MATH-300 Andrew Jones

Worksheet 4

Let R be a relation from A to B, let S be a relation from B to C, and let T be a relation from C to D.
Prove the following statements.

1. $IA \circ R = R$
2. $R \circ I_A = R$
3. $(R^{-1})^{-1} = R$
4. $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$
5. $(T \circ S) \circ R = T \circ (S \circ R)$
6. $DomR = RngR^{-1}$
7. $RngR = DomR^{-1}$

For Question 8–10, suppose that $A = B = C$.

8. If R and S are equivalence relations, then $S \circ R$ is an equivalence relation.
9. If R is a partial order, then $R \circ R$ is a partial order.
10. If R and S are partial orders, then it is not generally true that $S \circ R$ is a partial order.

Bonus Questions Give an example of two relations R and S on a set A such that

11. $R \circ S \neq S \circ R$.
12. $S \circ R$ is an equivalence relation, but neither R nor S is an equivalence relation.