On Savings Optimisation Strategies

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Outline

On Risk Measures

Final Wealth distribution

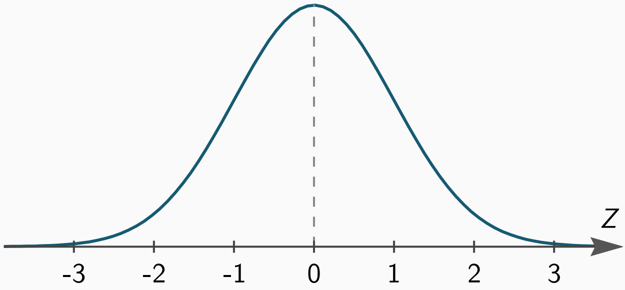


Figure 1: Theoretical normal distribution.

Volatility

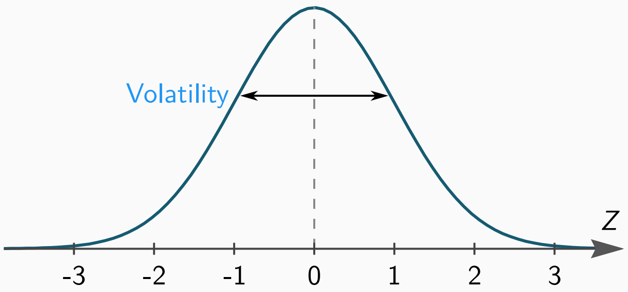


Figure 2: Volatility as measure for dispersion or risk.

Value at Risk (VaR)

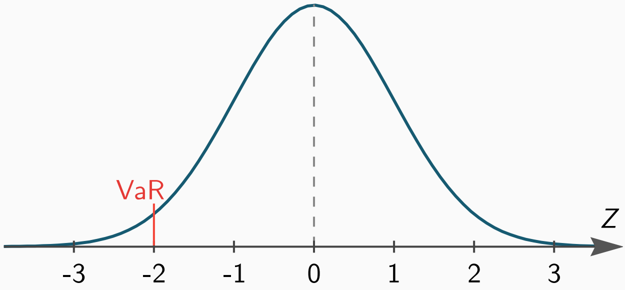


Figure 3: Value ar Risk (VaR) is the quantile 5%.

Expected Shortfall

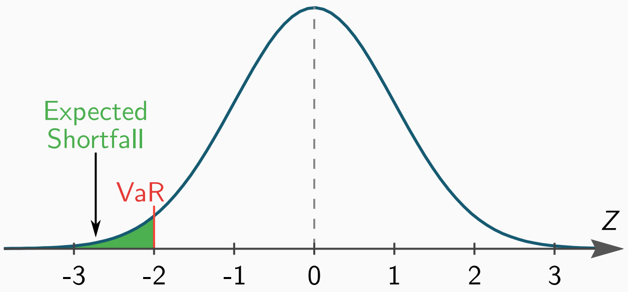


Figure 4: Expected Shortfall is the average of the beyond-the-VaR distribution.

Optimising Investment Strategies

Constant Proportion Portfolio Insurance (CPPI)

$$X_{t+1} = (1 + N(\alpha, \sigma))X_t\pi + (1 - \pi)X_t + C_t$$

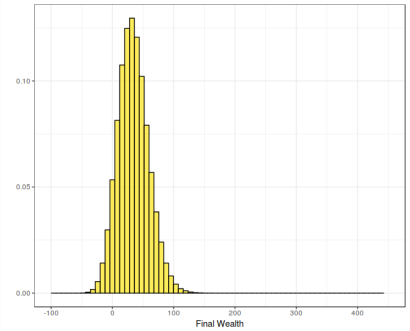


Figure 5: Results of the simulation for the CPPI model. Final wealth obtained for every simulation, over $100,000$ simulations, where $T = 60$, $\mu = 0.0343$, $\sigma = 0.1544$, $a = 10$, $\pi = 0.1$.

