Parsing Binomials & Multinomials in Probability

Andrew Jones

Introduction

Consider expanding the introduction to provide a more comprehensive overview of the document's purpose and to motivate the topic.

The use of the Binomial and Multinomial theorems in Probability can often not be intuitively obviously.

Rephrase for better flow: For example, "The application of Binomial and Multinomial theorems in probability can often lack intuitive clarity."

This text intends to highlight the relations between computing polynomials from binomials with combinations, total probability, and probability mass functions.

It would be helpful to briefly explain what the text intends to highlight. For example: "This text aims to clarify the connection between polynomial expansion of binomials, combinations, total probability, and probability mass functions."

1 Binomial Theorem

The binomial theorem expresses the expansion of two monomial terms such as $(x+y)^2 = x^2 + 2xy + y^2$. However, in probability the binomial theorem can express the probability of all combinations of two independent events.

Example 1. Let an unfair coin be flipped twice with $P(Tails) = 0.3$ and $P(Heads) = 0.7$
We know the probability must sum to 1. In two flips then, $(T+H)^2 = T^2 + 2TH + H^2$. This aligns with the outcomes of TT, TH, HT, and HH for 2 flips. Substituting in the probabilities we have $0.3^2 + 2 * 0.3 * 0.7 + 0.7^2 = 1$.

In the example, it would be helpful to explicitly state how each term in the expansion corresponds to a specific outcome $(e.g., T^2 = TT, 2TH = TH + HT, H^2 = HH)$ to make the connection clearer.

To compute the expansion of two monomials the theorem uses:

Definition 1 (Factorial n!)**.** Count everyway to permute a set of n distinct objects

$$n! = \prod_{i=1}^{n} i$$

where $0! = 1$ and $n \geq 0$.

Building on factorials, the theorem uses the

Definition 2 (Binomial Coefficient)**.** Count everyway to combine a set of n objects of k size

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Note that the binomial coeffecient can be used to determine the number of combinations of a particular size the binomial theorem produces.

Example 2. 3 flips of a coin yields:

$$
\begin{aligned}
&(T+H)^3 = TTT + TTH + THT + HTT + HHT + HTH + THH + HHH \\
&= T^3 + 3T^2H + 3H^2T + H^3 \\
&\binom{3}{3} = 1 \text{ hence } T^3 = TTT \text{ or } H^3 = HHH \\
&\binom{3}{2} = 3 \text{ hence } 3T^2H = TTH + THT + HTT \text{ or } 3H^2T = HHT + HTH + THH
\end{aligned}
$$

Remark 1. The total number of combinations of the binomial is 2^n.
$2^3 = T^3 + 3T^2H + 3H^2T + H^3 = 1 + 3 + 3 + 1 = 8$

Many combinations can be simplified by the use of Pascal's identity:

Pascal's Identity.

$$
\begin{aligned}
\binom{n-1}{k} + \binom{n-1}{k-1} &= \frac{(n-1)!}{k!(n-1-k)!} + \frac{(n-1)!}{(k-1)!(n-k)!} \\
&= (n-1)![\frac{n-k}{k!(n-k)!} + \frac{k}{k(n-k)!}] \\
&= (n-1)!\frac{n}{k!(n-k)!} \\
&= \frac{n!}{k!(n-k)!} \\
&= \binom{n}{k}
\end{aligned}
$$

□

Binomial Theorem. Assume that $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$ and by the the definition of the binomial coeffecient $n \geq 0$. For the case $(n = 0) \Rightarrow (a+b)^0 = 1$. For the case $n \geq 0$.

