Worksheet 1 Answer Key

We will prove 1., 2., 4., and 6., and disprove 3. and 5.

1. Put $n = 4$ and observe that $n + 1 = 5$.
2. Let $n = 0$ and note that $0 < 7$.
3. Fix $x \in \mathbb{R}$ and let $y = x + 1$. It follows that $x < y$.
4. Let $x = 1$ and choose $k \in \mathbb{N}$. We have $x^k = x$.
5. Put $x = 0$ and let $y \in \mathbb{R}$. Observe that $xy = 0 \neq 1$.
6. Let $x = 1$ and fix $y \in \mathbb{R}$. It follows that $xy = y$.
7. Let $P(m, n)$ be the property that $m = n$.
8. This is impossible.

   Proof of impossibility. Let $n_0 \in \mathbb{Z}$ be chosen so that $Q(k, n_0)$ is true for all $k \in \mathbb{Z}$. Fix $m \in \mathbb{Z}$ and put $n = n_0$. It follows by the condition on $n = n_0$ that $Q(m, n)$ is true. $\square$

9. Yes, it is true.

   Proof. Let $b \in B$ and fix $a \in A$. We are guaranteed that for every value $a' \in A$ and every value $b' \in B$ the condition $P(a', b')$ is true. In particular, it is true for $a' = a$ and $b' = b$. $\square$

10. Yes, true.

    Proof. Let $a_0 \in A$ be chosen so that there is a $b_0 \in B$ such that $P(a_0, b_0)$ is true. Put $b = b_0$ and $a = a_0$ and note that $P(a, b)$ is true. $\square$