ML_Assignment_2_Task_1_SalehAlkhalifa

October 9, 2024

1 Assignment 2 - Task 1

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1.0.1 1. Import Libraries and Data:

[344]:
```
import pandas as pd
import seaborn as sns
import matplotlib.pyplot as plt
from scipy import stats
```

[345]:
```
df = pd.read_csv("HousingData.csv")
```

1.0.2 2. Exploratory Data Analysis:

[346]:
```
df.head()
```

[346]:
```
      CRIM    ZN  INDUS  CHAS    NOX     RM   AGE     DIS  RAD  TAX  PTRATIO  \
0  0.00632  18.0   2.31   0.0  0.538  6.575  65.2  4.0900    1  296     15.3
1  0.02731   0.0   7.07   0.0  0.469  6.421  78.9  4.9671    2  242     17.8
2  0.02729   0.0   7.07   0.0  0.469  7.185  61.1  4.9671    2  242     17.8
3  0.03237   0.0   2.18   0.0  0.458  6.998  45.8  6.0622    3  222     18.7
4  0.06905   0.0   2.18   0.0  0.458  7.147  54.2  6.0622    3  222     18.7

        B  LSTAT  MEDV
0  396.90   4.98  24.0
1  396.90   9.14  21.6
2  392.83   4.03  34.7
3  394.63   2.94  33.4
```

```
4  396.90    NaN  36.2
```

[347]:
```python
# I cannot remember which feature is which, so I am renaming them for
↪additional context
column_mapping = {
    "CRIM": "crime_rate",
    "ZN": "residential_land",
    "INDUS": "business_land",
    "CHAS": "by_river",
    "NOX": "nox_concentration",
    "RM": "avg_rooms",
    "AGE": "old_homes",
    "DIS": "job_distance",
    "RAD": "highway_access",
    "TAX": "property_tax",
    "PTRATIO": "student_teacher_ratio",
    "B": "black_population",
    "LSTAT": "low_status_pct",
    "MEDV": "median_value"
}

df = df.rename(columns=column_mapping)
```

[348]:
```python
# Getting a sense of the rows and columns
df.shape
```

[348]:
```
(506, 14)
```

[349]:
```python
# Getting a sense of the data within the columns
df.describe()
```

[349]:
```
       crime_rate  residential_land  business_land    by_river  \
count  486.000000        486.000000     486.000000  486.000000
mean     3.611874         11.211934      11.083992    0.069959
std      8.720192         23.388876       6.835896    0.255340
min      0.006320          0.000000       0.460000    0.000000
25%      0.081900          0.000000       5.190000    0.000000
50%      0.253715          0.000000       9.690000    0.000000
75%      3.560263         12.500000      18.100000    0.000000
max     88.976200        100.000000      27.740000    1.000000

       nox_concentration   avg_rooms   old_homes  job_distance  \
count         506.000000  506.000000  486.000000    506.000000
mean            0.554695    6.284634   68.518519      3.795043
std             0.115878    0.702617   27.999513      2.105710
min             0.385000    3.561000    2.900000      1.129600
25%             0.449000    5.885500   45.175000      2.100175
```

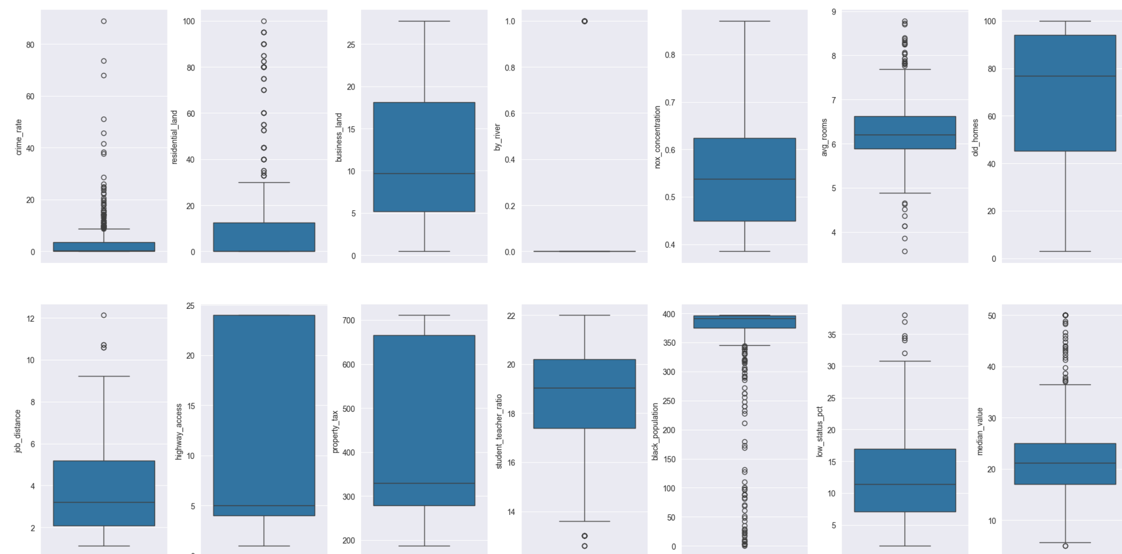
```
50%             0.538000    6.208500   76.800000      3.207450
75%             0.624000    6.623500   93.975000      5.188425
max             0.871000    8.780000  100.000000     12.126500

       highway_access  property_tax  student_teacher_ratio  black_population  \
count      506.000000    506.000000             506.000000        506.000000
mean         9.549407    408.237154              18.455534        356.674032
std          8.707259    168.537116               2.164946         91.294864
min          1.000000    187.000000              12.600000          0.320000
25%          4.000000    279.000000              17.400000        375.377500
50%          5.000000    330.000000              19.050000        391.440000
75%         24.000000    666.000000              20.200000        396.225000
max         24.000000    711.000000              22.000000        396.900000

       low_status_pct  median_value
count      486.000000    506.000000
mean        12.715432     22.532806
std          7.155871      9.197104
min          1.730000      5.000000
25%          7.125000     17.025000
50%         11.430000     21.200000
75%         16.955000     25.000000
max         37.970000     50.000000
```

[350]:

```python
# Understanding the data types
df.info()
```

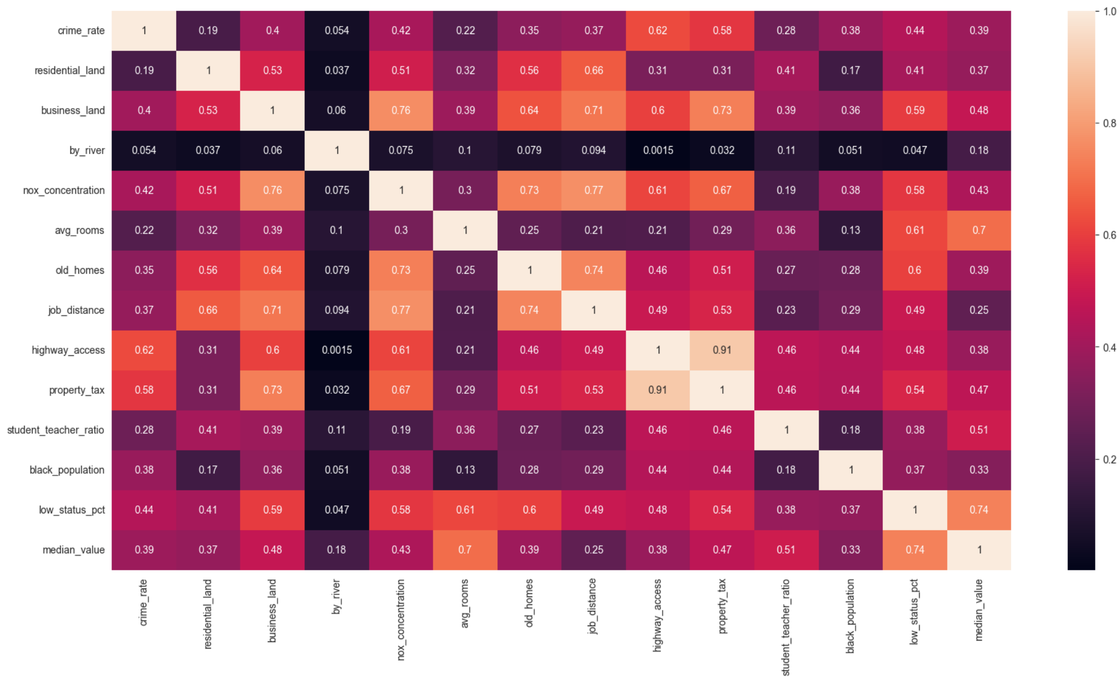
```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 506 entries, 0 to 505
Data columns (total 14 columns):
 #   Column                 Non-Null Count  Dtype
---  ------                 --------------  -----
 0   crime_rate             486 non-null    float64
 1   residential_land       486 non-null    float64
 2   business_land          486 non-null    float64
 3   by_river               486 non-null    float64
 4   nox_concentration      506 non-null    float64
 5   avg_rooms              506 non-null    float64
 6   old_homes              486 non-null    float64
 7   job_distance           506 non-null    float64
 8   highway_access         506 non-null    int64
 9   property_tax           506 non-null    int64
 10  student_teacher_ratio  506 non-null    float64
 11  black_population       506 non-null    float64
 12  low_status_pct         486 non-null    float64
 13  median_value           506 non-null    float64
dtypes: float64(12), int64(2)
memory usage: 55.5 KB
```

```
[351]: # Plotting the distributions of the features

fig, axis = plt.subplots(ncols=7, nrows=2, figsize=(20, 10))
idx = 0
axis = axis.flatten()
for key, val in df.items():
    sns.boxplot(y=key, data=df, ax=axis[idx])
    idx += 1
plt.tight_layout(pad=0.4, w_pad=0.5, h_pad=5.0)
```




```
[352]: plt.figure(figsize=(20, 10))
sns.heatmap(df.corr().abs(),  annot=True)
```