$$
\begin{aligned}
(a+b)^{n+1} &= (a+b)(a+b)^n = (a+b)\sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k} \\
&= \sum_{k=0}^{n} \binom{n}{k} a^{k+1} n n - k + \sum_{k=o}^{n} \binom{n}{k} a^k b^{n-k+1} \\
m &= k+1 \\
&= \sum_{m=1}^{n+1} \binom{n}{m-1} a^m b^{n-m+1} + \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k+1} \\
&= b^{n+1} + \sum_{k=1}^{n} [\binom{n}{k} + \binom{n}{k-1}] a^k b^{n-k+1} + a^{n+1} \\
&= b^{n+1} + \sum_{k=1}^{n} \binom{n+1}{k} a^k b^{n-k+1} + a^{n+1} \\
&= \sum_{k}^{n+1} \binom{n+1}{k} a^k b^{n+1-k}
\end{aligned}
$$

$\square$

2 Multinomial Theorem

While the binomial theorem works for 2 independent events, the multinomial theorem generalizes to any number of groups/events. It uses the multinomial coeffecient.

Rephrase for clarity: "While the binomial theorem applies to two independent events, the multinomial theorem generalizes this to any number of groups or events."

Definition 3 (Multinomial Coeffecient)**.**

$$\binom{N}{n_1..n_r} = \frac{N!}{n_1!..n_r!}$$

Where n_1 to n_r are different group sizes.

Example 3. For 13 items we want to know how many combinations of 5, 5, and 3 can be made

$$\begin{aligned}\binom{13}{5,5,3} &= \binom{13}{5}\binom{8}{5}\binom{3}{3} \\ &= \frac{13!}{5!(13-5)!}\frac{8!}{5!(8-5)!}\frac{3!}{3!(3-3)!} \\ &= \frac{13!}{5!5!3!}\end{aligned}$$

We now combine the multinomial coeffecients with monomials raised to a power to get:

Definition 4 (multinomial theorem)**.**

$$(x_1 + ... + x_r)^n = \sum_{(n1,...,nr)} \binom{n}{n1,...,nr} x_1^{n1}...x_r^{nr}$$
$$\text{where } n1 + ... + nr = n$$

Multinomial Theorem.

The proof for the Multinomial Theorem is incomplete. It only shows the base case and suggests using the binomial theorem. A full inductive proof is needed.

Fix $r = 1$ and observe that $(x_1)^n = \sum_{(n1=1)} \binom{n}{n1=1} x_1^{n1=1} = nx_1$
Fix $m = r+1$ and $(x_r + x_{r+1})^n = \sum_{(r,...,r+1)} \binom{n}{x_1,...,x_{r+1}} x_1^r x_{r+1}^{r+1}$ and observe that this is provable using the binomial theorem. $\square$

3 Possible Outcomes to Equations

Parsing Binomials & Multinomials in Probability

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Introduction

The use of the Binomial and Multinomial theorems in Probability can often not be intuitively obviously. This text intends to highlight the relations between computing polynomials from binomials with combinations, total probability, and probability mass functions.

Replace with 'intuitive'

'We intend'

1 Binomial Theorem

The binomial theorem expresses the expansion of two monomial terms such as $(x+y)^2 = x^2 + 2xy + y^2$. However, in probability the binomial theorem can express the probability of all combinations of two independent events.

This is a theorem, opportunity to have a proof for the most fundamental part of paper.

Example 1. Let an unfair coin be flipped twice with $P(Tails) = 0.3$ and $P(Heads) = 0.7$
We know the probability must sum to 1. In two flips then, $(T+H)^2 = T^2 + 2TH + H^2$. This aligns with the outcomes of TT, TH, HT, and HH for 2 flips. Substituting in the probabilities we have $0.3^2 + 2 * 0.3 * 0.7 + 0.7^2 = 1$.

To compute the expansion of two monomials the theorem uses:

Definition 1 (Factorial n!)**.** Count everyway to permute a set of n distinct objects

Use math mode for variables

$$n! = \prod_{i=1}^{n} i$$

where $0! = 1$ and $n \geq 0$.

