Basic Proof Language

Proposition 1. *For every $x \in \mathbb{R}$, there is a $y \in \mathbb{R}$ such that $x > y + 1$.*

Proof. $$\left\{\begin{matrix}\text{Let} \\ \text{Fix} \\ \text{Choose} \\ \text{Suppose that}\end{matrix}\right\} x \in \mathbb{R} \text{ and } \left\{\begin{matrix}\text{let} \\ \text{put}\end{matrix}\right\} y = x - 2. \left\{\begin{matrix}\text{Observe that} \\ \text{We have} \\ \text{It follows that}\end{matrix}\right\} x > y.$$ □

Proposition 2. *There is a $k \in \mathbb{Z}$ such that $k < \ell$ for all $\ell \in \mathbb{N}$.*

Proof. $$\left\{\begin{matrix}\text{Let} \\ \text{Put}\end{matrix}\right\} k = -1 \text{ and } \left\{\begin{matrix}\text{let} \\ \text{fix} \\ \text{choose} \\ \text{suppose that}\end{matrix}\right\} \ell \in \mathbb{N}. \left\{\begin{matrix}\text{Observe that} \\ \text{We have} \\ \text{It follows that}\end{matrix}\right\} k < \ell.$$ □

Proposition 3. *If $A \subseteq B \subseteq C$, then $(A \cap B) \subseteq (A \cap C)$*

Proof. $$\left\{\begin{matrix}\text{Let} \\ \text{Fix} \\ \text{Choose} \\ \text{Suppose that}\end{matrix}\right\} x \in A \cap B. \left\{\begin{matrix}\text{It follows that} \\ \text{Thus,} \\ \text{Hence,} \\ \text{Therefore,}\end{matrix}\right\} x \in A \text{ and } x \in B. \left\{\begin{matrix}\text{From} \\ \text{Since} \\ \text{As}\end{matrix}\right\} B \subseteq C,$$

$$\left\{\begin{matrix}\text{it follows that} \\ \text{we obtain} \\ \text{we deduce that} \\ \text{we have}\end{matrix}\right\} x \in C, \left\{\begin{matrix}\text{from which we conclude that} \\ \text{from which it follows that} \\ \text{from which} \\ \text{which yields} \\ \text{and we conclude that}\end{matrix}\right\} x \in A \cap C.$$ □