

# An Introduction to Category Theory

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## Introduction

One of math's most abstract fields: Category Theory arose from the habit of representing relations as diagrams on blackboards. While it's origins might be in the corporeal world of chalkboards and erasers, Category Theory is a field of mathematics that ephaises the abstract study of mathematics as form and relation over the applied use of mathematics as calculation.

## 1 Objects and Arrows

Fundamental to Category Theory are categories.

**Definition 1.** A category consists of a class of Objects  $\text{ob}(\mathcal{C})$ ; a class  $\text{mor}(\mathcal{C})$  of Arrows; a source of Objects to map from; a target of Objects to map to. Categories must satisfy three conditions:

1. Arrows must be associative
2. Arrows most compose with other Arrows
3. All Objects must have a left and right identity that is part of the Arrows

Arrows can be and usually are functions. Objects can be and usually are sets. Both arrows and objects are members of classes which allows Category Theory to abstract over collections that are often problematic to define in terms of their objects. Hence Category Theory can reason about large sets resulting in a field that is more about structure than content. "Category theory is extreme in the sense that it actively discourages us from looking inside the objects. An object in category theory is an abstract nebulous entity." - (Page 10 - Category Theory for Programmers - Bartosz Milewski). "Category Theory involves the next level of abstraction-i.e., comparing forests." - (Page 1, Herrlich Cat Theory 2nd).

**Theorem 1.** *The set of all sets is a category*

*Proof.* Let  $\Omega$  be the set of all sets. Observe that  $\Omega$  is a class containing Objects  $U(\Omega)$ . Let  $Id_\Omega$  be  $f : A \rightarrow A$ . □

*Example 1.* Use the Monoid  $A(\mathbb{Z}, +)$  here.

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## 2 Functors and Natural Transformations

This was previously established in [?] and [?, ?].