How does this formula count every possible permutation? Could just define a factorial and make a proof that it can be used that way. Consider rewriting definition to just say: "The *factorial* of a nonnegative integer n is given by $n! = \prod_{i=1}^{n} i$", then following with a theorem or proposition about how the number of permutations of a set of n distinct objects is equal to $n!$.

Building on factorials, the theorem uses the

Definition 2 (Binomial Coefficient)**.** Count everyway to combine a set of n objects of k size

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Note that the binomial coeffecient can be used to determine the number of combinations of a particular size the binomial theorem produces.

Similar issue to definition above: states a theorem and the formula for a definition without proving they are connected.

Example 2. 3 flips of a coin yields:

$$(T+H)^3 = TTT + TTH + THT + HTT + HHT + HTH + THH + HHH$$
$$= T^3 + 3T^2H + 3H^2T + H^3$$
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Remark 1. The total number of combinations of the binomial is 2^n.
$2^3 = T^3 + 3T^2H + 3H^2T + H^3 = 1 + 3 + 3 + 1 = 8$

Many combinations can be simplified by the use of Pascal's identity:

Should state what Pascal's Identity actual is as a lemma before proving it.

Pascal's Identity.

$$\begin{aligned}\binom{n-1}{k} + \binom{n-1}{k-1} &= \frac{(n-1)!}{k!(n-1-k)!} + \frac{(n-1)!}{(k-1)!(n-k)!} \\ &= (n-1)![\frac{n-k}{k!(n-k)!} + \frac{k}{k(n-k)!}] \\ &= (n-1)!\frac{n}{k!(n-k)!} \\ &= \frac{n!}{k!(n-k)!} \\ &= \binom{n}{k}\end{aligned}$$

□

Don't replace the word 'proof' from the proof with what you are proving.

Binomial Theorem. Assume that $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$

Where does this formula come from? This proof would only say that if $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$, then $(a+b)^{n+1} = \sum_{k}^{n+1} \binom{n+1}{k} a^k b^{n+1-k}$. Should prove the statement as a lemma before using it in a theorem proof.

and by the the definition of the binomial coeffecient $n \geq 0$. For the case $(n = 0) \Rightarrow (a+b)^0 = 1$. For the

State what the binomial theorem is before proving case $n \geq 0$.

$$
\begin{aligned}
(a+b)^{n+1} &= (a+b)(a+b)^n = (a+b)\sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k} \\
&= \sum_{k=0}^{n}\binom{n}{k}a^{k+1}nn-k + \sum_{k=o}^{n}\binom{n}{k}a^k b^{n-k+1} \\
m &= k+1 \\
&= \sum_{m=1}^{n+1}\binom{n}{m-1}a^m b^{n-m+1} + \sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k+1} \\
&= b^{n+1} + \sum_{k=1}^{n}[\binom{n}{k} + \binom{n}{k-1}]a^k b^{n-k+1} + a^{n+1} \\
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&= \sum_{k}^{n+1}\binom{n+1}{k}a^k b^{n+1-k}
\end{aligned}
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2 Multinomial Theorem

While the binomial theorem works for 2 independent events, the multinomial theorem generalizes to any number of groups/events. It uses the multinomial coeffecient.

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$$\binom{N}{n_1..n_r} = \frac{N!}{n_1!..n_r!}$$

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\begin{aligned}
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We now combine the multinomial coeffecients with monomials raised to a power to get:

Definition 4 (multinomial theorem)**.**

$$
\begin{aligned}
(x_1 + ... + x_r)^n = \sum_{(n1,...,nr)}\binom{n}{n1,...,nr}x_1^{n1}...x_r^{nr} \\
\text{where } n1 + ... + nr = n
\end{aligned}
$$

Use \begin{theorem}rather than definition. The proof comes right afterwards.

