CONTINUOUS RANDOM VARIABLES

1. X is a a delay in hours of flight from Chicago with p.d.f defined by $f(x) = 0.2 - 0.02x$ for $0 \leq X \leq 10$.

   Find

   a) The probability that the delay is less than 4 hours.
   b) The probability that the delay is between 2 and 6 hours.
   c) Probability of delay is exactly 5 hours.

2. A continuous random variable X is modelled by the p.d.f $f(x)$ given by

$$f(x) = \begin{cases} \frac{2}{75}x\,, & 0 \leq x \leq 5 \\ \frac{2}{15}, & 5 \leq x \leq 10 \\ 0, & elsewhere \end{cases}$$

   a) Sketch $f(x)$
   b) Find P(X ≥ 4)

3. A temporary round about is installed at crossroads as an experiment. The time T minutes which vehicles have to wait before entering the round about has a p.d.f defined by

$$f(t) = \begin{cases} 0.8 - 0.32t, & 0 \leq t \leq 2.5 \\ 0, & elsewhere \end{cases}$$

   a) Expected time
   b) Variance and standard deviation.

4. A continuous random variable X having values only between 0 and 4 has a density function given by $f(x) = \frac{1}{2} - rx$, where r is a real number . Find

   a) The value of r
   b) $E[X]$
   c) $Var(X)$
   d) $P(1 < X < 2)$

5. A certain student asked her friend to guess her height in metres. The student considered that the height guessed by a randomly selected friend can be modelled by the random variable Y with p.d.f defined by

$$f(y) = \begin{cases} \frac{3}{16}(4y - y^2), & 0 \leq y \leq 2 \\ 0, & elsewhere \end{cases}$$

   Find

   a) $E[Y]$
   b) $Var(Y)$ and standard deviation of Y

c) $P(Y < 1)$

SPECIAL PROBABILITY DISTRIBUTIONS

6. The probability of a defective item in a sample of 400 bolts is 0.1. Find.
   (a) The expectation
   (b) The standard deviation.
7. In an experiment, a fair die is rolled five times. Determine
   (a) The expected value for a 3 to appear twice.
   (b) The expected value for a 3 to appear at least two times.

8. The mean number of bacteria per millilitre of a liquid from lake Victoria is known to be 4. Assuming that the number of bacteria distribution in the lake follows a Poisson distribution, find the probability that in a one millilitre liquid there will be,
   (a) no bacterium
   (b) 4 bacteria,
   (c) less than 3 bacteria.
9. A factory packs bolts in boxes of 300. The probability that a bolt is defective is 0.001. Find the probability that a box chosen at random contains two defective bolts.
10. . On average, the institute's photocopier breaks down eight times during a working week (Monday to Friday). The number of breakdowns can be modeled by Poisson distribution, calculate the probability that it breaks down:
    (a) Five times in a given week.
    (b) Once on Wednesday.
11. The probability that the electric bulbs manufactured by a certain company are defective is 0.02. If 100 bulbs are selected at random, find the probability that
    (a) no bulb is defective
    (b) one bulb is defective
12. The length of wires produced by a machine are normally distributed with mean length of 150 cm and standard deviation of 10 cm. Find the probability that the length of a randomly selected strip is,
    (a) Shorter than 165 cm
    (b) Within 5 cm of the mean
13. An industry packs matchsticks in matchboxes with an average of 14 matchsticks in each box, where the standard deviation is 2.5. If there are 1,000 matchboxes, how many of them contain between 12 and 15 matchsticks?
14. At the Express House Delivery Service, providing high-quality service to customers is the top priority of the management. The company guarantees a refund of all charges if a package it is delivering does not arrive at its destination by the specified time. It is known from past data that despite all efforts, 2% of the packages mailed through this company do not arrive at their

destinations within the specified time. Suppose a corporation mails 10 packages through Express House Delivery Service on a certain day.

(a) Find the probability that exactly one of these 10 packages will not arrive at its destination within the specified time.

(b) Find the probability that at most one of these 10 packages will not arrive at its destination within the specified time.

15. A washing machine in a laundromat breaks down an average of three times per month. Using the Poisson probability distribution formula, find the probability that during the next month this machine will have
    (a) exactly two breakdowns
    (b) at most one breakdown