Alternative

$$\pi_t X_t = A(K + X_t + C_t)$$
$$X_{t+1} = (1 + N(\alpha, \sigma))X_t\pi_t + (1 - \pi_t)X_t + C_t$$

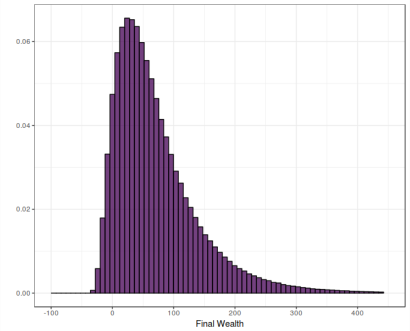
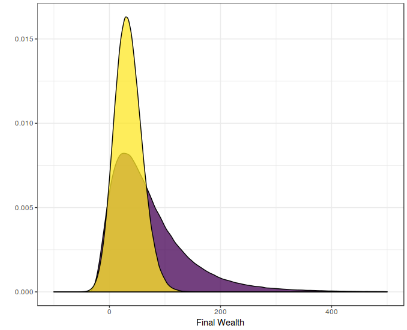
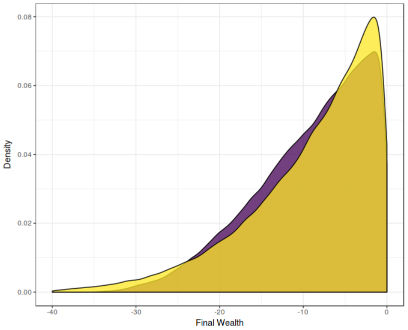


Figure 6: Results of 100,000 simulations for the *Alternative* model. Final wealth obtained for every simulation, where $T = 30$, $\mu = 0.0343$, $\sigma = 0.1544$, $a = 10$, $\pi = 0.1$.

Comparison



((a)) Whole distribution of the Final Wealth for both strategies.



((b)) Losses obtained from both strategies.

Figure 7: Results of the simulation for the both strategies. Final wealth obtained for every simulation. The yellow one is the CPPI, and the purple one is the Alternative. Results from over 100,000 simulations for each strategy, using $\alpha = 0.343$, $\sigma = 0.1544$, $a = 10$, $T = 60$, $A = 0.5$, $\pi = 0.1$, $K = 42$.

Comparison

Table 1: Results of $100,000$ simulations using different risk levels, with $\alpha = 0.343$, $\sigma = 0.1544$, $a = 10$, $T = 60$, $A = 0.5$ and therefore $ES/K = -3.3$. These results match approximately those presented in [1], on Table 1 at page 8. Any differences are attributable to the intrinsic randomness of simulations.

π	**ES**	K	**CPPI ret**	**Alt ret**	**diff**	**equiv** π
0.1	-12.47	40.59	0.33	0.51	0.18	0.17
0.2	-27.03	87.98	0.65	0.98	0.38	0.30
0.3	-42.99	139.95	0.95	1.42	0.46	0.45
0.4	-60.13	195.71	1.23	1.83	0.60	0.61
0.5	-78.25	255.02	1.5	2.22	0.71	0.69
0.6	-99.96	325.36	1.76	2.63	0.87	0.92
0.7	-120.61	392.56	2.00	3.00	1.00	*
0.8	-144.71	471.02	2.23	3.40	1.17	*
0.9	-172.94	562.92	2.39	3.81	1.42	*
1	-205.09	667.57	2.58	4.27	1.70	*

Comparison of Expected Shortfalls

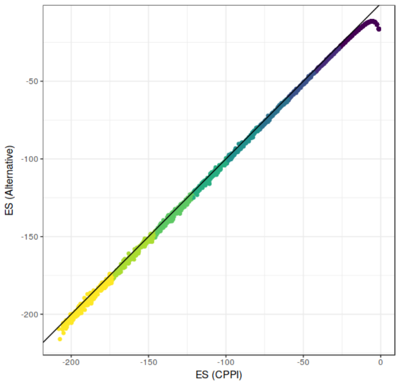


Figure 8: Plot that shows the relation between equivalent π and A. Results from $20,000,000$ simulations performed for many different values of π and A, with $\alpha = 0.343$, $\sigma = 0.1544$, $a = 10$, $T = 60$.

