Practice Midterm 1a

Name: _____

For each of the following statements,

i. express the statement in terms of quantifiers, (*1 pt.*)

ii. express the negation in terms of quantifiers, (*1 pt.*)

iii. indicate whether the statement is true or false, (*2 pt.*)

iv. either prove or disprove the statement (*3 pts.* for logical correctness, *3 pts.* for conventional writing.)

1. For every $k \in \mathbb{Z}$, there is an $\ell \in \mathbb{N}$ satisfying $\ell \leq k$.
2. For all $x \in \mathbb{R}$ there is a $y \in \mathbb{R}$ such that if $x < y$ then there is a $z \in \mathbb{R}$ with $x < z < y$.
3. Let A, B, and C be sets. If $A \cap B \subseteq C$, then $A \subseteq C$ or $B \subseteq C$.
4. If $f : A \to B$ and $g : B \to C$ are not surjective, then $g \circ f : A \to C$ is not surjective.
5. If $f : A \to B$ is a bijection, then there is a $g : B \to A$ such that $g \circ f = \mathrm{id}_A$ and $f \circ g = \mathrm{id}_B$.

Bonus Question. (*5 pts.*)

6. Prove that $\lim_{x \to 0} x^2 = 0$.