

Show all your work. A right answer is a correct result together with the correct steps used to obtain it:
Right Answer = Correct Result + Correct Steps

Solve the following problems from the book

Chapter 1

9. Prove in detail that if S and T are denumerable, then $S \cup T$ is denumerable.
12. Use Mathematical Induction to prove that if the set S has n elements, then $\mathcal{P}(S)$ has 2^n elements.

Chapter 2

12. Find all $x \in \mathbb{R}$ that satisfy the inequality $4 < |x + 2| + |x - 1| < 5$.
4. Let S be a nonempty bounded set in \mathbb{R} .
 - (a) Let $a > 0$, and let $aS := \{as : s \in S\}$. Prove that
$$\inf(aS) = a \inf S, \quad \sup(aS) = a \sup S.$$
 - (b) Let $b < 0$ and let $bS = \{bs : s \in S\}$. Prove that
$$\inf(bS) = b \sup S, \quad \sup(bS) = b \inf S.$$
5. Find the infimum and supremum, if they exist, of each of the following sets.
 - (a) $A := \{x \in \mathbb{R} : 2x + 5 > 0\},$
 - (b) $B := \{x \in \mathbb{R} : x + 2 \geq x^2\},$
 - (c) $C := \{x \in \mathbb{R} : x < 1/x\},$
 - (d) $D := \{x \in \mathbb{R} : x^2 - 2x - 5 < 0\}.$