[352]: <Axes: >



[353]:
```python
import pandas as pd
import seaborn as sns
import matplotlib.pyplot as plt
from sklearn.preprocessing import StandardScaler, MinMaxScaler, PowerTransformer
from sklearn.decomposition import PCA
import numpy as np
from sklearn.model_selection import train_test_split

class BostonHousingEDA:
    def __init__(self, data):
        """
        Initializes the EDA class with the data for Boston Housing
        """
        # Set attributes
        self.data = data
        self.scaled_data = None
        self.cleaned_data = None
        self.X = None
        self.y = None
        self.column_mapping = {
            "CRIM": "crime_rate",
            "ZN": "residential_land",
            "INDUS": "business_land",
            "CHAS": "by_river",
            "NOX": "nox_concentration",
```

```
            "RM": "avg_rooms",
            "AGE": "old_homes",
            "DIS": "job_distance",
            "RAD": "highway_access",
            "TAX": "property_tax",
            "PTRATIO": "student_teacher_ratio",
            "B": "black_population",
            "LSTAT": "low_status_pct",
            "MEDV": "median_value"
        }
        self.data = self.data.rename(columns=self.column_mapping)

        # Define X and y
        self.X = self.data.drop(columns=['median_value'])
        self.y = self.data['median_value']

    def handle_missing_values(self, strategy="mean"):
        """
        Handles missing values in the dataset with three methods
        """
        if strategy == "mean":
            self.data.fillna(self.data.mean(), inplace=True)
        elif strategy == "median":
            self.data.fillna(self.data.median(), inplace=True)
        elif strategy == "drop":
            self.data.dropna(inplace=True)
        else:
            raise ValueError("Error. Select value")

    def visualize_data_distribution(self):
        """
        Visualize the distribution of all features in the dataset to get a␣
↪sense of shape
        """
        # Create the fig
        plt.figure(figsize=(20, 15))

        # Iterate over the columns and plot each
        for i, col in enumerate(self.data.columns, 1):
            # Create a subplot
            plt.subplot(4, 4, i)
            # Plot the histplot
            sns.histplot(self.data[col], kde=True, bins=30)
            plt.title(f'Distribution of {col}')
        plt.tight_layout()
        plt.show()
```

```
    def visualize_correlation_matrix(self):
        """
        Plot the correlation matrix
        """
        # Get correlation
        corr_matrix = self.data.corr()
        # Create fig
        plt.figure(figsize=(12, 8))
        # Create heatmap
        sns.heatmap(corr_matrix, annot=True, cmap='coolwarm', linewidths=0.5)
        plt.title('Correlation Matrix')
        plt.show()

    def handle_multicollinearity(self, threshold=0.75):
        """
        Identify and remove highly correlated features based on a given␣
↪threshold.
        """
        # add target y
        self.y = self.data['median_value']

        # create a copy of the features
        self.X = self.data.drop(columns=['median_value'])

        # calc the correlation matrix of the features
        corr_matrix = self.X.corr().abs()
        upper_tri = corr_matrix.where(np.triu(np.ones(corr_matrix.shape), k=1).
↪astype(bool))

        # drop with high corr
        to_drop = [column for column in upper_tri.columns if␣
↪any(upper_tri[column] > threshold)]
        self.cleaned_data = self.X.drop(columns=to_drop)

        # update self.X with cleaned data
        self.X = self.cleaned_data

        print(f"The columns dropped due to MCL: {to_drop}")
        print(f"Remaining columns: {self.cleaned_data.columns.tolist()}")

        return self.cleaned_data, self.y



    def scale_data(self, method='standard'):
        """
        Scale the data using the selected method and return scaled X and y
```

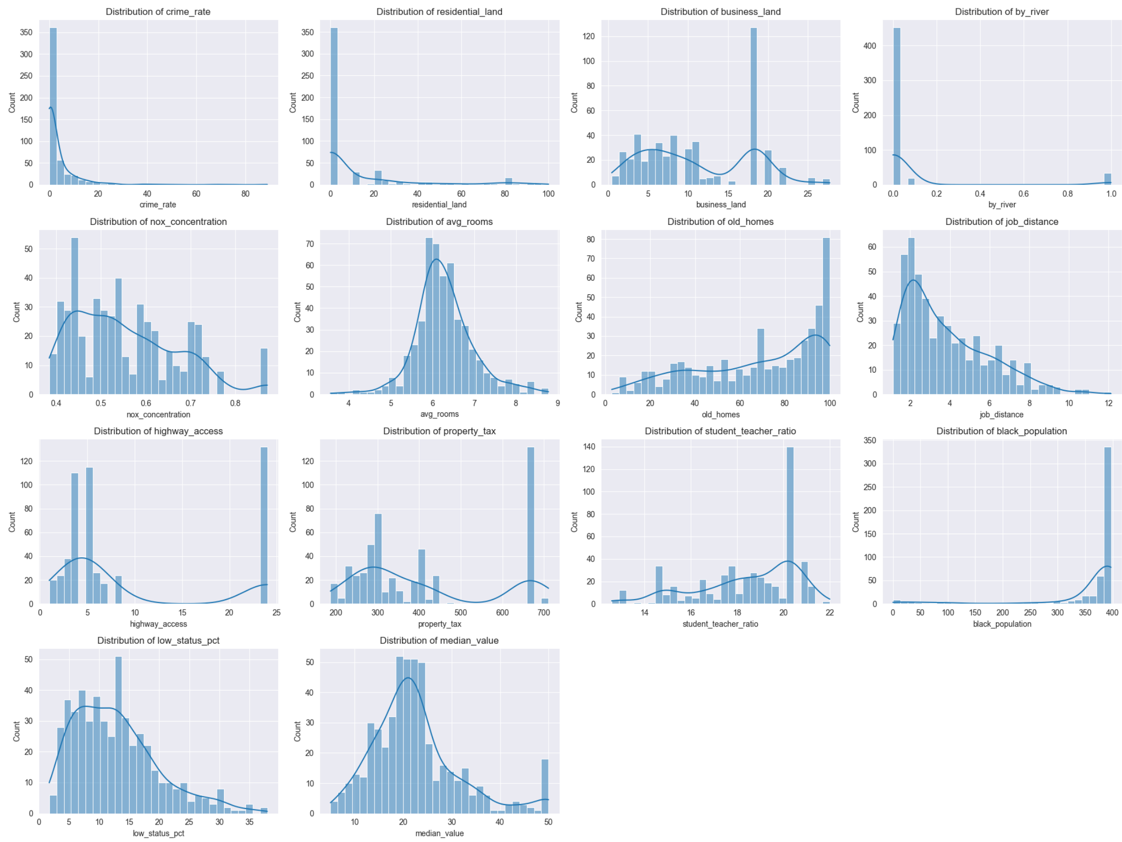
```
        """

        # If/else with three options to compare for assignemtn
        if method == 'standard':
            scaler = StandardScaler()
        elif method == 'minmax':
            scaler = MinMaxScaler()
        elif method == 'power':
            scaler = PowerTransformer()
        else:
            raise ValueError("Error. Pick one")

        # Use cleaned data
        data_to_scale = self.X if self.X is not None else self.data.
↪drop(columns=['median_value'])
        self.scaled_data = pd.DataFrame(scaler.fit_transform(data_to_scale),␣
↪columns=data_to_scale.columns)

        # return X and y for splitting
        return self.scaled_data, self.y

    def pca_visualization(self, n_components=2):
        """
        Apply the PCA algo to visualize the data
        """

        # Check for scaled data
        if self.scaled_data is None:
            print("Please scale the data first.")
            return

        # Apply PCA
        pca = PCA(n_components=n_components)
        pca_components = pca.fit_transform(self.scaled_data)

        # Plot results
        plt.figure(figsize=(8, 6))
        plt.scatter(pca_components[:, 0], pca_components[:, 1], c=self.y,␣
↪cmap='viridis')
        plt.colorbar(label='Median Value')
        plt.xlabel('Principal Component 1')
        plt.ylabel('Principal Component 2')
        plt.title('PCA Visualization (2 components)')
        plt.show()

    def summarize_data(self):
        """
```

```python
        Summarize data
        """
        print(self.data.describe())

    def show_r2(self, y_test, y_pred):
        """
        show the r2 plot
        """
        # calc r2 score
        r2 = r2_score(y_test, y_pred)
        print(f"R2 Score: {r2:.4f}")

        # Plot y_test vs y_pred
        plt.figure(figsize=(8, 6))
        plt.scatter(y_test, y_pred, edgecolor='k', alpha=0.7, label='Predicted␣
↪Vals vs Actual Vals')
        plt.plot([y_test.min(), y_test.max()], [y_test.min(), y_test.max()],␣
↪'r--', lw=2, label='Line of Perfect fit')
        plt.xlabel('Actual Values')
        plt.ylabel('Predicted Values')
        plt.title(f'R2 Correlation: {r2:.4f}')
        plt.legend()
        plt.grid(True)
        plt.show()
```