Comparison

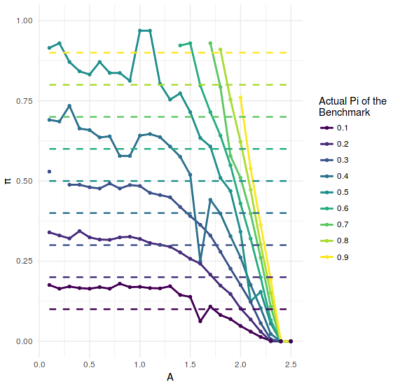


Figure 9: Plot that shows the relation between equivalent π and A. Results from $20,000,000$ simulations performed for many different values of π and A, with $\alpha = 0.343$, $\sigma = 0.1544$, $a = 10$, $T = 60$.

Pooled Funds

Simulating Mortality

The work of [2] proved that, given a Binomial Distribution, where

$$Pr(X = k) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k},$$

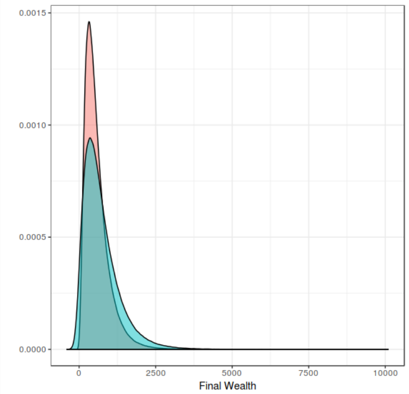
we can numerically generate k given p and n. (The algorithm to do so is packed in the base R library.)

Simulating Mortality

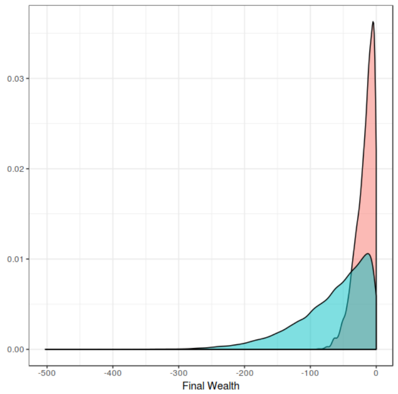
$$M = \frac{k}{n} w$$

$$\pi_t X_t = A(K + X_t + C_t)$$
$$X_{t+1} = (1 + N(\alpha, \sigma)) X_t \pi_t + (1 - \pi_t) X_t + C_t + M_t X_t$$

Comparing Strategies with Mortality



((a)) first caption.



((b)) second caption.

Figure 10: Results of the simulation for the both models with mortality. Final wealth obtained for every simulation. The blue one is the CPPI, and the red one is the Alternative. Results from over $100,000$ simulations for each strategy, using $\alpha = 0.343$, $\sigma = 0.1544$, $a = 10$, $T = 60$, $A = 0.5$ and $\pi = 0.1$.

Comparing Strategies with Mortality

Table 2: Results of 100,000 simulations of both CPPI and Alternative models in a Pooled Fund, using different risk levels, with $\alpha = 0.343$, $\sigma = 0.1544$, $a = 10$, $T = 60$, $A = 0.5$ and $w = 1$.