Multinomial Theorem. Fix $r = 1$ and observe that $(x_1)^n = \sum_{(n1=1)} \binom{n}{n1=1} x_1^{n1=1} = nx_1$
Fix $m = r + 1$ and $(x_r + x_{r+1})^n = \sum_{(r,\ldots,r+1)} \binom{n}{x_1,\ldots,x_{r+1}} x_1^r x_{r+1}^{r+1}$ and observe that this is provable using the binomial theorem. $\square$

3 Possible Outcomes to Equations

Empty section. Already have three sections so this can just be deleted.

Parsing Binomials & Multinomials in Probability

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There are a lot of newcommands, renewcommands, and newtheorems at the beginning of your tex.file that are unnecessary to have because you don't use them in your paper.

Introduction

The use of the Binomial and Multinomial theorems in Probability can often not be intuitively obviously. This text intends to highlight the relations between computing polynomials from binomials with combinations, total probability, and probability mass functions.

I believe that "theorems" should be capitalized and "Probability" should be lowercased. Also I think it the sentence would flow better if you said "intuitively obvious".

According to the AMS style guide section 3.1 the text that follows a section head should have an indent.

1 Binomial Theorem

The binomial theorem expresses the expansion of two monomial terms such as $(x + y)^2 = x^2 + 2xy + y^2$. However, in probability the binomial theorem can express the probability of all combinations of two independent events.

Example 1. Let an unfair coin be flipped twice with $P(Tails) = 0.3$ and $P(Heads) = 0.7$
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Definition 2 (Binomial Coefficient)**.** Count everyway to combine a set of n objects of k size

I would say "of size k" instead of "k size".

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Add a "." after the formula above.

Note that the binomial coeffecient can be used to determine the number of combinations of a particular size the binomial theorem produces.

Example 2. 3 flips of a coin yields:

$$(T+H)^3 = TTT + TTH + THT + HTT + HHT + HTH + THH + HHH$$
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Add a "." after the THH above.

Remark 1. The total number of combinations of the binomial is 2^n.
$2^3 = T^3 + 3T^2H + 3H^2T + H^3 = 1 + 3 + 3 + 1 = 8$

Many combinations can be simplified by the use of Pascal's identity:

Pascal's Identity.

$$\begin{aligned}\binom{n-1}{k} + \binom{n-1}{k-1} &= \frac{(n-1)!}{k!(n-1-k)!} + \frac{(n-1)!}{(k-1)!(n-k)!} \\ &= (n-1)![\frac{n-k}{k!(n-k)!} + \frac{k}{k(n-k)!}] \\ &= (n-1)!\frac{n}{k!(n-k)!} \\ &= \frac{n!}{k!(n-k)!} \\ &= \binom{n}{k}\end{aligned}$$

□

Add a "." at the end of your proof after $\binom{n}{k}$.

Binomial Theorem. Assume that $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$ and by the the definition of the binomial coeffecient $n \geq 0$. For the case $(n = 0) \Rightarrow (a+b)^0 = 1$. For the case $n \geq 0$.

$$\begin{aligned}(a+b)^{n+1} &= (a+b)(a+b)^n = (a+b)\sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k} \\ &= \sum_{k=0}^{n}\binom{n}{k}a^{k+1}nn-k + \sum_{k=o}^{n}\binom{n}{k}a^k b^{n-k+1} \\ m &= k+1 \\ &= \sum_{m=1}^{n+1}\binom{n}{m-1}a^m b^{n-m+1} + \sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k+1} \\ &= b^{n+1} + \sum_{k=1}^{n}[\binom{n}{k} + \binom{n}{k-1}]a^k b^{n-k+1} + a^{n+1} \\ &= b^{n+1} + \sum_{k=1}^{n}\binom{n+1}{k}a^k b^{n-k+1} + a^{n+1} \\ &= \sum_{k}^{n+1}\binom{n+1}{k}a^k b^{n+1-k}\end{aligned}$$

□

Add a "." at the end of your proof

2 Multinomial Theorem

While the binomial theorem works for 2 independent events, the multinomial theorem generalizes to any number of groups/events. It uses the multinomial coeffecient.

