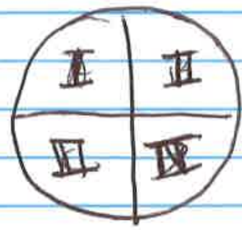
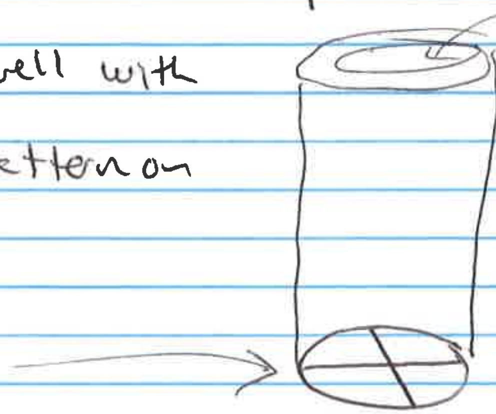
Intuition?

EX

Consider a related problem: Drop 4 coins down a deep wishing well with the following grid pattern on the bottom





Assume the probability that a coin lands in Quad i is $1/4$ (each area is $1/4$ total area)

What is the probability that the four coins land in different quadrants (assume each coin drop is an independent event).

$P(A_i)$ = probability that coin i lands in an "unoccupied" quadrant (given that $i-1$ quadrants are occupied)

First coin: $P(A_1) = 1$

2nd coin: $P(A_2) = \frac{3}{4}$ (one occupied)

$P(A_3) = 1/2$ (two occupied)

$P(A_4) = 1/4$

$$P(A_1)\cdot P(A_2)\,P(A_3)\cdot P(A_4) = \frac{3}{4\cdot 4\cdot 2} = \frac{3}{32}$$

$\boxed{= 0.09375}$

i.e. this is also

$P = \frac{4!}{4^4}$ ← # of ways for the four coins to be separate

← total number of 4 coin position outcomes

* Why is this less likely than the four aces in 4 piles problem?

intuition?

EX

Consider another related problem.

Consider a nonstandard deck of cards with $4N$ cards including four aces. None of the other cards are aces. Divide this deck into 4 piles of N cards. What is the probability that ~~three are~~ each pile has an ace?

Let's use the same definitions of E_1, E_2, E_3, E_4 (see notes, p. 62).

$$P(E_1E_2E_3E_4) = P(E_4|E_1E_2E_3) \cdot P(E_3|E_2E_1)P(E_2|E_1)P(E_1)$$

$$P(E_1) = 1$$

$$P(E_2|E_1) = 1 - \underbrace{\frac{N-1}{4N-1}}_{\text{probability that } A_2 \text{ is in pile 1 given } A_1 \text{ is in pile 1.}}$$

$N-1$ — # of spots left in pile 1

$4N-1$ — # of cards that could go in pile

$$P(E_3|E_1E_2) = 1 - \frac{2(N-1)}{4N-2} = 1 - \text{prob. that } A_3 \text{ is in piles 1 or 2 given } A_1 \text{ in pile 1, } A_2 \text{ in pile 2}$$

$$P(E_4|E_1E_2E_3) = 1 - \frac{3(N-1)}{4N-3} = 1 - \text{prob. } A_4 \text{ is in piles 1,2,3 given } A_1 \text{ in pile 1, } A_2 \text{ in pile 2, } A_3 \text{ in pile 3}$$

So

$$P(E_1E_2E_3E_4) = \left(1 - \frac{3(N-1)}{4N-3}\right)\left(1 - \frac{2(N-1)}{4N-2}\right)\left(1 - \frac{N-1}{4N-1}\right)\cdot 1$$

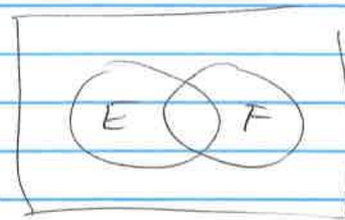
$$= \frac{N}{(4N-3)} \cdot \frac{2N}{(4N-2)} \cdot \frac{3N}{(4N-1)}$$

	N	P(all aces separate)
(4 cards)	1	1
(8 cards)	2	$\frac{2}{5}\cdot\frac{4}{6}\cdot\frac{6}{7} = \frac{8}{35} \approx 0.229$
(12 cards)	3	$\frac{3}{\cancel{9}}\cdot\frac{6}{10}\cdot\frac{\cancel{9}}{11} = \frac{18}{110} = \frac{9}{55} \approx 0.164$
⋮		
(52 cards)	13	$\frac{13}{49}\cdot\frac{26}{50}\cdot\frac{39}{51} \approx 0.105498\ldots$
(104 cards)	26	$\frac{26}{101}\cdot\frac{52}{102}\cdot\frac{78}{103} \approx 0.09938$
↓ ∞	$N \to \infty$ ↓	$\frac{1}{4}\cdot\frac{2}{4}\cdot\frac{3}{4} = \frac{3}{32} = 0.09375$

