Edge Detection from Spectral Phase Data

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Objectives

Spectral data are collected in many imaging applications. Reconstruction schemes typically rely on priors about edges in the image, hence the need for good edge detectors. However, the full set of data are rarely fully available in practice. The magnitude of Fourier data can be corrupt or lost to noise, leading to inaccurate edge detection and signal reconstructions. This study set out to explore:

- What edge information is unique to phase and magnitude data?
- What detection methods clearly separate phase and magnitude?
- Are phase-only edge detecting schemes resilient to noisy or intermittent data?

Introduction

The N-th *Fourier partial sum* of a function f is a harmonic approximation to f given by

$$S_N f(x) = \sum_{k=-N}^{N} \hat{f}_k e^{ikx},$$

where the *Fourier coefficients* $\hat{f}_k$ are given by the *Fourier transform* of f, viz.,

$$\hat{f}_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-ikx}\, \mathrm{d}x.$$

Define the *jump function* of f, $[f](x)$, as the difference between the right and left hand limits of f at every point x,

$$[f](x) = f(x^+) - f(x^-).$$

Dirichlet devised a relationship between these limits and the Fourier partial sum approximation of f,

$$S_N f(x) = \sum_{k=-N}^{N} \hat{f}_k e^{ikx} \longrightarrow \frac{1}{2}(f(x^+) + f(x^-)).$$

The concentration factor edge detection method [1] introduces a *concentration factor* σ which modifies the partial sum to concentrate along the singular support of the underlying function by

$$S_N^\sigma[f](x) = \sum_{k=-N}^{N} i\,\mathrm{sgn}(k)\sigma(k)\hat{f}_k e^{ikx} \longrightarrow [f](x).$$

Figure 1: Analytical concentration factors with associated jump approximations.

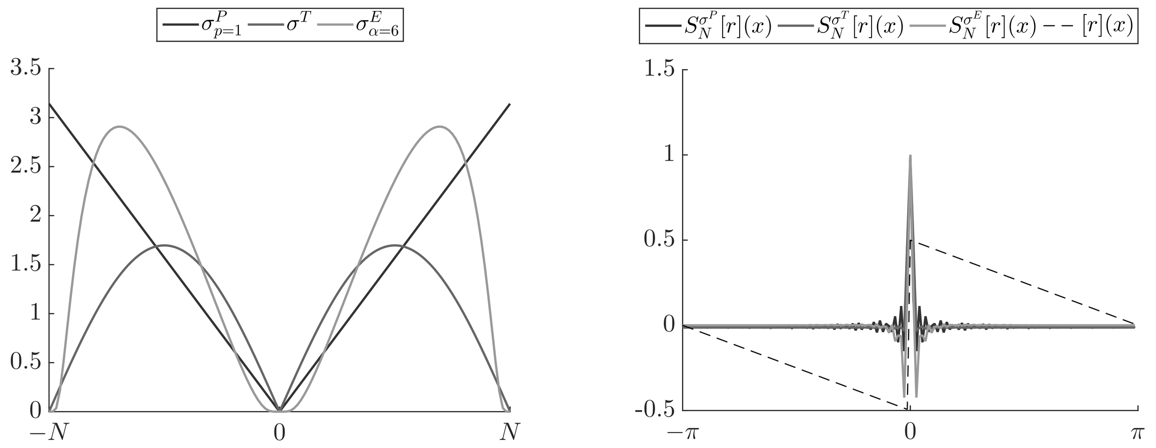
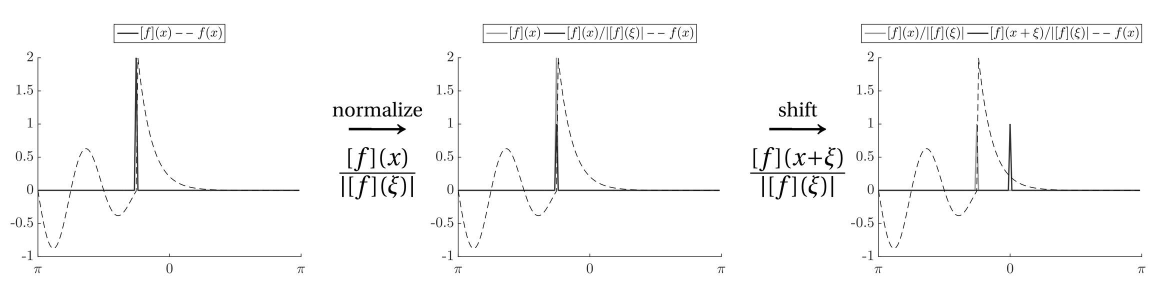