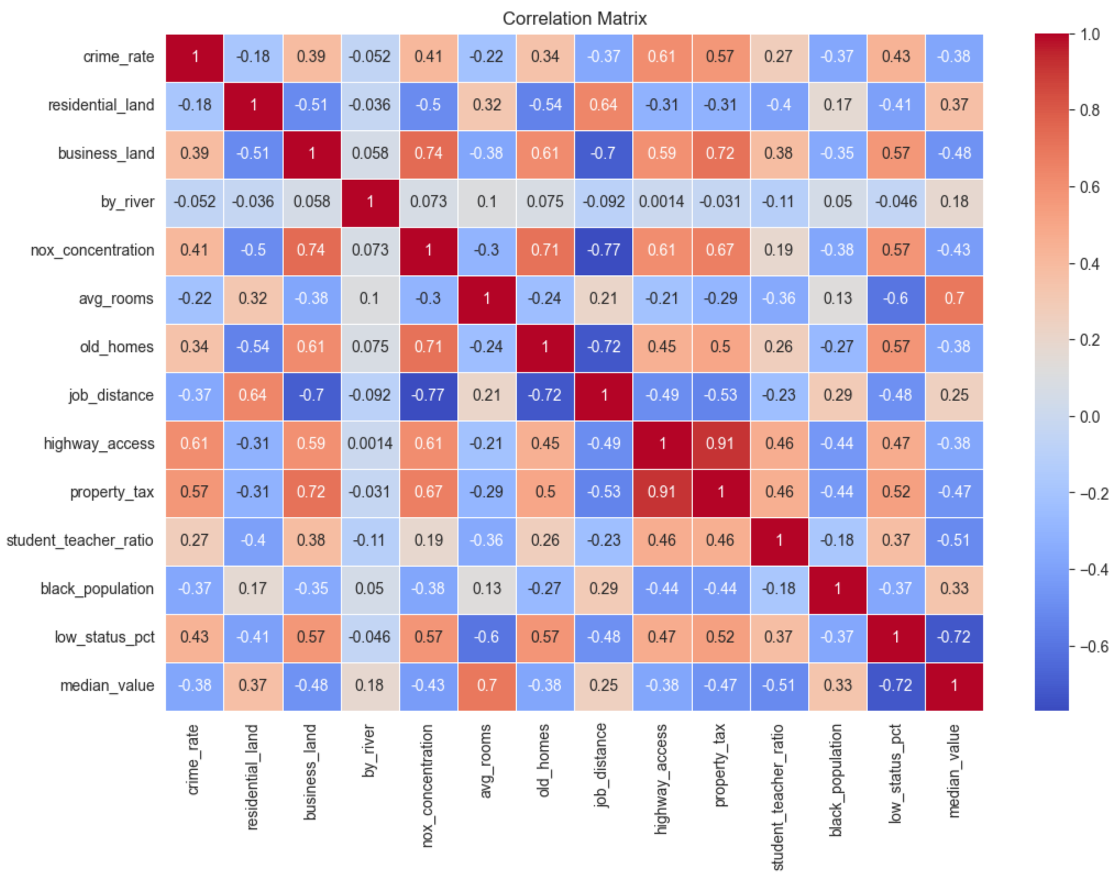
[354]:
```python
# Load the dataset
df = pd.read_csv("HousingData.csv")
```

[355]:
```python
# Initialize the EDA class
eda = BostonHousingEDA(df)

# Handle missing values
eda.handle_missing_values(strategy='mean')
```

[356]:
```python
# Visualize the data distribution
eda.visualize_data_distribution()
```



[357]:
```
# Visualize the correlation matrix
eda.visualize_correlation_matrix()
```



1.1 3. Data Preprocessing:

[358]:
```
eda.summarize_data()
```

```
        crime_rate  residential_land  business_land    by_river  \
count   506.000000        506.000000     506.000000  506.000000
mean      3.611874         11.211934      11.083992    0.069959
std       8.545770         22.921051       6.699165    0.250233
min       0.006320          0.000000       0.460000    0.000000
25%       0.083235          0.000000       5.190000    0.000000
50%       0.290250          0.000000       9.900000    0.000000
75%       3.611874         11.211934      18.100000    0.000000
max      88.976200        100.000000      27.740000    1.000000

        nox_concentration   avg_rooms   old_homes  job_distance  \
count          506.000000  506.000000  506.000000    506.000000
mean             0.554695    6.284634   68.518519      3.795043
std              0.115878    0.702617   27.439466      2.105710
min              0.385000    3.561000    2.900000      1.129600
25%              0.449000    5.885500   45.925000      2.100175
```

```
50%             0.538000    6.208500   74.450000      3.207450
75%             0.624000    6.623500   93.575000      5.188425
max             0.871000    8.780000  100.000000     12.126500

       highway_access  property_tax  student_teacher_ratio  black_population  \
count      506.000000    506.000000             506.000000        506.000000
mean         9.549407    408.237154              18.455534        356.674032
std          8.707259    168.537116               2.164946         91.294864
min          1.000000    187.000000              12.600000          0.320000
25%          4.000000    279.000000              17.400000        375.377500
50%          5.000000    330.000000              19.050000        391.440000
75%         24.000000    666.000000              20.200000        396.225000
max         24.000000    711.000000              22.000000        396.900000

       low_status_pct  median_value
count      506.000000    506.000000
mean        12.715432     22.532806
std          7.012739      9.197104
min          1.730000      5.000000
25%          7.230000     17.025000
50%         11.995000     21.200000
75%         16.570000     25.000000
max         37.970000     50.000000
```

