Midterm 1

Name: _____

For each of the following statements,

i. express the statement in terms of quantifiers, (*1 pt.*)

ii. express the negation in terms of quantifiers, (*1 pt.*)

iii. indicate whether the statement is true or false, (*2 pt.*)

iv. either prove or disprove the statement (*3 pts.* for logical correctness, *3 pts.* for conventional writing.)

1. For all $k \in \mathbb{Z}$ there is an $\ell \in \mathbb{Z}$ such that $k\ell = 1$.

   i. statement in quantifiers:

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   ii. negation in quantifiers:

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   iii. the statement is: true false (circle one)

   iv. proof or disproof:

2. There is an $x \in \mathbb{R}$ such that $x < y^2$ for all $y \in \mathbb{R}$.

   i. statement in quantifiers:

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   ii. negation in quantifiers:

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   iii. the statement is: true false

   iv. proof or disproof:

3. Let A, B, and C be sets. If $A \subseteq B \subseteq C$, then $(A \cap B) \subseteq (A \cap C)$.

   i. statement in quantifiers:

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   ii. negation in quantifiers:

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   iii. the statement is: true false

   iv. proof or disproof:

4. If $f : A \to B$ and $g : B \to C$ are injective, then $g \circ f$ is injective.

   i. statement in quantifiers:

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   ii. negation in quantifiers:

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   iii. the statement is: true false

   iv. proof or disproof:

5. If $f : A \to B$ is surjective and $g : B \to C$ is injective, then $g \circ f : A \to C$ is bijective.

   i. statement in quantifiers:

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   ii. negation in quantifiers:

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   iii. the statement is: true false

   iv. proof or disproof:

Bonus Question. (*5 pts.*)

6. Prove that $\lim_{x \to \infty} x^2 = \infty$.