Parsing Binomials & Multinomials in Probability

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Introduction

Note that

i. first point
ii. second point
iii. third point

1 Binomial Theorem

The binomial theorem expresses the expansion of two monomial terms such as $(x+y)^2 = x^2 + 2xy + y^2$. However, in probability the binomial theorem can express the total probability of two independent events.

Example 1. Let an unfair coin be flipped twice with $P(Tails) = 0.3$ and $P(Heads) = 0.7$
We know the probability must sum to 1. In two flips then, $(T+H)^2 = T^2 + 2TH + H^2$. This aligns with the outcomes of TT, TH, HT, and HH for 2 flips. Substituting in the probabilities we have $0.3^2 + 2 * 0.3 * 0.7 + 0.7^2 = 1$.

To compute the expansion of two monomials the theorem uses:

Definition 1 (Factorial n!)**.** Count everyway to permute a set of n distinct objects

$$n! = \prod_{i=1}^{n} i$$

where $0! = 1$ and $n \geq 0$.

Building on factorials, the theorem uses the

Definition 2 (Binomial Coefficient)**.** Count everyway to combine a set of n objects of k size

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Note that the binomial coeffecient can be used to determine the number of combinations of a particular size the binomial theorem produces.

Example 2. 3 flips of a coin yields:

$$\begin{aligned}
&(T+H)^3 = TTT + TTH + THT + HTT + HHT + HTH + THH + HHH \\
&= T^3 + 3T^2H + 3H^2T + H^3 \\
&\binom{3}{3} = 1 \text{ hence } T^3 = TTT \text{ or } H^3 = HHH \\
&\binom{3}{2} = 3 \text{ hence } 3T^2H = TTH + THT + HTT \text{ or } 3H^2T = HHT + HTH + THH
\end{aligned}$$

Many combinations can be simplified by the use of Pascal's identity:

Pascal's Identity.

$$\begin{aligned}
\binom{n-1}{k} + \binom{n-1}{k-1} &= \frac{(n-1)!}{k!(n-1-k)!} + \frac{(n-1)!}{(k-1)!(n-k)!} \\
&= (n-1)![\frac{n-k}{k!(n-k)!} + \frac{k}{k(n-k)!}] \\
&= (n-1)!\frac{n}{k!(n-k)!} \\
&= \frac{n!}{k!(n-k)!} \\
&= \binom{n}{k}
\end{aligned}$$

□

Binomial Theorem. □

2 Multinomial Theorem

Lemma 1. *We have*

$$\int_0^\pi \sin(3x)\,\mathrm{d}x = \frac{2}{3}.$$

Proof. A direct computation yields

$$\begin{aligned}
\int_0^\pi \sin(3x)\,\mathrm{d}x &= \frac{1}{3}\int_0^{3\pi} \sin u\,\mathrm{d}u, \qquad u = 3x, \\
&= \frac{1}{3}\big[-\cos u\big]_0^{3\pi} \\
&= \frac{1}{3}\big[1-(-1)\big] \\
&= \frac{2}{3}.
\end{aligned}$$

□

Remark 1. This is interesting since…

3 Possible Outcomes to Equations