[399]:
```python
# Handle multicollinearity
X, y = eda.handle_multicollinearity(threshold=0.75)

X.head()
```

```
The columns dropped due to MCL: ['job_distance', 'property_tax']
Remaining columns: ['crime_rate', 'residential_land', 'business_land',
'by_river', 'nox_concentration', 'avg_rooms', 'old_homes', 'highway_access',
'student_teacher_ratio', 'black_population', 'low_status_pct']
```

[399]:
```
   crime_rate  residential_land  business_land  by_river  nox_concentration  \
0     0.00632              18.0           2.31       0.0              0.538
1     0.02731               0.0           7.07       0.0              0.469
2     0.02729               0.0           7.07       0.0              0.469
3     0.03237               0.0           2.18       0.0              0.458
4     0.06905               0.0           2.18       0.0              0.458

   avg_rooms  old_homes  highway_access  student_teacher_ratio  \
0      6.575       65.2               1                   15.3
1      6.421       78.9               2                   17.8
2      7.185       61.1               2                   17.8
3      6.998       45.8               3                   18.7
4      7.147       54.2               3                   18.7
```

```
   black_population  low_status_pct
0            396.90        4.980000
1            396.90        9.140000
2            392.83        4.030000
3            394.63        2.940000
4            396.90       12.715432
```

[399]:

```

```

[400]:

```
# sxcale data
X, y = eda.scale_data(method='minmax')

print(X.shape)

X.head()
```

```
(506, 11)
```

[400]:

```
   crime_rate  residential_land  business_land  by_river  nox_concentration  \
0    0.000000              0.18       0.067815       0.0           0.314815
1    0.000236              0.00       0.242302       0.0           0.172840
2    0.000236              0.00       0.242302       0.0           0.172840
3    0.000293              0.00       0.063050       0.0           0.150206
4    0.000705              0.00       0.063050       0.0           0.150206

   avg_rooms  old_homes  highway_access  student_teacher_ratio  \
0   0.577505   0.641607        0.000000               0.287234
1   0.547998   0.782698        0.043478               0.553191
2   0.694386   0.599382        0.043478               0.553191
3   0.658555   0.441813        0.086957               0.648936
4   0.687105   0.528321        0.086957               0.648936

   black_population  low_status_pct
0          1.000000        0.089680
1          1.000000        0.204470
2          0.989737        0.063466
3          0.994276        0.033389
4          1.000000        0.303130
```

[401]:

```
print(X.head())
```

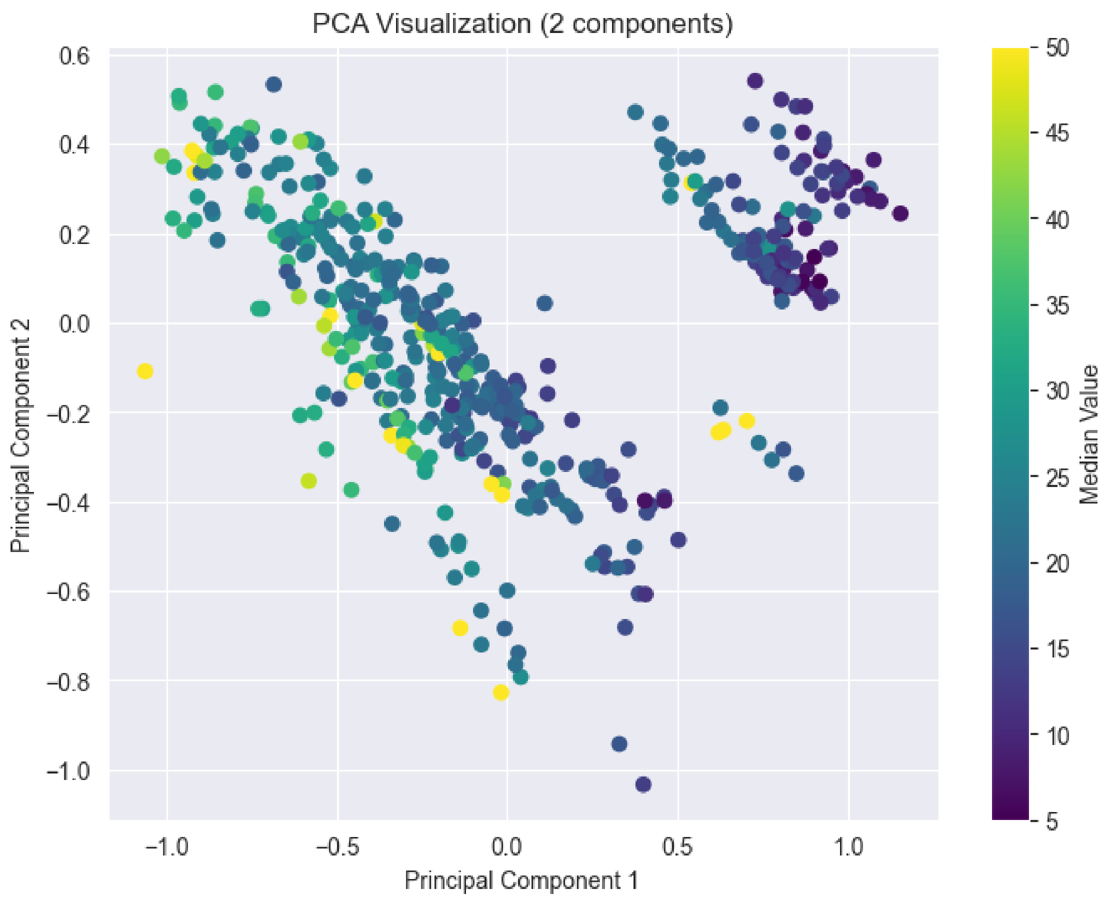
```
   crime_rate  residential_land  business_land  by_river  nox_concentration  \
0    0.000000              0.18       0.067815       0.0           0.314815
1    0.000236              0.00       0.242302       0.0           0.172840
2    0.000236              0.00       0.242302       0.0           0.172840
3    0.000293              0.00       0.063050       0.0           0.150206
4    0.000705              0.00       0.063050       0.0           0.150206
```

```
   avg_rooms  old_homes  highway_access  student_teacher_ratio  \
0   0.577505   0.641607        0.000000               0.287234   
1   0.547998   0.782698        0.043478               0.553191   
2   0.694386   0.599382        0.043478               0.553191   
3   0.658555   0.441813        0.086957               0.648936   
4   0.687105   0.528321        0.086957               0.648936   

   black_population  low_status_pct  
0          1.000000        0.089680  
1          1.000000        0.204470  
2          0.989737        0.063466  
3          0.994276        0.033389  
4          1.000000        0.303130  
```

[402]:
```python
len(y)
```

[402]: 506

[403]:
```python
# visualize again
eda.pca_visualization(n_components=2)
```



[404]:
```
# describe
eda.summarize_data()
```

```
       crime_rate  residential_land  business_land    by_river  \
count  506.000000        506.000000     506.000000  506.000000
mean     3.611874         11.211934      11.083992    0.069959
std      8.545770         22.921051       6.699165    0.250233
min      0.006320          0.000000       0.460000    0.000000
25%      0.083235          0.000000       5.190000    0.000000
50%      0.290250          0.000000       9.900000    0.000000
75%      3.611874         11.211934      18.100000    0.000000
max     88.976200        100.000000      27.740000    1.000000

       nox_concentration   avg_rooms   old_homes  job_distance  \
count         506.000000  506.000000  506.000000    506.000000
mean            0.554695    6.284634   68.518519      3.795043
std             0.115878    0.702617   27.439466      2.105710
min             0.385000    3.561000    2.900000      1.129600
25%             0.449000    5.885500   45.925000      2.100175
50%             0.538000    6.208500   74.450000      3.207450
75%             0.624000    6.623500   93.575000      5.188425
max             0.871000    8.780000  100.000000     12.126500

       highway_access  property_tax  student_teacher_ratio  black_population  \
count      506.000000    506.000000             506.000000        506.000000
mean         9.549407    408.237154              18.455534        356.674032
std          8.707259    168.537116               2.164946         91.294864
min          1.000000    187.000000              12.600000          0.320000
25%          4.000000    279.000000              17.400000        375.377500
50%          5.000000    330.000000              19.050000        391.440000
75%         24.000000    666.000000              20.200000        396.225000
max         24.000000    711.000000              22.000000        396.900000

       low_status_pct  median_value
count      506.000000    506.000000
mean        12.715432     22.532806
std          7.012739      9.197104
min          1.730000      5.000000
25%          7.230000     17.025000
50%         11.995000     21.200000
75%         16.570000     25.000000
max         37.970000     50.000000
```