Definition 3 (Multinomial Coeffecient)**.**

$$\binom{N}{n_1..n_r} = \frac{N!}{n_1!..n_r!}$$

Where n_1 to n_r are different group sizes.

I believe that "Where" should not be capitalized and you shouldn't start you sentence with the math formula above. Consider giving more details to the Multinomial Coeffecient before just saying what the formula is after the "Definition 3". Same critique for "Defintion 4".

Example 3. For 13 items we want to know how many combinations of 5, 5, and 3 can be made

$$\begin{aligned}\binom{13}{5,5,3} &= \binom{13}{5}\binom{8}{5}\binom{3}{3} \\ &= \frac{13!}{5!(13-5)!}\frac{8!}{5!(8-5)!}\frac{3!}{3!(3-3)!} \\ &= \frac{13!}{5!5!3!}\end{aligned}$$

Add a "." at the end of your example after $\frac{13!}{5!5!3!}$.

We now combine the multinomial coeffecients with monomials raised to a power to get:

Definition 4 (multinomial theorem)**.**

$$(x_1 + ... + x_r)^n = \sum_{(n1,...,nr)} \binom{n}{n1,...,nr} x_1^{n1}...x_r^{nr}$$
$$\text{where } n1 + ... + nr = n$$

Multinomial Theorem. Fix $r = 1$ and observe that $(x_1)^n = \sum_{(n1=1)} \binom{n}{n1=1} x_1^{n1=1} = nx_1$
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3 Possible Outcomes to Equations

Obviously need to add information to this section.

Parsing Binomials & Multinomials in Probability

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Introduction

The use of the Binomial and Multinomial theorems in Probability can often not be intuitively obviously. This text intends to highlight the relations between computing polynomials from binomials with combinations, total probability, and probability mass functions.

probability should be lowercase

1 Binomial Theorem

The binomial theorem expresses the expansion of two monomial terms such as $(x + y)^2 = x^2 + 2xy + y^2$. However, in probability the binomial theorem can express the probability of all combinations of two independent events.

Example 1. Let an unfair coin be flipped twice with $P(Tails) = 0.3$ and $P(Heads) = 0.7$
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display mode for the math here could clean up the example and make it more easily read

$(T + H)^2 = T^2 + 2TH + H^2$. This aligns with the outcomes of TT, TH, HT, and HH for 2 flips. Substituting in the probabilities we have

here as well, display mode for the math here could clean up the example and make it more easily read

$0.3^2 + 2 * 0.3 * 0.7 + 0.7^2 = 1$.

To compute the expansion of two monomials the theorem uses:

Make sure to have consistent use of tabs and indentation

Definition 1 (Factorial n!)**.** Count everyway to permute a set of n distinct objects

$$n! = \prod_{i=1}^{n} i$$

where $0! = 1$ and $n \geq 0$.

Building on factorials, the theorem uses the

is this sentence finished or is it leading to the next definition? it is a little unclear here what the sentence does.

Definition 2 (Binomial Coefficient)**.** Count everyway to combine a set of n objects of k size

overall tex comment, consider using spaces between e.g. the tex code so the tex is more easily read. separating the definitions and paragraphs can help clarify sections so you can easily see where it is within the code

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Note that the binomial coeffecient can be used to determine the number of combinations of a particular size the binomial theorem produces.

indentation issue

Example 2. 3 flips of a coin yields:

i would rewrite this to align these equations, you are using the align function but the equations are not really aligned in a logical manner. maybe rewriting the equations since the "or" is throwing me off in reading them. not sure how they should be centered.

$$(T+H)^3 = TTT + TTH + THT + HTT + HHT + HTH + THH + HHH$$
$$= T^3 + 3T^2H + 3H^2T + H^3$$
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consistent indent

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&= b^{n+1} + \sum_{k=1}^{n}\binom{n+1}{k}a^k b^{n-k+1} + a^{n+1} \\
&= \sum_{k}^{n+1}\binom{n+1}{k}a^k b^{n+1-k}
\end{aligned}$$

□

the second section here the tex is more organized and clearly read since you used more blank lines between sections like after definition 3 and before the example. i can look at it and see which part of the code does what, instead of trying to look through a big block of text.

