

Worksheet 3

Let A , B , and C be sets. Prove or disprove the following statements.

1. If $A \cap B = \emptyset$ and $B \cap C = \emptyset$, then $A \cap C = \emptyset$

Proof. Assume that $A = \{a\}$ and $C = \{a\}$ and $B = \{b\}$ It follows that:
 $A \cap B = \emptyset$ and $B \cap C = \emptyset$ however $A \cap C = \{a\}$ There for: $\exists A : \exists C :$
 $A \cap C \neq \emptyset$ and $A \cap B = \emptyset$ and $B \cap C = \emptyset$ \square

2. If $A \not\subseteq B$ and $B \not\subseteq C$, then $A \not\subseteq C$

Proof. Assume that $A = \{a\}$ and $C = \{a, c\}$ and $B = \{b\}$ It follows that:
 $A \not\subseteq B$ and $B \not\subseteq C$ however $A \subseteq C$ There for: $\exists A : \exists C : A \subseteq C$ and
 $A \not\subseteq B$ and $B \not\subseteq C$ \square

3. If $A \subseteq \emptyset$, then $a = \emptyset$

Proof. Assume the negation $A \subseteq \emptyset$ and $A \neq \emptyset$. If $A \neq \emptyset$ then $A \not\subseteq \emptyset$ by definition of \emptyset \square

4. If $A \subseteq C$ and $B \subseteq C$, then $A \cap B \subseteq C$

Proof. Assume that $A \subseteq C$ and $B \subseteq C$ there for 2 cases can occur for
 $A \cap B \subseteq C$

Case 1: $A \cap B = \emptyset$ there for $A \cap C \subseteq C$ as $\emptyset \subseteq C$

Case 2: $A \cap B \neq \emptyset$ then $\forall e \in A \cap B : e \in C$ there for $A \cap B \subseteq C$ \square

5. If $f : A \rightarrow B$ is injective and $g : B \rightarrow C$ is injective, then $g \circ f : A \rightarrow C$ is injective.

Proof. Assume that $\forall x, y \in A$ if $f(x) = f(y)$ then $x = y$ and the same for g . $f(A) \subseteq B$ and $g(B) \subseteq C$ there for as both f and g are injective, the subset of B passed from f to g will also be injective. Hence $g \circ f$ is injective. \square

6. If $f : A \rightarrow B$ is surjective and $g : B \rightarrow C$ is surjective, then $g \circ f : A \rightarrow C$ is surjective

Proof. By the definition of surjective f maps to all values in B , similarly g maps to all values in C . Hence $g \circ f$ maps to all values in C and is surjective. \square

7. Give an example of a function $f : A \rightarrow A$ that is injective but not surjective.

Proof. $g : b \rightarrow 2b$ maps to only the even co-domain □

8. Give an example of a function $g : A \rightarrow A$ that is surjective but not injective.

Proof. □

9. Let $f : A \rightarrow B$ and $g : B \rightarrow A$. If $g \circ f = id_a$, then both f and g are bijections.

10. If $f : A \rightarrow A$ is surjective, and if A is a finite set, then f is injective.

11. If $f : A \rightarrow A$ satisfies the property that $f \circ f = id_a$ then f is a bijection.

Proof. As previous proof state the composition of two surjective functions are surjective and the same for injective, hence $f \circ f$ is a bijection. □