π	**ES**	K	**CPPI ret**	**Alt ret**	**diff**	**equiv** π
10	140.32	18.11	1.84	1.97	0.13	0.15
20	88.75	88.41	2.13	2.79	0.65	0.34
30	58.50	96.45	2.66	2.80	0.13	0.32
40	20.85	165.75	3.13	3.59	0.46	0.49
50	-28.11	232.00	3.46	4.17	0.70	0.64
60	-73.18	294.04	3.83	4.67	0.83	1.06
70	-118.58	381.23	4.24	5.40	1.15	1.63
80	-153.69	424.03	4.77	5.85	1.07	*
90	-223.58	524.09	4.88	6.77	1.88	*
100	-261.12	618.38	5.16	6.79	1.62	*

Comparison of Expected Shortfalls

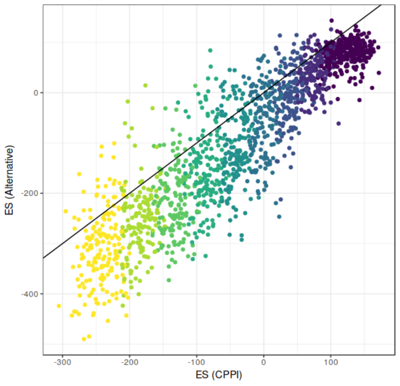


Figure 11: Plot that shows the relation between equivalent π and A. Results from $20,000,000$ simulations performed for many different values of π and A, with $\alpha = 0.343$, $\sigma = 0.1544$, $a = 10$, $T = 60$.

Comparison

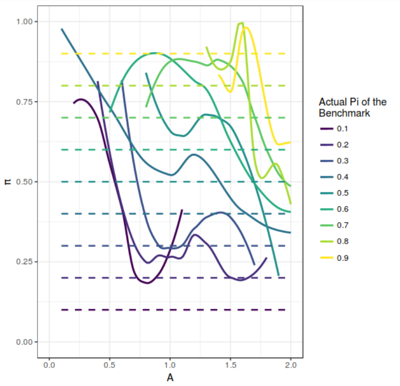


Figure 12: Plot that shows the relation between equivalent π and A. Results from $20,000,000$ simulations performed for many different values of π and A, with $\alpha = 0.343$, $\sigma = 0.1544$, $a = 10$, $T = 60$.

Tails Analysis as Risk Assessment

What is a Tail?

$$Z = (Y - u | Y > u)$$

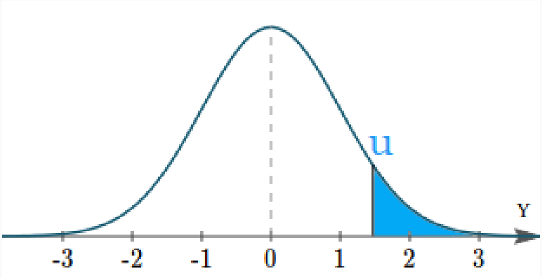


Figure 13: The value u from which the Tail begins is not well defined, and os is up to every case to set it.

General Pareto Distribution (GPD)

The Pickands-Balkema-de Haan theorem states that for a large class of distributions Z, exists u such as that F_u is well approximated by the Generalized Pareto Distribution (GPD), whose standard cumulative distribution function is:

$$F_\xi(y) = \begin{cases} 1 - (1 + \xi \frac{y}{\psi})^{-1\xi} & : \xi \neq 0 \\ 1 - e^{-\frac{y}{\psi}} & : \xi = 0 \end{cases}$$

- $\xi > 0$. The tail does not converge to zero, ever. Power-lawl
- $\xi = 0$. The tail converges to zero at infinity. Exponential.
- $\xi < 0$. The tail sharply converges to zero in a finite point.