2 Multinomial Theorem

While the binomial theorem works for 2 independent events, the multinomial theorem generalizes to any number of groups/events. It uses the multinomial coeffecient.

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tab issue

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probability, the binomial...

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$$n! = \prod_{i=1}^{n} i$$

where $0! = 1$ and $n \geq 0$.

"where $0! = 1$" is more of a note as it is has yet to be used. You could replace 0 by n and define what $n!$ is.

Building on factorials, the theorem uses the

Definition 2 (Binomial Coefficient)**.** Count everyway to combine a set of n objects of k size

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Note that the binomial coefficient can be used to determine the number of combinations of a particular size the binomial theorem produces.

Can use the remark theorem style to make "note" look like "Example 2." below.

Example 2. 3 flips of a coin yields:

$$\begin{aligned}
&(T+H)^3 = TTT + TTH + THT + HTT + HHT + HTH + THH + HHH \\
&= T^3 + 3T^2H + 3H^2T + H^3 \\
&\binom{3}{3} = 1 \text{ hence } T^3 = TTT \text{ or } H^3 = HHH \\
&\binom{3}{2} = 3 \text{ hence } 3T^2H = TTH + THT + HTT \text{ or } 3H^2T = HHT + HTH + THH
\end{aligned}$$

Remark 1. The total number of combinations of the binomial is 2^n.
$2^3 = T^3 + 3T^2H + 3H^2T + H^3 = 1 + 3 + 3 + 1 = 8$

Many combinations can be simplified by the use of Pascal's identity:

This entire section could be put under as a note.

Pascal's Identity.

$$\begin{aligned}\binom{n-1}{k} + \binom{n-1}{k-1} &= \frac{(n-1)!}{k!(n-1-k)!} + \frac{(n-1)!}{(k-1)!(n-k)!} \\ &= (n-1)![\frac{n-k}{k!(n-k)!} + \frac{k}{k(n-k)!}] \\ &= (n-1)!\frac{n}{k!(n-k)!} \\ &= \frac{n!}{k!(n-k)!} \\ &= \binom{n}{k}\end{aligned}$$

□

add a period when you are done with proofs. As so, $\binom{n}{k}$.

Binomial Theorem. Assume that $(a+b)^n = \sum_{k=0}^{n}\binom{n}{k}a^kb^{n-k}$ and by the the definition of the binomial coefficient $n \geq 0$. For the case $(n = 0) \Rightarrow (a+b)^0 = 1$. For the case $n \geq 0$.

$$\begin{aligned}(a+b)^{n+1} = (a+b)(a+b)^n &= (a+b)\sum_{k=0}^{n}\binom{n}{k}a^kb^{n-k} \\ &= \sum_{k=0}^{n}\binom{n}{k}a^{k+1}nn-k + \sum_{k=o}^{n}\binom{n}{k}a^kb^{n-k+1} \\ m = k+1 & \\ &= \sum_{m=1}^{n+1}\binom{n}{m-1}a^mb^{n-m+1} + \sum_{k=0}^{n}\binom{n}{k}a^kb^{n-k+1} \\ &= b^{n+1} + \sum_{k=1}^{n}[\binom{n}{k} + \binom{n}{k-1}]a^kb^{n-k+1} + a^{n+1} \\ &= b^{n+1} + \sum_{k=1}^{n}\binom{n+1}{k}a^kb^{n-k+1} + a^{n+1} \\ &= \sum_{k}^{n+1}\binom{n+1}{k}a^kb^{n+1-k}\end{aligned}$$

□

2 Multinomial Theorem

While the binomial theorem works for 2 independent events, the multinomial theorem generalizes to any number of groups/events. It uses the multinomial coefficient.

works for two...

