4. If A_1, \dots, A_n are sets with $A_1 \cap \dots \cap A_n = \emptyset$, then $A_i \cap A_j = \emptyset$ for some $1 \leq i, j \leq n$.
5. If $f : A \rightarrow B$ is surjective, then for every subset $S \subseteq A$ and every $b \in B$, there is an $s \in S$ with $f(s) = b$.

Bonus Question. (*5 pts.*)

6. Prove that $\lim_{x \rightarrow 0} \frac{1}{x^2} = \infty$.