Andrew Jones Math-351

Homework 1

1. How many 5 card poker hands in a Standard Deck are there w/ one pair?

1A How many one pairs in a Standard Deck?

13 unique ranks

4 suits → S, H, C, D

1 pair 2S → [2H, 2C, 2D] = 3 pairs

$\binom{4}{2}$ 2H → [2C, 2D] = 2 pairs

2C → [2D] = 1

$\frac{4!}{(4-2)!2!}$ $\frac{12}{2} = 6$ 3! = 6 total pairs per rank.

13 · 6 = 60 + 18 = 78 total unique pairs

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1B How many 5 card hands w/ 3 cards that are not paired.

First we lose one rank because of the existing one pair so 12 ranks we choose 3 cards

$\binom{12}{3}$ Next we need only one card from each suit so

$\binom{12}{3}\binom{4}{1}^3$

put this together

$\binom{13}{1}\binom{4}{2}\binom{12}{3}\binom{4}{1}^3$

$13 \quad \frac{4!}{2!2!} = \frac{12}{2} = 6 \quad \frac{12!}{9!3!} = \frac{12\cdot 11\cdot 10}{6} \quad \frac{4!}{3!1!} = 4$

$78 \cdot 220 \cdot 4^3 = 1{,}098{,}240$

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2 How many five card poker hands are there w/ one pair if you remove all the diamonds from the deck?

4 suits − 1 suit = 3 suits

$\binom{13}{1}\binom{3}{2} \cdot \binom{12}{3}\binom{3}{1}^3$

$13 \cdot \frac{3!}{1!2!}$ 220 $\frac{3!}{2!1!} = 3$

$13 \cdot 3 \cdot 220 \cdot 3^3$

$39 \cdot 220 \cdot 27 = 231{,}660$

3 How many Full House hands?

3 of a kind + 1 pair

One Pair 3 of a kind

$\binom{13}{1}\binom{4}{2} \cdot \binom{12}{1}\binom{4}{3}$

$13 \cdot 6$ $12 \cdot 4$

$78 \cdot 48 = 3744$

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4

16 kids total

4 kids swim

3 kids bike

3 kids running

2 kids learning

4 kids shoe tie

$$\frac{16!}{4!\,3!\,3!\,2!\,4!}$$

$$\binom{16}{4}\cdot\binom{12}{3}\cdot\binom{9}{3}\binom{6}{2}\binom{4}{4}$$

$$\frac{16\ 15\ 14\ 13\ \cancel{12}\ 11\ 10\ 9\ 8\ 7\ \cancel{6}\ 5}{(4\cdot3\cdot2)\ (\cancel{3\cdot2})\ (\cancel{3\cdot2})\ (\cancel{2})}$$

$$\frac{16\cdot15\cdot14\cdot13\cdot11\cdot10\cdot9\cdot7\cdot5}{3}$$

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$$\frac{16\cdot 15\cdot 14\cdot 13\cdot 11\cdot 10\cdot \overset{3}{\cancel{9}}\cdot 7\cdot 5}{\cancel{3}}$$

$$16\cdot 15\cdot 14\cdot 13\cdot 11\cdot 10\cdot 3\cdot 7\cdot 5 = 1{,}203{,}048{,}000$$

$$\binom{16}{4}\cdot\binom{12}{3}\cdot\binom{9}{3}\binom{6}{2}\binom{4}{4}$$

$$\frac{16!}{\cancel{12!}\,4!}\cdot\frac{\cancel{12!}}{\cancel{9!}\,3!}\cdot\frac{\cancel{9!}}{\cancel{6!}\,3!}\cdot\frac{\cancel{6!}}{\cancel{4!}\,2!}\;\frac{\cancel{4!}}{0!\,4!}$$

4A

$$\boxed{\frac{16!}{4!\,3!\,3!\,2!\,4!} = 504{,}504{,}000}$$

4B

16 Kids total
4 Kids swim
2 Kids bike
3 Kids running
2 Kids learning
4 Kids shoe tie
1 Fixer

$$\frac{16!}{4!\,2!\,3!\,2!\,4!\,1!} =$$

$$\boxed{1{,}513{,}512{,}000}$$

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5 Small party can only hold 4
12 friends ⊃ 3 friends w/ dogs
1 bad dog so 11 elligible friends
2 friends w/ dogs — dog friend A, dog friend B
dogs don't get along.
3 groups here
Group 1 contains 10 friends including

dog friend A — dog friend B — No dogs

$$\binom{1}{1}\binom{9}{3} + \binom{1}{1}\binom{9}{3} + \binom{9}{4}$$

1 df + 9 nondog — 1 df 9 nondog — 9 nondog

$$\frac{9!}{6!3!} + \frac{9!}{6!3!} + \frac{9!}{5!4!}$$

$$\frac{9\cdot 8\cdot 7}{3\cdot 2} + \frac{9\cdot 8\cdot 7}{3\cdot 2} + \frac{9\cdot 8\cdot 7\cdot 6}{4\cdot 3\cdot 2}$$

$$\frac{504}{6} + \frac{504}{6} + \frac{3024}{24}$$

$84 + 84 + 126 = 294$ choices for the party.

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5 total $\binom{11}{4} - \binom{2}{2}\binom{9}{2}$

$$\frac{11!}{7!\,4!} - \frac{9!}{7!\,2!}$$

$$\frac{11\cdot 10\cdot 9\cdot 8}{4\cdot 3\cdot 2} - \frac{9\cdot 8}{2}$$

$$\boxed{330 - 36 = 294}$$

294 Choices for the party.

6 12 rowers 8 seats 1 coxswain

6A order is important (permutations)

$$\frac{12!}{(12-1)!} = 12 \qquad \frac{11!}{(11-8)!} = \frac{11!}{3!}$$

$$\boxed{12 \cdot \frac{11!}{3!} = 79{,}833{,}600}$$

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6 | 6B

$$\frac{12!}{11!} \binom{11}{8}$$

coxswain rowers

$$\frac{12!}{(12-10)!} = 12 \cdot \frac{11 \cdot 10 \cdot 9}{3 \cdot 2 \cdot 1}$$

$$12 \cdot \frac{11 \cdot 10 \cdot 9}{6} = 2 \cdot 11 \cdot 10 \cdot 9 = \boxed{1980}$$

6C

$\binom{12}{9}$ if coxswain & order doesn't matter

$$\frac{12!}{3!9!} \quad \frac{12 \cdot 11 \cdot 10}{3 \cdot 2} = 220$$

$\binom{12}{3}$ Get left behind at the boat house