Definition 3 (Multinomial Coeffecient).

Multinomial Coefficient

$$\binom{N}{n_1..n_r} = \frac{N!}{n_1!..n_r!}$$

Where n_1 to n_r are different group sizes.

Example 3. For 13 items we want to know how many combinations of 5, 5, and 3 can be made

A bit confusing, maybe say combinations of 5 items, another 5 items, and 3 items.

$$\begin{aligned}\binom{13}{5,5,3} &= \binom{13}{5}\binom{8}{5}\binom{3}{3} \\ &= \frac{13!}{5!(13-5)!}\frac{8!}{5!(8-5)!}\frac{3!}{3!(3-3)!} \\ &= \frac{13!}{5!5!3!}\end{aligned}$$

We now combine the multinomial coeffecients with monomials raised to a power to get:

Definition 4 (multinomial theorem)**.**

$$(x_1 + ... + x_r)^n = \sum_{(n1,...,nr)} \binom{n}{n1,...,nr} x_1^{n1}...x_r^{nr}$$

$$\text{where } n1 + ... + nr = n$$

Multinomial Theorem. Fix $r = 1$ and observe that $(x_1)^n = \sum_{(n1=1)} \binom{n}{n1=1} x_1^{n1=1} = nx_1$
Fix $m = r + 1$ and $(x_r + x_{r+1})^n = \sum_{(r,...,r+1)} \binom{n}{x_1,...,x_{r+1}} x_1^r x_{r+1}^{r+1}$ and observe that this is provable using the binomial theorem. □

3 Possible Outcomes to Equations

Not sure what happened here but this whole section seems to be missing?

Parsing Binomials & Multinomials in Probability

Andrew Jones

General comments: Feedback by Shakeeb Uddin Quick note that I want to mention before I get started is that all the feedback I give are just mere suggestions.
Your paper looks amazing just the way it is and also helps mine out as well.

Introduction

The use of the Binomial and Multinomial theorems in Probability can often not be intuitively obviously. This text intends to highlight the relations between computing polynomials from binomials with combinations, total probability, and probability mass functions.

1 Binomial Theorem

The binomial theorem expresses the expansion of two monomial terms such as $(x+y)^2 = x^2 + 2xy + y^2$. However, in probability the binomial theorem can express the probability of all combinations of two independent events.

Example 1. Let an unfair coin be flipped twice with $P(Tails) = 0.3$ and $P(Heads) = 0.7$
We know the probability must sum to 1. In two flips then, $(T+H)^2 = T^2 + 2TH + H^2$. This aligns with the outcomes of TT, TH, HT, and HH for 2 flips. Substituting in the probabilities we have $0.3^2 + 2 * 0.3 * 0.7 + 0.7^2 = 1$.

To compute the expansion of two monomials the theorem uses:

Definition 1 (Factorial n!)**.**

For defintions, I noticed in class and in the template the professor gave us, we would have the word we are defining in the text of the definiton as well, but also emphasized as well. Kind of like this: The *defintion* is ...

Count everyway to permute a set of n distinct objects

$$n! = \prod_{i=1}^{n} i$$

where $0! = 1$ and $n \geq 0$.

Building on factorials, the theorem uses the

Definition 2 (Binomial Coefficient)**.** Count everyway to combine a set of n objects of k size

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Note that the binomial coeffecient can be used to determine the number of combinations of a particular size the binomial theorem produces.

