Practice Midterm 1b

Name: _____

For each of the following statements,

    i. express the statement in terms of quantifiers, (*1 pt.*)

    ii. express the negation in terms of quantifiers, (*1 pt.*)

    iii. indicate whether the statement is true or false, (*2 pt.*)

    iv. either prove or disprove the statement (*3 pts.* for logical correctness, *3 pts.* for conventional writing.)

1. If $m, n \in \mathbb{Z}$ and $m \leq n$, then $m^2 \leq n^2$.

2. There is a $k \in \mathbb{Z}$ such that $kx \notin \mathbb{Z}$ for any $x \in \mathbb{R}$.

3. Let $A \subseteq B \subseteq C$ be sets. If $A \cap C \neq \varnothing$, then $A \cap B \neq \varnothing$.

4. If $f : A \to B$ and $g : B \to C$ are not injective, then $g \circ f : A \to C$ is not injective.

5. If $f : A \to B$ and $g : B \to A$ satisfy $g \circ f = \mathrm{id}_A$ and $f \circ g = \mathrm{id}_B$, then f and g are bijections.

Bonus Question. (*5 pts.*)

6. Let $f : \mathbb{R} \to \mathbb{R}$ be a function. Prove that if $\lim\limits_{x \to \infty} f(x) = \infty$, then $\lim\limits_{x \to \infty} \dfrac{1}{f(x)} = 0$.