An Introduction to Category Theory

Andrew L Jones

Introduction

One of the most abstract fields of mathematics: Category Theory arose from the practice of representing relations as diagrams on blackboards. Although its origins might be in the corporeal world of chalkboards and erasers, Category Theory is a field of mathematics that emphasizes the abstract study of mathematics as form and relation over the applied use of mathematics as calculation. As Lawvere states, it "has long been felt in advanced algebra and topology, namely that the substance of mathematics resides not in Substance (as it is made to seem when $\in$ is the irreducible predicate, with the accompanying necessity of defining all concepts in terms of a rigid elementhood relation) but in Form" [2]. Category theory is a language that allows mathematicians to study math as form.

Nice Introduction... But the first section was confusing; Maybe you could split it up and also maybe clarify what you mean by "form and relation" earlier on, since that's a key point.

1 Objects and Arrows

Fundamental to this study of mathematical form is the concept of a Category.

Definition 1. A category consists of a Class of Objects ob(c); a Class $mor(C)$ of Arrows; a source of Objects to map from $dom(C)$; a target of Objects to map to $cod(C)$. Categories must satisfy three conditions:

1. Arrows must be associative
2. Arrows most compose with other Arrows
3. All Objects must have a left and right identity that is part of the Arrows

1.1 Concrete Categories

Arrows can be and usually are functions. Objects can be and usually are sets. Categories in which every Object is a set are called **small** categories. Categories in which every arrow is id are called **discrete** categories.

Theorem 1. *The Group $A(\mathbb{R}, +, *)$ is a **small discrete** category.*

Proof. Let C be a category with $ob(C) = \{\mathbb{R}\}$ with $dom(C) = \mathbb{R}$ and $cod(C) = \mathbb{R}$. Let $a, b, c \in R$. For $+$ fix $e = 0$ and observe that $a + e = a$ and $e + a = a$. Observe that $(a + b) + c = a + (b + c)$. Likewise, for $*$ fix $e = 1$ and observe $a * 1 = a$ and $1 * a = a$. Observe that $(a * b) * c = a * (b * c)$. Hence, the Group $A(\mathbb{R}, +, *)$ is a category. □

This proof can be better formatted maybe you can use :. so you can make it more defined and it show-cases each math ideals etc.

1.2 Comparing Forests

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$$\begin{tikzcd} A \arrow[r, "f"] \arrow[dr, "h"'] & B \arrow[d, "g"] \\ & C \end{tikzcd}$$

In the above diagram we have $f : A \to B$, $g : B \to C$, $h = g \circ f$. As following f to g has the same result as h, we can claim this diagram commutes.

Woah! This is a really cool diagram that showcases how it works. Nice Example!

Definition 2. A diagram **commutes** when all parallel arrows obtained by composing arrows in the diagram agree.

2 Functors

Categories commute by composition of arrows hence Category Theory is the study of the form of those arrows. Therefore, the field contains many different types of Arrows. Functors are a type of Arrow that maps between Categories while preserving structure.

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Proof. Fix $a, b \in \mathbb{R}$ and $f : x \mapsto ax$ and $g : x \mapsto bx$. Therefore $g \circ f = bax$. Hence, $F(g \circ f) = F(f) \circ F(g)$. Observe that $|bax| = |b|ax||$. $\square$

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[dr, "h"'] & B \arrow[d, "g"] \\ & C \end{tikzcd} \qquad \begin{tikzcd} F(A) \arrow[r, "F(f)"] \arrow[dr, "F(h)"'] & F(B) \arrow[d, "F(g)"] \\ & F(C) \end{tikzcd}$$

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General comments: Very solid for the most part

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A little too vague—spell out exactly what associativity, composability, and identities mean

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For (a), I would try to also give the explicit law: $F(id_X) = id_{\mathrm{F(X)}}$ for every object $X \in A$.

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Proof. Fix $a, b \in \mathbb{R}$ and $f : x \mapsto ax$ and $g : x \mapsto bx$. Therefore $g \circ f = bax$. Hence, $F(g \circ f) = F(f) \circ F(g)$. Observe that $|bax| = |b|ax||$. □

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ob(c) should be italicized for consistency

Needs more specific language, "source" is not clear

typo, most should be "must."

