Show all your work. A right answer is a correct result together with the correct steps used to obtain it:
Right Answer = Correct Result + Correct Steps

Chapter 3

7. Let $x_n := 1/\ln(n+1)$ for $n \in \mathbb{N}$.
   (a) Use the definition of limit to show that $\lim(x_n) = 0$.
   (b) Find a specific value of $K(\varepsilon)$ as required in the definition of limit for each of (i) $\varepsilon = 1/2$, and (ii) $\varepsilon = 1/10$.

8. Prove that $\lim(x_n) = 0$ if and only if $\lim(|x_n|) = 0$. Give an example to show that the convergence of $(|x_n|)$ need not imply the convergence of (x_n).

9. Show that if $x_n \geq 0$ for all $n \in \mathbb{N}$ and $\lim(x_n) = 0$, then $\lim(\sqrt{x_n}) = 0$.

10. Prove that if $\lim(x_n) = x$ and if $x > 0$, then there exists a natural number M such that $x_n > 0$ for all $n \geq M$.

2. Give an example of two divergent sequences X and Y such that:
   (a) their sum $X + Y$ converges, (b) their product XY converges.

3. Show that if X and Y are sequences such that X and $X + Y$ are convergent, then Y is convergent.

4. Show that if X and Y are sequences such that X converges to $x \neq 0$ and XY converges, then Y converges.

7. If (b_n) is a bounded sequence and $\lim(a_n) = 0$, show that $\lim(a_n b_n) = 0$. Explain why Theorem 3.2.3 *cannot* be used.