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Coherent Risk Measures

$$
\begin{aligned}
\rho(\lambda X) &= \lambda \rho(X) \\
\rho(\lambda + X) &= \lambda + \rho(X) \\
P(X_1 < X_2) = 1 &\implies \rho(X_1) < \rho(X_2) \\
\rho(X_1 + X_2) &\leq \rho(X_1) + \rho(X_2).
\end{aligned} \tag{1}
$$

Scale and Location free Risk Measures

$$
\begin{aligned}
\rho(\lambda X) &= \rho(X) \\
\rho(\lambda + X) &= \rho(X) \\
P(X_1 < X_2) = 1 &\implies \rho(X_1) < \rho(X_2) \\
\rho(X_1 + X_2) &\leq \rho(X_1) + \rho(X_2).
\end{aligned} \tag{2}
$$

Fitting a GPD

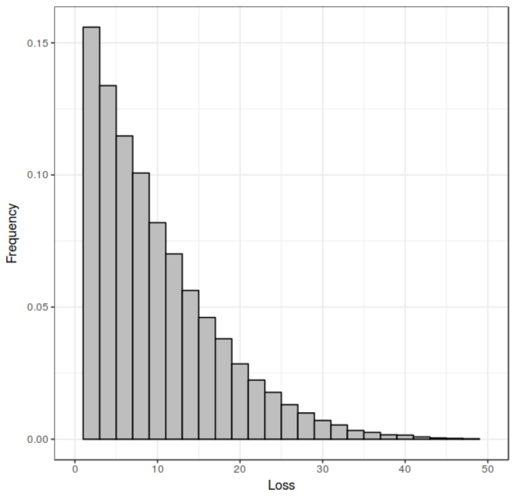


Figure 14: Frequency Histogram of the losses of the CPPI strategy. Using $\pi = 0.1$, $T = 60$, $\alpha = 0.0343$, $\sigma = 0.1544$, $a = 10$ and $1,000,000$ simulations.

Fitting a GPD

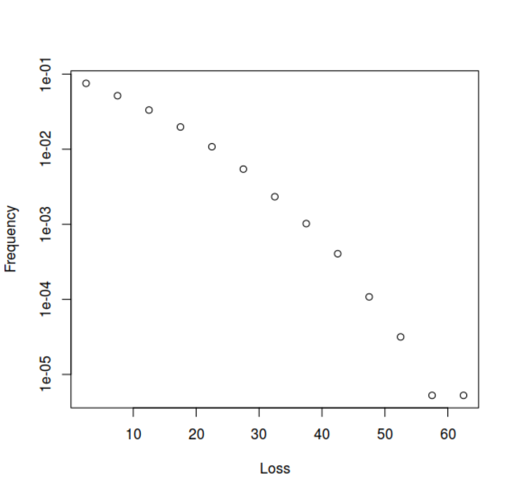


Figure 15: Density Distribution of the losses of the CPPI strategy in a logarithmic scale. Using $\pi = 0.1$, $T = 60$, $\alpha = 0.0343$, $\sigma = 0.1544$, $a = 10$ and $1,000,000$ simulations.

Fitting a GPD

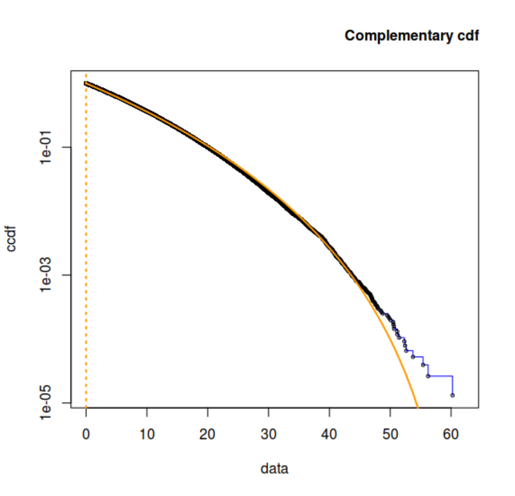
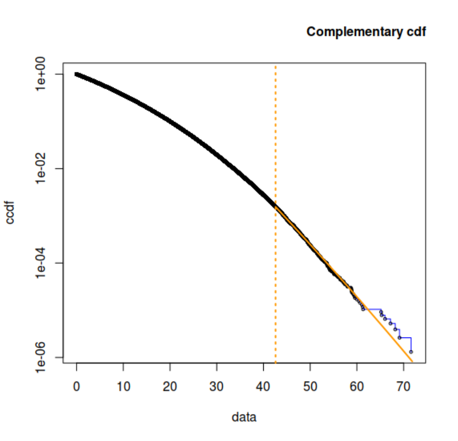