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Descriptor 'concrete' is not used, did you mean discrete?

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this proof does not fit the definition of a discrete category, a discrete category does not have any arithmetic operations.

$\square$

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After copy and pasting the tex.file from the WA2 drafts into overleaf it was giving me an error and wouldn't compile the page. I changed your "math" package to "amsmath" and it compiled, so maybe that's something that you should change.

Make sure you use the package "fullpage" as required by the instructions of WA2. After using fullpage see that more content must be added to meet the 3 page minimum requirement without the bibliography.

Introduction

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The Group $A(R,+,*)$ is a **small discrete** category. Let C be a category with $ob(C) = \{R\}$ with $dom(C) = R$ and $cod(C) = R$. Let $a, b, c \in R$. For $+$ fix $e = 0$ and observe that $a + e = a$ and $e + a = a$. Observe that $(a + b) + c = a + (b + c)$. Likewise, for $*$ fix $e = 1$ and observe $a * 1 = a$ and $1 * a = a$. Observe that $(a * b) * c = a * (b * c)$. Hence, the Group $A(R, +, *)$ is a category.

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References

Just to let you know, I'm not sure that your topic "An Introduction to Category Theory" would recieve fullmarks in the "ambitiousness of topic" category in the rubric.

Make sure that you put your citations in the end bibliography. Also make sure that you define some of your things like definitions, theorems, proofs, etc in the preamble so they don't show up red in the tex.file and transfer over to the pdf.

I enjoy your use of diagrams in your paper, they are visually appealing and help understanding for the reader.

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The introduction is engaging, and the definition of a category is precise.

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The explanation of categories using sets and arrows is very clear, and the diagram is helpful.

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The definitions and proofs in this section are clear and concise.

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General comments: Feedback by Shakeeb Uddin
Quick note that I want to mention before I get started is that all the feedback I give are just mere suggestions.
Your paper looks amazing just the way it is and also helps mine out as well

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Try using bold or emph to make it clear which word you are trying to define.

For the words you have in bold, do you want to try giving them their own definitions?

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2 Functors

Categories commute by composition of arrows hence Category Theory is the study of the form of those arrows. Therefore, the field contains many different types of Arrows. Functors are a type of Arrow that maps between Categories while preserving structure.

Definition 3. Assume that A and B are categories. A Functor F is a map between Categories such that:

1. Each Object from A maps to an Object in B
2. Each Arrow from A maps to an Arrow in B such that:
   (a) The identity arrows Id_A and Id_B hold
   (b) Preserves composition such that $F(f \circ g) = F(f) \circ F(g)$ where $f \in mor(A)$ and $g \in mor(B)$.

Theorem 2. *The function $F : x \mapsto |x|$ is a functor from $\mathbb{Z}$ to $\mathbb{Z}_{\geq \nvdash}$*

Proof. Fix $a, b \in \mathbb{R}$ and $f : x \mapsto ax$ and $g : x \mapsto bx$. Therefore $g \circ f = bax$. Hence, $F(g \circ f) = F(f) \circ F(g)$. Observe that $|bax| = |b|ax||$. ☐

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[dr, "h"'] & B \arrow[d, "g"] \\ & C \end{tikzcd} \qquad \begin{tikzcd} F(A) \arrow[r, "F(f)"] \arrow[dr, "F(h)"'] & F(B) \arrow[d, "F(g)"] \\ & F(C) \end{tikzcd}$$

Therefore the diagram commutes.

Awesome paper so far. I believe that there are a few more things that you would like to add. Don’t forget about your refs!

This is amazing! I love how you were able to do this. For the F(h), you might want to move it a bit away from the arrow that way it doesn’t col-

References

[1] Horst Herrlich. *Category theory : An introduction*. Berlin : Heldermann, 1979.

[2] F. William Lawvere. An elementary theory of the category of sets (long version) with commentary. *Reprints in Theory and Applications of Categories*, 2005.

[3] Bartosz Milewski. *Category Theory for Programmers*. Igal Tabachnik, 2017. Version 0.1.