Example 2. 3 flips of a coin yields:

I would recommend having the align be at the = sign.

$$(T+H)^3 = TTT + TTH + THT + HTT + HHT + HTH + THH + HHH$$
$$= T^3 + 3T^2H + 3H^2T + H^3$$
$$\binom{3}{3} = 1 \text{ hence } T^3 = TTT \text{ or } H^3 = HHH$$
$$\binom{3}{2} = 3 \text{ hence } 3T^2H = TTH + THT + HTT \text{ or } 3H^2T = HHT + HTH + THH$$

Remark 1. The total number of combinations of the binomial is 2^n.
$2^3 = T^3 + 3T^2H + 3H^2T + H^3 = 1 + 3 + 3 + 1 = 8$

Many combinations can be simplified by the use of Pascal's identity:

Pascal's Identity.

$$\begin{aligned}
\binom{n-1}{k} + \binom{n-1}{k-1} &= \frac{(n-1)!}{k!(n-1-k)!} + \frac{(n-1)!}{(k-1)!(n-k)!} \\
&= (n-1)![\frac{n-k}{k!(n-k)!} + \frac{k}{k(n-k)!}] \\
&= (n-1)!\frac{n}{k!(n-k)!} \\
&= \frac{n!}{k!(n-k)!} \\
&= \binom{n}{k}
\end{aligned}$$

□

Binomial Theorem. Assume that $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$ and by the the definition of the binomial coeffecient $n \geq 0$. For the case $(n = 0) \Rightarrow (a+b)^0 = 1$. For the case $n \geq 0$.

Another thing I would recommend is for equations that might require more space is to give them thier own displace style. You could do something like:
Assume that

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$

and by ...
There by giving the readers more room to read.

$$
\begin{aligned}
(a+b)^{n+1} &= (a+b)(a+b)^n = (a+b)\sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k} \\
&= \sum_{k=0}^{n}\binom{n}{k}a^{k+1}nn-k + \sum_{k=o}^{n}\binom{n}{k}a^k b^{n-k+1} \\
m &= k+1 \\
&= \sum_{m=1}^{n+1}\binom{n}{m-1}a^m b^{n-m+1} + \sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k+1} \\
&= b^{n+1} + \sum_{k=1}^{n}[\binom{n}{k} + \binom{n}{k-1}]a^k b^{n-k+1} + a^{n+1} \\
&= b^{n+1} + \sum_{k=1}^{n}\binom{n+1}{k}a^k b^{n-k+1} + a^{n+1} \\
&= \sum_{k}^{n+1}\binom{n+1}{k}a^k b^{n+1-k}
\end{aligned}
$$

□

2 Multinomial Theorem

While the binomial theorem works for 2 independent events, the multinomial theorem generalizes to any number of groups/events. It uses the multinomial coeffecient.

Definition 3 (Multinomial Coeffecient)**.**

$$\binom{N}{n_1..n_r} = \frac{N!}{n_1!..n_r!}$$

Where n_1 to n_r are different group sizes.

Example 3. For 13 items we want to know how many combinations of 5, 5, and 3 can be made

$$
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\binom{13}{5,5,3} &= \binom{13}{5}\binom{8}{5}\binom{3}{3} \\
&= \frac{13!}{5!(13-5)!}\frac{8!}{5!(8-5)!}\frac{3!}{3!(3-3)!} \\
&= \frac{13!}{5!5!3!}
\end{aligned}
$$

We now combine the multinomial coeffecients with monomials raised to a power to get:

Definition 4 (multinomial theorem)**.**

$$(x_1 + ... + x_r)^n = \sum_{(n1,...,nr)}\binom{n}{n1,...,nr}x_1^{n1}...x_r^{nr}$$
$$\text{where } n1 + ... + nr = n$$

Multinomial Theorem. Fix $r = 1$ and observe that $(x_1)^n = \sum_{(n1=1)}\binom{n}{n1=1}x_1^{n1=1} = nx_1$
Fix $m = r+1$ and $(x_r + x_{r+1})^n = \sum_{(r,...,r+1)}\binom{n}{x_1,...,x_{r+1}}x_1^r x_{r+1}^{r+1}$ and observe that this is provable using the binomial theorem. □

3 Possible Outcomes to Equations