Figure 16: Fitted GPD using $u = 0$ as a threshold. Using $\pi = 0.1$, $T = 60$, $\alpha = 0.0343$, $\sigma = 0.1544$, $a = 10$ and $1,000,000$ simulations.

Fitting a GPD



Turned out to be $u = 42.59$ and $\xi = 0.14$

Figure 17: Fitted GPD using Using $\pi = 0.1$, $T = 60$, $\alpha = 0.0343$, $\sigma = 0.1544$, $a = 10$ and $1,000,000$ simulations.
Turned out to be $u = 42.59$ and $\xi = 0.14$

Extreme Value Index (EVI) ξ

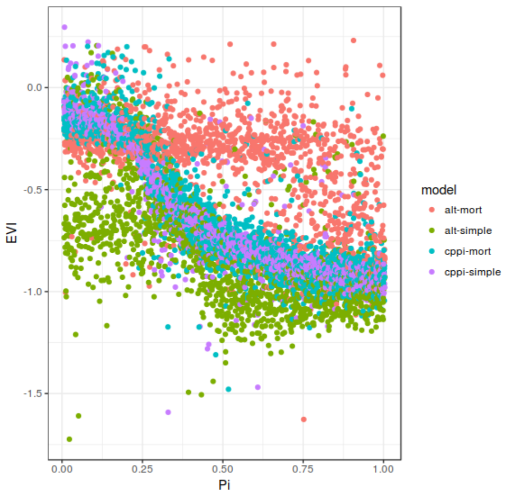


Figure 18: Extreme Value Index measured by varying values of π and A. Using $\alpha = 0.0343$, $\sigma = 0.1544$, and 1000000 simulations.

Extreme Value Index (EVI) ξ

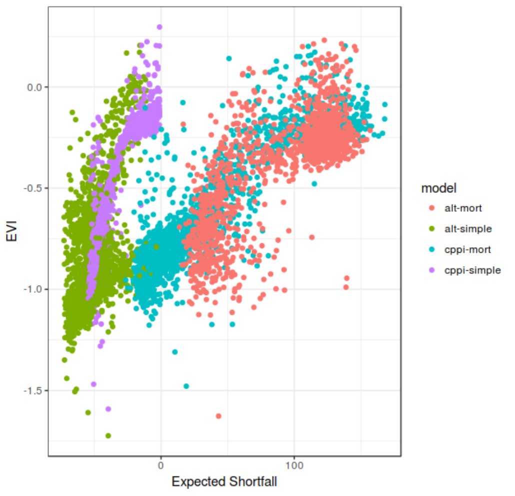


Figure 19: Extreme Value Index vs Expected Shortfall. Scatter plot of $10,000,000$ simulations of all four strategies, iterating over all values of $0 <= \pi <= 1$ and $0.1 <= A <= 2$.

Conclusion

Summary

- It is optimal to define a K value in order to set the risk level of the investor.
- Pooled Funds can dramatically increase the return of savings strategies without compromising the risk aversion levels.
- Parameters that define the shape of losses tails can serve as location and scale free risk measures.

'All money is a matter of belief.' Adam Smith

Questions?

R. Gerrard, M. Guillén, J. P. Nielsen, and A. M. Pérez-Marín, "Long-run savings and investment strategy optimization," *The Scientific World Journal*, vol. 2014, 2014.

V. Kachitvichyanukul and B. W. Schmeiser, "Binomial random variate generation.," *AMC*, 1988.