[405]:
```
from sklearn.model_selection import train_test_split

# Split the X and y from each other
```

```python
# Test this on different scalers to see what works best
X, y = eda.scale_data(method='minmax')
```

[406]:

```python
X
```

[406]:

```
     crime_rate  residential_land  business_land  by_river  nox_concentration  \
0      0.000000              0.18       0.067815       0.0           0.314815
1      0.000236              0.00       0.242302       0.0           0.172840
2      0.000236              0.00       0.242302       0.0           0.172840
3      0.000293              0.00       0.063050       0.0           0.150206
4      0.000705              0.00       0.063050       0.0           0.150206
..          …                 …              …       …                 …
501    0.000633              0.00       0.420455       0.0           0.386831
502    0.000438              0.00       0.420455       0.0           0.386831
503    0.000612              0.00       0.420455       0.0           0.386831
504    0.001161              0.00       0.420455       0.0           0.386831
505    0.000462              0.00       0.420455       0.0           0.386831

     avg_rooms  old_homes  highway_access  student_teacher_ratio  \
0     0.577505   0.641607        0.000000               0.287234
1     0.547998   0.782698        0.043478               0.553191
2     0.694386   0.599382        0.043478               0.553191
3     0.658555   0.441813        0.086957               0.648936
4     0.687105   0.528321        0.086957               0.648936
..         …          …              …                     …
501   0.580954   0.681771        0.000000               0.893617
502   0.490324   0.760041        0.000000               0.893617
503   0.654340   0.907312        0.000000               0.893617
504   0.619467   0.889804        0.000000               0.893617
505   0.473079   0.675783        0.000000               0.893617

     black_population  low_status_pct
0            1.000000        0.089680
1            1.000000        0.204470
2            0.989737        0.063466
3            0.994276        0.033389
4            1.000000        0.303130
..                …               …
501          0.987619        0.303130
502          1.000000        0.202815
503          1.000000        0.107892
504          0.991301        0.131071
505          1.000000        0.169702

[506 rows x 11 columns]
```

[407]:
```
# Split the data in training and testing sets
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2,␣
↪random_state=42)
```

[408]:
```
print("X_train.shape", X_train.shape)
print("X_test.shape", X_test.shape)
print("y_train.shape", y_train.shape)
print("y_test.shape", y_test.shape)
```

```
X_train.shape (404, 11)
X_test.shape (102, 11)
y_train.shape (404,)
y_test.shape (102,)
```

[409]:
```
X_train
```

[409]:
```
     crime_rate  residential_land  business_land  by_river  nox_concentration  \
477    0.168788          0.000000       0.646628       0.0           0.471193
15     0.006981          0.000000       0.281525       0.0           0.314815
332    0.000319          0.112119       0.205279       0.0           0.108848
423    0.079174          0.000000       0.646628       0.0           0.471193
19     0.008087          0.000000       0.281525       0.0           0.314815
..          …                 …              …       …                  …
106    0.001853          0.000000       0.296921       0.0           0.277778
270    0.003291          0.200000       0.238270       0.0           0.162551
348    0.000098          0.800000       0.056818       0.0           0.102881
435    0.125369          0.000000       0.646628       0.0           0.730453
102    0.002500          0.000000       0.296921       0.0           0.277778

     avg_rooms  old_homes  highway_access  student_teacher_ratio  \
477   0.333972   0.972194        1.000000               0.808511
15    0.435524   0.552008        0.130435               0.893617
332   0.473271   0.210093        0.000000               0.457447
423   0.487066   0.675783        1.000000               0.808511
19    0.415022   0.685891        0.130435               0.893617
..         …          …               …                      …
106   0.435907   0.916581        0.173913               0.882979
270   0.439739   0.403708        0.086957               0.638298
348   0.589002   0.276004        0.130435               0.468085
435   0.587852   0.944387        1.000000               0.808511
102   0.544932   0.849640        0.173913               0.882979

     black_population  low_status_pct
477          0.880428        0.639625
15           0.996772        0.185982
332          0.912628        0.168322
423          0.005547        0.594923
19           0.984997        0.263521
```

```
..             …             …
106          0.996898       0.467163
270          0.979197       0.310982
348          0.984972       0.117550
435          0.276186       0.594371
102          0.177720       0.245585

[404 rows x 11 columns]
```

[410]:
```
X_train.head()
```

[410]:
```
     crime_rate  residential_land  business_land  by_river  nox_concentration  \
477    0.168788          0.000000       0.646628       0.0           0.471193
15     0.006981          0.000000       0.281525       0.0           0.314815
332    0.000319          0.112119       0.205279       0.0           0.108848
423    0.079174          0.000000       0.646628       0.0           0.471193
19     0.008087          0.000000       0.281525       0.0           0.314815

     avg_rooms  old_homes  highway_access  student_teacher_ratio  \
477   0.333972   0.972194        1.000000               0.808511
15    0.435524   0.552008        0.130435               0.893617
332   0.473271   0.210093        0.000000               0.457447
423   0.487066   0.675783        1.000000               0.808511
19    0.415022   0.685891        0.130435               0.893617

     black_population  low_status_pct
477          0.880428        0.639625
15           0.996772        0.185982
332          0.912628        0.168322
423          0.005547        0.594923
19           0.984997        0.263521
```

[411]:
```
y_train
```

[411]:
```
477    12.0
15     19.9
332    19.4
423    13.4
19     18.2
       …
106    19.5
270    21.1
348    24.5
435    13.4
102    18.6
Name: median_value, Length: 404, dtype: float64
```

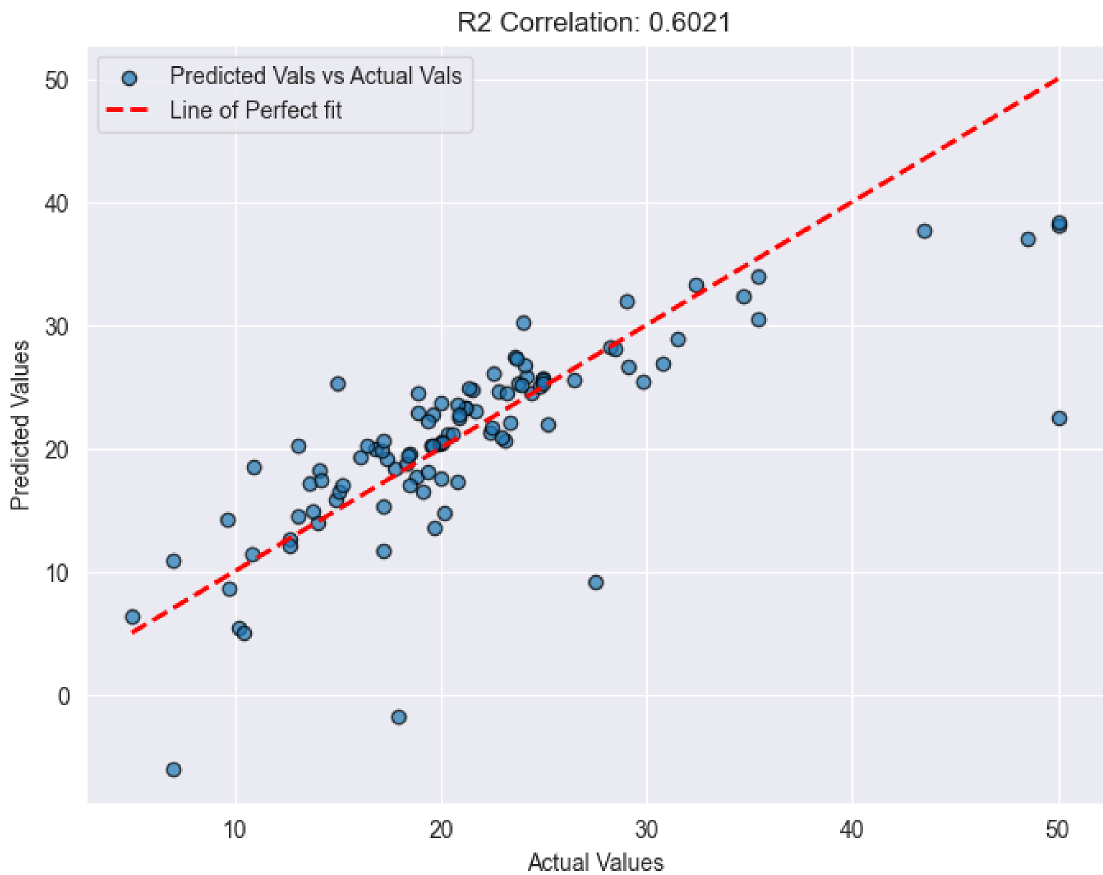
1.1.1 4. OLS regression:

[412]:
```python
import numpy as np

class OLSRegression:
    def __init__(self):
        self.theta = None

    def fit(self, X, y):
        """
        Fit method that computes OLS
        """
        # add bias term to X
        ones_column = np.ones((X.shape[0], 1))
        X_bias = np.concatenate((ones_column, X), axis=1)

        # Compute X^T
        X_transpose = X_bias.T

        # multiply X^T by X
        X_transpose_X = X_transpose.dot(X_bias)

        # inverse of (X^T * X)
        X_transpose_X_inv = np.linalg.inv(X_transpose_X)

        # multiply the inverse by X^T
        X_transpose_X_inv_X_transpose = X_transpose_X_inv.dot(X_transpose)

        # multiply by y to get the coefficiens
        self.theta = X_transpose_X_inv_X_transpose.dot(y)


    def predict(self, X):
        """
        Make predictions
        """
        # add bias term to X
        ones_column = np.ones((X.shape[0], 1))
        X_bias = np.concatenate((ones_column, X), axis=1)

        # multiply X_bias by theta
        y_pred = X_bias.dot(self.theta)

        # Return the pred values
        return y_pred
```

[413]:

```python
from sklearn.metrics import mean_squared_error, r2_score

# Initialize and train the OLS regression model
ols_model = OLSRegression()

# Fit the model on training data
ols_model.fit(X_train, y_train)

# Make predictions on the test set
y_pred = ols_model.predict(X_test)

# Evaluate the model
mse = mean_squared_error(y_test, y_pred)
r2 = r2_score(y_test, y_pred)

print(f"OLS Regression Mean Squared Error: {mse}")
print(f"OLS Regression R2 Score: {r2}")

eda.show_r2(y_test, y_pred)
```

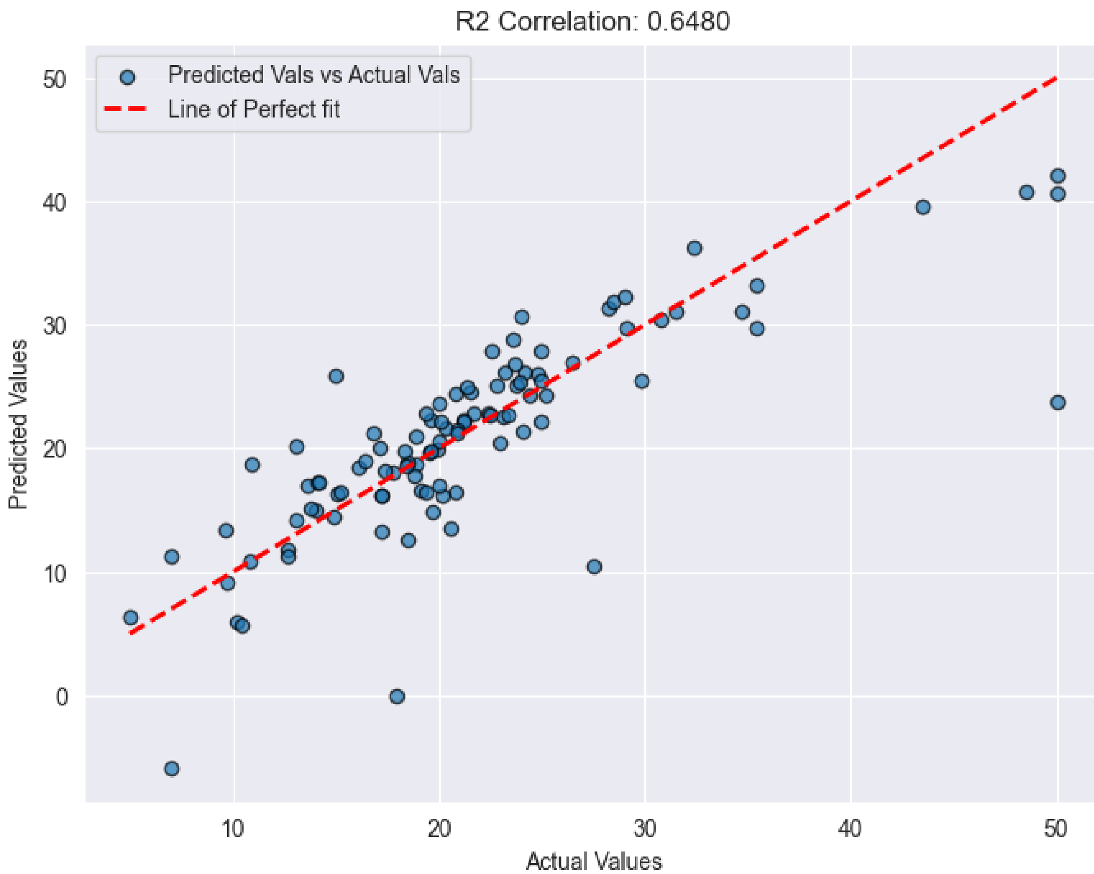
```
OLS Regression Mean Squared Error: 29.17862654453569
OLS Regression R2 Score: 0.6021120786753205
R2 Score: 0.6021
```



[337]:

```
from sklearn.linear_model import LinearRegression

# Testing Linear Regression from SKLearn to compare

# Init the linreg model
lr_model = LinearRegression()
lr_model.fit(X_train, y_train)

# predict
y_pred_lr = lr_model.predict(X_test)

# Evaluate like previously using MSE and R2
mse_lr = mean_squared_error(y_test, y_pred_lr)
r2_lr = r2_score(y_test, y_pred_lr)

print(f"SLK LinearRegression Mean Squared Error: {mse_lr}")
print(f"SKL LinearRegression R2 Score: {r2_lr}")

eda.show_r2(y_test, y_pred)
```

```
SLK LinearRegression Mean Squared Error: 25.812604382189612
SKL LinearRegression R2 Score: 0.6480120993382017
R2 Score: 0.6480
```



1.1.2 5. Ridge Regression:

[338]:
```python
class RidgeRegression:
    def __init__(self, alpha=1.0):
        """
        Init the Ridge Regression model
        """
        self.alpha = alpha
        self.theta = None

    def fit(self, X, y):
        """
        Fit method for Ridge Regression using the normal equation
        """
        # add bias term to X
```

```
        X_bias = np.c_[np.ones((X.shape[0], 1)), X]

        # Calcuate theta using the normal equation with L2 reg
        identity = np.eye(X_bias.shape[1])
        identity[0, 0] = 0
        self.theta = np.linalg.inv(X_bias.T.dot(X_bias) + self.alpha *␣
 ↪identity).dot(X_bias.T).dot(y)

    def predict(self, X):
        """
        predict method for ridge reg model
        """
        # add bias term (intercept)
        X_bias = np.c_[np.ones((X.shape[0], 1)), X]
        return X_bias.dot(self.theta)
```

[339]:
```
alphas = [0.5, 1, 1.5, 2]

#iterate over multiple alpha vals
for alpha in alphas:
    print("-----------------------")
    print(f"\nAlpha = {alpha}:")
    ridge = RidgeRegression(alpha=alpha)
    ridge.fit(X_train, y_train)
    y_pred_ridge = ridge.predict(X_test)
    print(f"Ridge MSE: {mean_squared_error(y_test, y_pred_ridge)}")
    print(f"Ridge R2: {r2_score(y_test, y_pred_ridge)}")
```

```
-----------------------

Alpha = 0.5:
Ridge MSE: 25.19079436852606
Ridge R2: 0.6564912747859506
-----------------------

Alpha = 1:
Ridge MSE: 24.832196260159254
Ridge R2: 0.6613812189960132
-----------------------

Alpha = 1.5:
Ridge MSE: 24.63624181400551
Ridge R2: 0.6640533086893179
-----------------------

Alpha = 2:
Ridge MSE: 24.548498714424547
Ridge R2: 0.665249798162514
```

