1 Homework

Prove or Disprove the following statements:

1. $\exists n \in \mathbb{Z} : n + 1 = 5$

There exists a number n in the integers such that n + 1 = 5. Set n equal to 4. Observe that $4 + 1 = 5 \in \mathbb{Z}$.

2. $\forall n \in \mathbb{Z} : n > 7$

For all numbers n in the integers, n is greater than 7. Set n equal to 5. Observe that $5 \in \mathbb{Z}$.

3. $\exists x \in \mathbb{R} : \forall y \in \mathbb{R} : x \geq y$

There exists a real number x such that for all real numbers $y : n \geq y$. Fix x to 5. Fix y to 7. Observe that $5, 7 \in \mathbb{R}$. Observe that $y \geq n$ where y = 7 and x = 5.

4. $\exists x \in \mathbb{R} : \forall k \in \mathbb{N} : x^k = x$

There exists a real number x such that for all integers y: $x^k = x$. Fix x to 1. Observe that $1^y = 1$. There for the proposition is true.

5. $\forall x \in \mathbb{R} : \exists y \in \mathbb{R} : xy = 1$

For all x in the reals there exists a real number y such that xy = 1. Put $y = \frac{1}{x}$. Observe that $x * \frac{1}{x} = 1$. $\frac{1}{x} \in \mathbb{R}$. Therefore the proposition is true.

6. $\exists x \in \mathbb{R} : \forall y \in \mathbb{R} : xy = y$

There exists a real number x such that for all real numbers xy = y. Put x = 1 Observe that 1*y=y. $1 \in \mathbb{R}$.

7. Give an example of a proposition P for which:

$$\forall m \in \mathbb{Z} : \exists n \in \mathbb{Z} : P(m, n) \text{ is true and } \exists n \in \mathbb{Z} : \forall m \in \mathbb{Z} : P(m, n) \text{ is false}$$

For all integers m there exists an integer n such that P(m,n) is true. There exists an integer n such that for all integers m P(m,n) is false. Let P = m = n. Put n = m. Observe that n, $m \in \mathbb{Z}$. Observe that for all integers m there exists an integer n such that m = n. Observe that there is not a single integer n such that for all integers m m = n

8. Find a Proposition Q for which:

$$\forall m \in \mathbb{Z} : \exists n \in \mathbb{Z} : Q(m, n) \text{ is false and } \exists n \in \mathbb{Z} : \forall m \in \mathbb{Z} : Q(m, n) \text{ is true}$$

For all integers m there exists an integer n such that Q(m,n) is false. There exists an integer n such that for all integers m Q(m,n) is true. Put Q equal to m ¡ n. Put n = (m + 1). Observe that m, (m + 1) $in\mathbb{Z}$. Observe that for all integers m there is an integer m - 1 such that m ¡ (m + 1). Put m = 5 and n = 4. Observe that 5 < 4 is false. Observe that there does not exist a single integer n such that all integers m are greater than it.

9. Is the statement $\forall a \in \mathbb{A} : \forall b \in \mathbb{B} : P(a, b)$ communative and there for $\forall b \in \mathbb{B} : \forall a \in \mathbb{A} : P(a, b)$ is also true?

For all numbers a in A such that for all numbers b in B satisfy P(a,b). For all numbers b in B such that for all numbers a in A satisfy P(a,b). As the order of the arguments to the proposition does not change, I would assume that switching the order of the for all statements should not effect the value of the proposition.

10. $\exists a \in \mathbb{A} : \exists b \in \mathbb{B} : P(a, b)$ does it follow that $\exists b \in \mathbb{B} : \exists a \in \mathbb{A} : P(a, b)$?

There exists a number a in Set A such that there exists a number B in set B that satisifies P(a,b). There exists a number b in Set B such that there exists a number A in Set A that satisfies P(a,b). I would also assume here that exists is communative and what matters is switching the order of the parameters to the Property.