MATH-300 Andrew Jones

Worksheet 4

Let R be a relation from A to B, let S be a relation from B to C, and let T be a relation from C to D.
Prove the following statements.

1. $I_A \circ R = R$

   Proof. □

2. $R \circ I_A = R$

   Proof. □

3. $(R^{-1})^{-1} = R$

   Proof. □

4. $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$

   Proof. □

5. $(T \circ S) \circ R = T \circ (S \circ R)$

   Proof. □

6. $DomR = RngR^{-1}$

   Proof. □

7. $RngR = DomR^{-1}$

   Proof. □

For Question 8–10, suppose that $A = B = C$.

8. If R and S are equivalence relations, then $S \circ R$ is an equivalence relation.

   Proof. □

9. If R is a partial order, then $R \circ R$ is a partial order.

   Proof. □

10. If R and S are partial orders, then it is not generally true that $S \circ R$ is a partial order.

    Proof. □

Bonus Questions Give an example of two relations R and S on a set A such that

11. $R \circ S \neq S \circ R$.

    Proof. □

12. $S \circ R$ is an equivalence relation, but neither R nor S is an equivalence relation.

    Proof. □