

Worksheet 3

Let A , B , and C be sets. Prove or disprove the following statements.

1. If $A \cap B = \emptyset$ and $B \cap C = \emptyset$, then $A \cap C = \emptyset$

Proof. Assume that $A = \{a\}$ and $C = \{a\}$ and $B = \{b\}$ It follows that:
 $A \cap B = \emptyset$ and $B \cap C = \emptyset$ however $A \cap C = \{a\}$ There for: $\exists A : \exists C :$
 $A \cap C \neq \emptyset$ and $A \cap B = \emptyset$ and $B \cap C = \emptyset$ \square

2. If $A \not\subseteq B$ and $B \not\subseteq C$, then $A \not\subseteq C$

Proof. Assume that $A = \{a\}$ and $C = \{a, c\}$ and $B = \{b\}$ It follows that:
 $A \not\subseteq B$ and $B \not\subseteq C$ however $A \subseteq C$ There for: $\exists A : \exists C : A \subseteq C$ and
 $A \not\subseteq B$ and $B \not\subseteq C$ \square

3. If $A \subseteq \emptyset$, then $a = \emptyset$
4. If $A \subseteq C$ and $B \subseteq C$, then $A \cap B \subseteq C$
5. If $f : A \rightarrow B$ is injective and $g : B \rightarrow C$ is injective, then $g \circ f : A \rightarrow C$ is injective.
6. If $f : A \rightarrow B$ is surjective and $g : B \rightarrow C$ is surjective, then $g \circ f : A \rightarrow C$ is surjective
7. Give an example of a function $f : A \rightarrow A$ that is injective but not surjective.
8. Give an example of a function $g : A \rightarrow A$ that is surjective but not injective.
9. Let $f : A \rightarrow B$ and $g : B \rightarrow A$. If $g \circ f = id_a$, then both f and g are bijections.
10. If $f : A \rightarrow A$ is surjective, and if A is a finite set, then f is injective.
11. If $f : A \rightarrow A$ satisfies the property that $f \circ f = id_a$ then f is a bijection.