Figure 2: Concentration factor σ^P applied to create an edge map.

Concentration Factor Design

To design other concentration factors, we need to find a concentration factor σ so that $S_N^\sigma[f](x) \longrightarrow [f](x)$. Suppose f has one jump at ξ.

Figure 3: Design motivation.



This scaled and shifted version is identical to the jump of the ramp $r(x)$ as shown in Figure 1, where

$$r(x) = (-x + \mathrm{sgn}(x)\pi)/2\pi, \quad [r](x \neq 0) = 0, \quad [r](0) = 1.$$

Note that the Fourier coefficients of a 'nice' function f with one jump at $x = \xi$ are related to those of the ramp as

$$\hat{f}_k = \frac{[f](\xi)}{2\pi i k} e^{-ik\xi} + O(k^{-2}) \approx \hat{r}_k [f](\xi) e^{-ik\xi}$$

where

$$\hat{r}_k = \frac{1}{2\pi i k} \text{ for } k \neq 0, \quad \hat{r}_0 = 0.$$

Thus the solution σ to $W_0^\sigma(x) := S_N^\sigma[r](x) \longrightarrow [r](x)$ can be computed as $[r](x)$ and $\hat{r}_k$ are known. Since the Fourier coefficients of f contain the necessary scaling factor and phase shift, the solution σ will also satisfy $S_N^\sigma[f](x) \longrightarrow [f](x)$. Convex optimization is used to solve for concentration factors σ satisfying constraints we wish to impose. For a typical example, consider the problem formulation given by

$$\min_\sigma \ \|W_0^\sigma(x) - [r](x)\|_2 + \lambda \|W_0^\sigma(x)\|_1$$
$$\text{subject to } \ W_0^\sigma(x)|_{x=0} = 1.$$

Spectral Phase Data

Fourier coefficients $\hat{f}_k$ are complex numbers; they can be written in *phasor form*

$$\hat{f}_k = A_k e^{i\phi_k}$$

where $A_k \geq 0$ is the *amplitude* and ϕ_k is the *phase*. If the amplitude is removed from the data, then only the phase ϕ_k is known. However, importantly, phase data contains most of the information of features [2], as demonstrated in Figure 4.

Figure 4: Swapping spectral magnitude of two images.

Spectral Phase Data and Discontinuities

The locations and signs of jumps are entirely encoded within the phase of the spectral data. Thus, the analytical [1] and designed [3] concentration factors can be modified to approximate the jump function with exact jump locations even when only phase information from the spectral data are given.

Post-Processing Schemes

A few post-processing schemes for the concentration factor method have been developed [4], [5] utilizing multiple reconstructions from different concentration factors. In [6], the covariance between two jump approximations in noisy data is constructed; recent work shows that the covariance is maximized when the concentration factors are orthogonal. The design method is utilized to create orthogonal concentration factors, and then post-processed to create an enhanced [4] edge detector.

Jump Detection from Spectral Phase Data

Figure 5: Jump approximations from: (i) standard, (ii) intermittent, (iii) noisy intermittent, (iv) post-processed, noisy intermittent phase data.