3.3 Bayes's Formula

Let E and F be events. Then

$$E = EF \cup EF^c$$



since EF and EF^c are mutually exclusive events

by Axiom 3

$$P(E) = P(EF) + P(EF^c)$$

$$= P(E|F) \cdot P(F) + P(E|F^c)P(F^c)$$

but $P(F^c) + P(F) = 1$ so

$$\boxed{P(E) = P(E/F)P(F) + P(E|F^c)(1 - P(F))}$$

EX

Recall Celine.

$P(F) = 1/2$ probability she takes French
$P(C) = 1/2$ probability she takes Chem. } coin flip

$P(A|C) = \frac{2}{3}$ ← given she takes Chem probability of an A

$P(A|F) = 1/2$ ← given she takes French probability of an A.

Suppose after the semester she tells you she got an A.

What is the probability that she took chemistry?

$= P(C|A)$ = probability she took Chem given she got an A (different from $P(A|C)$).

$$= \frac{P(AC)}{P(A)} = \frac{P(A|C)P(C)}{P(AC) + P(AC^c)}$$

$$P(C|A) = \frac{P(A|C)P(C)}{P(A|C)P(C) + P(A|C^c)P(C^c)}$$

note $C^c = F$

so

$$P(C|A) = \frac{P(A|C)P(C)}{P(A|C)P(C) + P(A|F)P(F)}$$

$$= \frac{(2/3)(1/2)}{(2/3)(1/2) + (1/2)(1/2)} = \frac{\frac{2}{3}}{\frac{2}{3} + \frac{1}{2}} = \frac{1}{1 + 3/4} = \frac{4}{7}$$

probability she took chemistry

$\boxed{\text{Ch. 3 Problem 3.18}}$

46% of voters in a city classify themselves as <u>independents</u>.
30% of " " " " " " " <u>liberals</u>
24% " " " " " <u>conservatives</u>.

In a recent election...

35% of independents voted
62% of liberals voted
58% of conservatives voted.

A voter is chosen at random. Given that this person voted...

what is the probability that he/she is

a) independent? b) liberal? c) conservative?

d) What percentage of voters participated in the election?

a) ans. $P(I|V)$ = probability of I = indep. given that they voted, V.

$$P(I|V) = \frac{P(IV)}{P(V)}$$

$$P(V) = P(IV) + P(LV) + P(CV)$$

(from $V = \underbrace{IV \cup LV \cup CV}_{\text{union of mutually exclusive events}}$)

so $$P(I|V) = \frac{P(IV)}{P(IV) + P(LV) + P(CV)}$$

$P(IV) = P(V|I)P(I) = (.35)(.46) = 0.161$
$P(LV) = P(V|L) \cdot P(L) = (.62)(.3) = 0.186$
$P(CV) = P(V|C)\,P(C) = (.58)(.24) = 0.1392$
0.4862

$$P(I|V) = \frac{0.161}{0.161 + 0.186 + 0.1392} = \frac{0.161}{0.4862} \approx .331$$

b) $P(L|V) = \dfrac{P(LV)}{P(V)} = \dfrac{0.186}{0.4862} \approx .383$

c) $P(C|V) = \dfrac{P(CV)}{P(V)} = \dfrac{0.1392}{0.4862} \approx .286$

d) $P(V) = 0.4862$

Generalized Version of Bayes's Formula.

Suppose events $F_1, F_2, \ldots, F_n$ are mutually exclusive (i.e. $F_iF_j = \phi$ if $i \neq j$) and together they make up the whole sample space. So

$$\bigcup_{i=1}^{n} F_i = S$$

Then we can write

$$E = EF_1 \cup EF_2 \cup \ldots \cup EF_n$$

~~and then~~, since EF_i and EF_j are mutually exclusive for $i \neq j$

$$P(E) = P(EF_1) + P(EF_2) + \ldots + P(EF_n)$$

$$= \sum_{i=1}^{n} P(EF_i)$$

$$\boxed{P(E) = \sum_{i=1}^{n} P(E|F_i)P(F_i)}$$

Ross ch. 3.3 p. 69 eq. (3.4)