1.1.3 5. Lasso Regression:

[340]:
```
class LassoRegression:
    def __init__(self, alpha=1.0, n_iterations=1000, learning_rate=0.01):
        """
        Init the Lasso Regression model
        """
        self.alpha = alpha
        self.n_iterations = n_iterations
        self.learning_rate = learning_rate
        self.theta = None

    def fit(self, X, y):
        """
        Fir method using coordinate desent
        """
        # add bias term to X
        X_bias = np.c_[np.ones((X.shape[0], 1)), X]

        # set samples and feats
        n_samples, n_features = X_bias.shape

        # init theta
        self.theta = np.zeros(n_features)

        # implement gradient descent
        for _ in range(self.n_iterations):

            # set pred
            y_pred = X_bias.dot(self.theta)

            # set gradient via bias times dot
            gradients = X_bias.T.dot(y_pred - y) / n_samples

            # update theta with L1 reg
            for j in range(1, n_features):
                self.theta[j] -= self.learning_rate * (gradients[j] + self.
↪alpha * np.sign(self.theta[j]))

            # update intercept separately
            self.theta[0] -= self.learning_rate * gradients[0]

    def predict(self, X):
        """
        pred new data with Lasso reg
        """
        # Add bias term
```

```
        X_bias = np.c_[np.ones((X.shape[0], 1)), X]

        # return product
        return X_bias.dot(self.theta)
```

[341]:
```
from sklearn.metrics import mean_squared_error, r2_score

#iterate over multiple alpha vals
alphas = [0.5, 1, 1.5, 2]
for alpha in alphas:
    print("-----------------------")
    print(f"\nalpha = {alpha}:")
    lasso = LassoRegression(alpha=alpha, learning_rate=0.01, n_iterations=1000)
    lasso.fit(X_train, y_train)
    y_pred_lasso = lasso.predict(X_test)
    print(f"Lasso MSE: {mean_squared_error(y_test, y_pred_lasso)}")
    print(f"Lasso R2: {r2_score(y_test, y_pred_lasso)}")
```

```
-----------------------

alpha = 0.5:
Lasso MSE: 49.00844114265968
Lasso R2: 0.33170717463850197
-----------------------

alpha = 1:
Lasso MSE: 62.05883125966329
Lasso R2: 0.15374840100653564
-----------------------

alpha = 1.5:
Lasso MSE: 66.47302374946402
Lasso R2: 0.09355523627984508
-----------------------

alpha = 2:
Lasso MSE: 70.73738742806061
Lasso R2: 0.03540517917350694
```

[342]:
```
from sklearn.linear_model import Ridge, Lasso

alphas = [0.5, 0.75, 1, 1.5, 2]

# iterate over multiple alpha vals
for alpha in alphas:
    # ridge model
    print(f"\nSklearn Ridge alpha = {alpha}:")
    sklearn_ridge = Ridge(alpha=alpha)
```

```python
    sklearn_ridge.fit(X_train, y_train)
    y_pred_ridge_sklearn = sklearn_ridge.predict(X_test)
    print(f"Ridge MSE: {mean_squared_error(y_test, y_pred_ridge_sklearn)}")
    print(f"Ridge R²: {r2_score(y_test, y_pred_ridge_sklearn)}")

    # lasso model
    print(f"\nSklearn Lasso alpha = {alpha}:")
    sklearn_lasso = Lasso(alpha=alpha, max_iter=1000)
    sklearn_lasso.fit(X_train, y_train)
    y_pred_lasso_sklearn = sklearn_lasso.predict(X_test)
    print(f"Lasso MSE: {mean_squared_error(y_test, y_pred_lasso_sklearn)}")
    print(f"Lasso R²: {r2_score(y_test, y_pred_lasso_sklearn)}")
```

```
Sklearn Ridge alpha = 0.5:
Ridge MSE: 25.19079436852541
Ridge R²: 0.6564912747859595

Sklearn Lasso alpha = 0.5:
Lasso MSE: 33.62830965344012
Lasso R²: 0.5414349539294651

Sklearn Ridge alpha = 0.75:
Ridge MSE: 24.986408159234436
Ridge R²: 0.6592783423622244

Sklearn Lasso alpha = 0.75:
Lasso MSE: 44.60185159749377
Lasso R²: 0.39179666348339326

Sklearn Ridge alpha = 1:
Ridge MSE: 24.83219626015898
Ridge R²: 0.6613812189960169

Sklearn Lasso alpha = 1:
Lasso MSE: 57.565892476652685
Lasso R²: 0.21501537223571798

Sklearn Ridge alpha = 1.5:
Ridge MSE: 24.636241814005345
Ridge R²: 0.6640533086893202

Sklearn Lasso alpha = 1.5:
Lasso MSE: 75.04543037399255
Lasso R²: -0.023340500652033302

Sklearn Ridge alpha = 2:
Ridge MSE: 24.548498714424312
```

```
Ridge R²: 0.6652497981625172

Sklearn Lasso alpha = 2:
Lasso MSE: 75.04543037399255
Lasso R²: -0.023340500652033302
```

1.2 6. Compare Results:

