Math 351 - Spring 2025: Homework 5

Due: Wednesday, April 9, 2025 (but if you need until Friday April 11
that's ok too)

Instructions: Be sure to give explanations to your answers. I'm interested
not only in whether you get the correct answer but also how you obtained
it and your thought process along the way. Don't just write down a number
even if the answer seems obvious.

1. Suppose that X is a uniform random variable on the interval $[-1, 1]$. Find
   the probability density functions of X, $|X|$, and e^X.

2. The median of a continuous random variable having distribution function F
   is that value of m such that $F(m) = \frac{1}{2}$. That is, a random variable is just as
   likely to be larger than its median as it is to be smaller. Find the median of
   X if X is (**a**) uniformly distributed over (a, b), (**b**) normal with parameters
   μ, σ^2, and (**c**) exponential with parameter λ.

3. Suppose X is a normal random variable with parameters $\mu = 2$ and $\sigma^2 = 9$.
   (a) Find $P\{1 \le X \le 4\}$
   (b) What is the probability distribution of $Y = 5X + 1$. That is, what kind
   of random variable is this and what are its parameters?

4. An anonymous math professor drilled some holes in a standard six-sided
   die in an attempt to create a loaded die. In 328 independent rolls of this
   die, the die landed on a six 68 times. Was the attempt at loading this die
   successful? Explain your reasoning.

5. At the GMU/Fairfax shuttle bus stop, a bus to the Vienna Metro Station arrives every 30 minutes. Twelve minutes after the arrival of each bus to the Metro Station, another bus arrives and departs to the Manassas campus. A passenger arrives at the station at a uniformly distributed time on [0,30] with the intention of going to the Metro Station. Let X be the time the passenger will have to wait for the next bus to the Metro Station.

(a) What kind of random variable is X?

(b) What is $P\{X > 5\}$?

(c) If the passenger is absent-minded and just gets on the first bus that arrives, what is the probability that the passenger will be on a bus to the Metro Station?