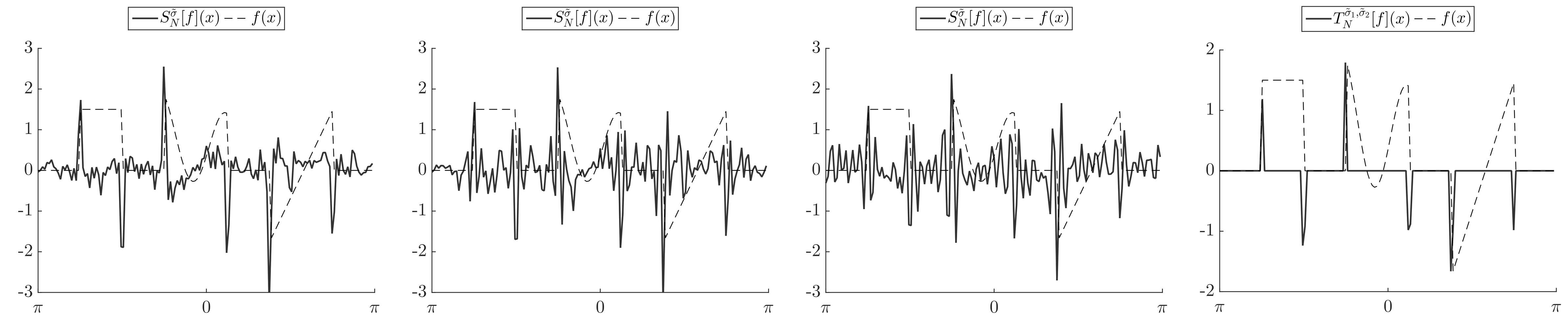


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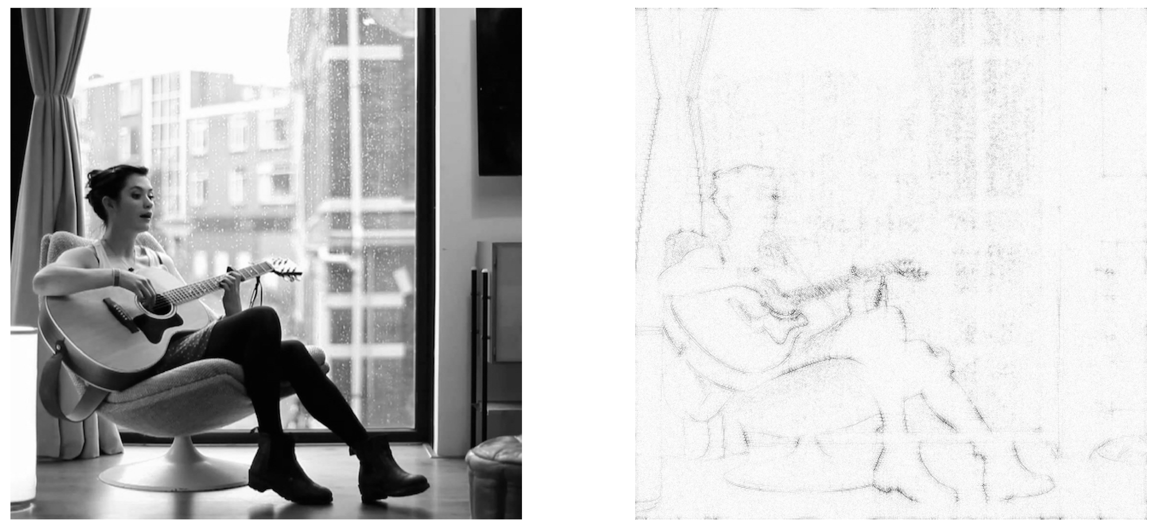
Figure 6: Enhanced edge map from intermittent phase data.



Conclusions and Future Work

A design approach allows for heavy flexibility in tailoring edge detection to the data given, including when data is missing and when magnitude information is removed. Further avenues of work lie ahead for edge detection, especially in the statistical realm, including post-processing schemes which rely less on human intervention.

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