16. Cynthia's Mail Order Company provides free examination of its products for 7 days. If not completely satisfied, a customer can return the product within that period and get a full refund. According to past records of the company, an average of 2 of every 10 products sold by this company are returned for a refund. Using the Poisson probability distribution formula, find the probability that exactly 6 of the 40 products sold by this company on a given day will be returned for a refund.
17. An average of 1.4 private airplanes arrive per hour at an airport. a.
    (a) Find the probability that during a given hour no private airplane will arrive at this airport.
    (b) Let X denote the number of private airplanes that will arrive at this airport during a given hour. Write the probability distribution of X.
18. A racing car is one of the many toys manufactured by Mack Corporation. The assembly times for this toy follow a normal distribution with a mean of 55 minutes and a standard deviation of 4 minutes. The company closes at 5 P.M. every day. If one worker starts to assemble a racing car at 4 P.M., what is the probability that she will finish this job before the company closes for the day?
19. Suppose the life span of a calculator manufactured by Texas Instruments has a normal distribution with a mean of 54 months and a standard deviation of 8 months. The company guarantees that any calculator that starts malfunctioning within 36 months of the purchase will be replaced by a new one. About what percentage of calculators made by this company are expected to be replaced?
20. A construction zone on a highway has a posted speed limit of 40 miles per hour. The speeds of vehicles passing through this construction zone are normally distributed with a mean of 46 miles per hour and a standard deviation of 4 miles per hour. Find the percentage of vehicles passing through this construction zone that are
    (a) Exceeding the posted speed limit
    (b) Traveling at speeds between 50 and 57 miles per hour

21. According to a 2004 survey by the telecommunications division of British Gas. Britons spend an average of 225 minutes per day communicating electronically (on a fixed landline phone, on a mobile phone, by emailing, by texting, and so on). Assume that currently such times for all Britons are normally distributed with a mean of 225 minutes per day and a standard deviation of 62 minutes per day. What percentage of Britons communicate electronically for
    (a) less than 60 minutes per day
    (b) More than 360 minutes per day
    (c) Between 120 and 180 minutes per day
    (d) Between 240 and 300 minutes per day?

22. Hupper Corporation produces many types of soft drinks, including Orange Cola. The filling machines are adjusted to pour 12 ounces of soda into each 12-ounce can of Orange Cola. How ever, the actual amount of soda poured into each can is not exactly 12 ounces; it varies from can to can. It has been observed that the net amount of soda in such a can has a normal distribution with a mean of 12 ounces and a standard deviation of .015 ounce.
    (a) What is the probability that a randomly selected can of Orange Cola contains 11.97 to 11.99 ounces of soda?
    (b) What percentage of the Orange Cola cans contain 12.02 to 12.07 ounces of soda?
23. Let X denote the time taken to run a road race. Suppose X is approximately normally distributed with a mean of 190 minutes and a standard deviation of 21 minutes. If one runner is selected at random, what is the probability that this runner will complete this road race
    (a) in less than 160 minutes?
    (b) in 215 to 245 minutes?
24. The mean diameter of 500 nuts produced by a certain factory is 151 mm and the standard deviation is 15 mm. If the diameters are normally distributed, find the number of nuts that have diameters.
    (a) Between 120 mm and 155 mm inclusive.
    (b) More than 185 mm.
    (c) At most 185 mm.

6.21 Determine the following probabilities for the standard normal distribution.

a. $P(-1.83 \le z \le 2.57)$ **b.** $P(0 \le z \le 2.02)$
c. $P(-1.99 \le z \le 0)$ **d.** $P(z \ge 1.48)$

6.22 Determine the following probabilities for the standard normal distribution.

a. $P(-2.46 \le z \le 1.88)$ **b.** $P(0 \le z \le 1.96)$
c. $P(-2.58 \le z \le 0)$ **d.** $P(z \ge .73)$

6.23 Find the following probabilities for the standard normal distribution.

a. $P(z < -2.34)$ **b.** $P(.67 \le z \le 2.59)$
c. $P(-2.07 \le z \le -.93)$ **d.** $P(z < 1.78)$

6.24 Find the following probabilities for the standard normal distribution.

a. $P(z < -1.31)$ **b.** $P(1.23 \le z \le 2.89)$
c. $P(-2.24 \le z \le -1.19)$ **d.** $P(z < 2.02)$

6.25 Obtain the following probabilities for the standard normal distribution.

a. $P(z > -.98)$ **b.** $P(-2.47 \le z \le 1.29)$
c. $P(0 \le z \le 4.25)$ **d.** $P(-5.36 \le z \le 0)$
e. $P(z > 6.07)$ **f.** $P(z < -5.27)$

6.26 Obtain the following probabilities for the standard normal distribution.

a. $P(z > -1.86)$ **b.** $P(-.68 \le z \le 1.94)$
c. $P(0 \le z \le 3.85)$ **d.** $P(-4.34 \le z \le 0)$
e. $P(z > 4.82)$ **f.** $P(z < -6.12)$

6.33 Let x be a continuous random variable that is normally distributed with a mean of 25 and a standard deviation of 6. Find the probability that x assumes a value

a. between 29 and 36 **b.** between 22 and 35

6.34 Let x be a continuous random variable that has a normal distribution with a mean of 40 and a standard deviation of 4. Find the probability that x assumes a value

a. between 29 and 35 **b.** from 34 to 50

6.35 Let x be a continuous random variable that is normally distributed with a mean of 80 and a standard deviation of 12. Find the probability that x assumes a value

a. greater than 69 **b.** less than 73
c. greater than 101 **d.** less than 87

6.36 Let x be a continuous random variable that is normally distributed with a mean of 65 and a standard deviation of 15. Find the probability that x assumes a value

a. less than 45 **b.** greater than 79
c. greater than 54 **d.** less than 70