1 Homework

Prove or Disprove the following statements:

1. $\exists$ n $\in \mathbb{Z}$: n + 1 = 5
   - There exists a number n in the integers such that n + 1 = 5
   - Set n equal to 4
   - Observe that 4 + 1 = 5 $\in \mathbb{Z}$
2. $\forall$ n $\in \mathbb{Z}$: $n > 7$
   - For all numbers n in the integers, n is greater than 7
   - Set n equal to 5
   - Observe that 5 $\in \mathbb{Z}$
3. $\exists$ x $\in \mathbb{R}$: $\forall$ y $\in \mathbb{R}$: $x \geq y$
   - There exists a real number x such that for all real numbers y: n $\geq$ y
   - Fix x to 5.
   - Fix y to 7.
   - Observe that 5, 7 $\in \mathbb{R}$
   - Observe that y $\geq$ n where y = 7 and x = 5
4. $\exists$ x $\in \mathbb{R}$: $\forall$ k $\in \mathbb{N}$: $x^k = x$
   - There exists a real number x such that for all integers y: $x^k = x$
   - Fix x to 1.
   - Observe that $1^y = 1$
5. $\forall$ x $\in \mathbb{R}$: $\exists$ y $\in \mathbb{R}$: xy = 1
   - For all x in the reals there exists a real number y such that xy = 1.
   - Put $y = \frac{1}{x}$
   - Observe that $x * \frac{1}{x} = 1$
   - $\frac{1}{x} \in \mathbb{R}$
6. $\exists$ x $\in \mathbb{R}$: $\forall$ y $\in \mathbb{R}$: xy = y

- There exists a real number x such that for all real numbers xy = y.
- Put x = 1
- Observe that 1*y=y
- $1 \in \mathbb{R}$

7. Give an example of a proposition P for which:

   $\forall$ m $\in \mathbb{Z}$: $\exists$ n $\in \mathbb{Z}$: P(m,n) is true and $\exists$ n $\in \mathbb{Z}$: $\forall$ m $\in \mathbb{Z}$: P(m,n) is false

   - For all integers m there exists an integer n such that P(m,n) is true
   - There exists an integer n such that for all integers m P(m,n) is false
   - Let P = m = n
   - Put n = m
   - Observe that n, m $\in \mathbb{Z}$
   - Observe that for all integers m there exists an integer n such that m = n
   - Observe that there is not a single integer n such that for all integers m m = n

8. Find a Proposition Q for which:

   $\forall$ m $\in \mathbb{Z}$: $\exists$ n $\in \mathbb{Z}$: Q(m,n) is false and $\exists$ n $\in \mathbb{Z}$: $\forall$ m $\in \mathbb{Z}$: Q(m,n) is true

   - For all integers m there exists an integer n such that Q(m,n) is false
   - There exists an integer n such that for all integers m Q(m,n) is true
   - Put Q equal to m ¡ n
   - Put n = (m + 1)
   - Observe that m, (m + 1) $in\mathbb{Z}$
   - Observe that for all integers m there is an integer m - 1 such that m ¡ (m + 1)
   - Put m = 5 and n = 4
   - Observe that 5 ¡ 4 is false
   - Observe that there does not exist a single integer n such that all integers m are greater than it.

9. Is the statement $\forall$ a $\in \mathbb{A}$: $\forall$ b $\in \mathbb{B}$: P(a,b) communative and there for $\forall$ b $\in \mathbb{B}$: $\forall$ a $\in \mathbb{A}$: P(a,b) is also true?

- For all numbers a in A such that for all numbers b in B satisfy P(a,b)
- For all numbers b in B such that for all numbers a in A satisfy P(a,b)
- As the order of the arguments to the proposition does not change, I would assume that switching the order of the for all statements should not effect the value of the proposition.

10. $\exists$ a $\in \mathbb{A}$: $\exists$ b $\in \mathbb{B}$: P(a,b) does it follow that $\exists$ b $\in \mathbb{B}$: $\exists$ a $\in \mathbb{A}$: P(a,b)?

    - There exists a number a in Set A such that there exists a number B in set B that satisifies P(a,b)
    - There exists a number b in Set B such that there exists a number A in Set A that satisfies P(a,b)
    - I would also assume here that exists is communative and what matters is switching the order of the parameters to the Property.