[343]:
```python
import pandas as pd
from sklearn.preprocessing import StandardScaler, MinMaxScaler, PowerTransformer
from sklearn.metrics import mean_squared_error, r2_score
from sklearn.linear_model import Ridge, Lasso

# Initialize and train the OLS regression model
def evaluate_models(X_train, X_test, y_train, y_test, alpha=None):
    # Initialize result dictionary
    results = {
        "Model": [],
        "alpha": [],
        "scaler": [],
        "Custom R2": [],
        "Custom MSE": [],
        "Sklearn R2": [],
        "Sklearn MSE": []
    }

    # custom OLS
    if alpha is None:
        ols_model = OLSRegression()
        ols_model.fit(X_train, y_train)
        y_pred_ols = ols_model.predict(X_test)
        mse_ols = mean_squared_error(y_test, y_pred_ols)
        r2_ols = r2_score(y_test, y_pred_ols)

        # Store OLS
        results["Model"].append("OLS")
        results["alpha"].append(None)
        results["scaler"].append(current_scaler_name)
        results["Custom R2"].append(r2_ols)
        results["Custom MSE"].append(mse_ols)
        results["Sklearn R2"].append(None)
        results["Sklearn MSE"].append(None)

    else:
        #Custom ridge
        custom_ridge = RidgeRegression(alpha=alpha)
        custom_ridge.fit(X_train, y_train)
```

```
        y_pred_custom_ridge = custom_ridge.predict(X_test)
        mse_custom_ridge = mean_squared_error(y_test, y_pred_custom_ridge)
        r2_custom_ridge = r2_score(y_test, y_pred_custom_ridge)

        # SKL Ridge
        sklearn_ridge = Ridge(alpha=alpha)
        sklearn_ridge.fit(X_train, y_train)
        y_pred_sklearn_ridge = sklearn_ridge.predict(X_test)
        mse_sklearn_ridge = mean_squared_error(y_test, y_pred_sklearn_ridge)
        r2_sklearn_ridge = r2_score(y_test, y_pred_sklearn_ridge)

        # Save results
        results["Model"].append("Ridge")
        results["alpha"].append(alpha)
        results["scaler"].append(current_scaler_name)
        results["Custom R2"].append(r2_custom_ridge)
        results["Custom MSE"].append(mse_custom_ridge)
        results["Sklearn R2"].append(r2_sklearn_ridge)
        results["Sklearn MSE"].append(mse_sklearn_ridge)

        # custom lasso
        custom_lasso = LassoRegression(alpha=alpha, learning_rate=0.01,␣
↪n_iterations=1000)
        custom_lasso.fit(X_train, y_train)
        y_pred_custom_lasso = custom_lasso.predict(X_test)
        mse_custom_lasso = mean_squared_error(y_test, y_pred_custom_lasso)
        r2_custom_lasso = r2_score(y_test, y_pred_custom_lasso)

        # sklearn lasso
        sklearn_lasso = Lasso(alpha=alpha, max_iter=1000)
        sklearn_lasso.fit(X_train, y_train)
        y_pred_sklearn_lasso = sklearn_lasso.predict(X_test)
        mse_sklearn_lasso = mean_squared_error(y_test, y_pred_sklearn_lasso)
        r2_sklearn_lasso = r2_score(y_test, y_pred_sklearn_lasso)

        # store results
        results["Model"].append("Lasso")
        results["alpha"].append(alpha)
        results["scaler"].append(current_scaler_name)
        results["Custom R2"].append(r2_custom_lasso)
        results["Custom MSE"].append(mse_custom_lasso)
        results["Sklearn R2"].append(r2_sklearn_lasso)
        results["Sklearn MSE"].append(mse_sklearn_lasso)

    return results

scalers = {
```

```
    "StandardScaler": StandardScaler(),
    "MinMaxScaler": MinMaxScaler(),
    "PowerTransformer": PowerTransformer()
}

# alpha values
alphas = [0.5, 1, 1.5, 2]

# Create DF
comparison_df = pd.DataFrame()

# Scaler mapping for original class
scaler_mapping = {
    "StandardScaler": "standard",
    "MinMaxScaler": "minmax",
    "PowerTransformer": "power"
}

# Loop through each scaler
for current_scaler_name, current_scaler in scalers.items():
    # Scale data
    X, y = eda.scale_data(method=scaler_mapping[current_scaler_name])

    # split to train and test
    X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2,␣
↪random_state=42)

    # evaulate OLS
    ols_results = evaluate_models(X_train, X_test, y_train, y_test, alpha=None)
    comparison_df = pd.concat([comparison_df, pd.DataFrame(ols_results)],␣
↪ignore_index=True)

    # loop through alphas and calc ridge and lasso
    for alpha in alphas:
        current_results = evaluate_models(X_train, X_test, y_train, y_test,␣
↪alpha)
        comparison_df = pd.concat([comparison_df, pd.
↪DataFrame(current_results)], ignore_index=True)

# SHow results
comparison_df
```

```
/var/folders/r4/9ncp61z543v94_5sltg11_bm0000gn/T/ipykernel_24506/2601374963.py:1
19: FutureWarning: The behavior of DataFrame concatenation with empty or all-NA
entries is deprecated. In a future version, this will no longer exclude empty or
all-NA columns when determining the result dtypes. To retain the old behavior,
exclude the relevant entries before the concat operation.
```

```
  comparison_df = pd.concat([comparison_df, pd.DataFrame(current_results)],
ignore_index=True)
/var/folders/r4/9ncp61z543v94_5sltg11_bm0000gn/T/ipykernel_24506/2601374963.py:1
14: FutureWarning: The behavior of DataFrame concatenation with empty or all-NA
entries is deprecated. In a future version, this will no longer exclude empty or
all-NA columns when determining the result dtypes. To retain the old behavior,
exclude the relevant entries before the concat operation.
  comparison_df = pd.concat([comparison_df, pd.DataFrame(ols_results)],
ignore_index=True)
/var/folders/r4/9ncp61z543v94_5sltg11_bm0000gn/T/ipykernel_24506/2601374963.py:1
14: FutureWarning: The behavior of DataFrame concatenation with empty or all-NA
entries is deprecated. In a future version, this will no longer exclude empty or
all-NA columns when determining the result dtypes. To retain the old behavior,
exclude the relevant entries before the concat operation.
  comparison_df = pd.concat([comparison_df, pd.DataFrame(ols_results)],
ignore_index=True)
```

[343]:

```
    Model  alpha            scaler  Custom R2  Custom MSE  Sklearn R2  \
0     OLS    NaN    StandardScaler   0.648012   25.812604         NaN
1   Ridge    0.5    StandardScaler   0.648107   25.805644    0.648107
2   Lasso    0.5    StandardScaler   0.621555   27.752815    0.621365
3   Ridge    1.0    StandardScaler   0.648199   25.798907    0.648199
4   Lasso    1.0    StandardScaler   0.619499   27.903557    0.619385
5   Ridge    1.5    StandardScaler   0.648288   25.792387    0.648288
6   Lasso    1.5    StandardScaler   0.606851   28.831112    0.606378
7   Ridge    2.0    StandardScaler   0.648374   25.786075    0.648374
8   Lasso    2.0    StandardScaler   0.580056   30.796118    0.580000
9     OLS    NaN      MinMaxScaler   0.648012   25.812604         NaN
10  Ridge    0.5      MinMaxScaler   0.656491   25.190794    0.656491
11  Lasso    0.5      MinMaxScaler   0.331707   49.008441    0.541435
12  Ridge    1.0      MinMaxScaler   0.661381   24.832196    0.661381
13  Lasso    1.0      MinMaxScaler   0.153748   62.058831    0.215015
14  Ridge    1.5      MinMaxScaler   0.664053   24.636242    0.664053
15  Lasso    1.5      MinMaxScaler   0.093555   66.473024   -0.023341
16  Ridge    2.0      MinMaxScaler   0.665250   24.548499    0.665250
17  Lasso    2.0      MinMaxScaler   0.035405   70.737387   -0.023341
18    OLS    NaN  PowerTransformer   0.702784   21.795955         NaN
19  Ridge    0.5  PowerTransformer   0.702713   21.801170    0.702713
20  Lasso    0.5  PowerTransformer   0.678410   23.583409    0.680138
21  Ridge    1.0  PowerTransformer   0.702638   21.806670    0.702638
22  Lasso    1.0  PowerTransformer   0.668776   24.289904    0.669681
23  Ridge    1.5  PowerTransformer   0.702560   21.812436    0.702560
24  Lasso    1.5  PowerTransformer   0.655367   25.273234    0.655860
25  Ridge    2.0  PowerTransformer   0.702478   21.818449    0.702478
26  Lasso    2.0  PowerTransformer   0.630445   27.100889    0.631805

    Sklearn MSE
```

```
0           NaN
1     25.805644
2     27.766770
3     25.798907
4     27.911920
5     25.792387
6     28.865816
7     25.786075
8     30.800179
9           NaN
10    25.190794
11    33.628310
12    24.832196
13    57.565892
14    24.636242
15    75.045430
16    24.548499
17    75.045430
18          NaN
19    21.801170
20    23.456674
21    21.806670
22    24.223564
23    21.812436
24    25.237088
25    21.818449
26    27.001166
```

- The custom models and sklearn models show nearly identical robustness and performance in terms of correlation R2 and MSE confirming the correctness of the custom implementations above and correctness of code.
- PowerTransformer scaler consistently shows the best performance across both Ridge and Lasso. We observe this with the highest R2 correlations and lowest MSE values, which means PowerTransformer is doing a better job at transforming the data making the models more effective
- StandardScaler and MinMaxScaler underperform slightly compared to PowerTransformer, showing lower R2 and higher MSE values across the models.
- While both StandardScaler and MinMaxScaler can stabilize the data they don't seem to provide the optimal transformation that PowerTransformer does in this dataset as seen above in the results
- Ridge regression is more stable across different values of alpha. As alpha increases there's only a slight decrease in R2 and a slightly increase in MSE in the different iterations in the loop which shows Ridge is handling regularization well due to its L2 penalty. This consistency makes Ridge reliable when working with different levels of regularization.
- Lasso regression is more sensitive to changes in alpha, showing significant performance degra-

dation (we see lower R2, higher MSE) as alpha increases. This sensitivity comes from the L1 regularization which aggressively shrinks coefficients and can set them to zero, leading to underfitting if alpha becomes too large like we saw in class

- Ridge generally outperforms Lasso in terms of R2 and MSE, particularly when using PowerTransformer. Ridge retains more information by shrinking coefficients without eliminating them, which is especially useful when most features contribute to the predictive power of the model.
- Lasso is more aggressive in shrinking coefficients, which can result in underfitting as alpha increases. THis is especially true when some features are important but are driven to zero because of the model. This explains why Lasso struggles as alpha goes up leading to worse performance compared to Ridge like we see in the table above.
- Regularization has a greater impact on Lasso due to L1 red, while Ridge L2 regularization offers a more balanced trade-off between bias and variance. This is particularly true in datasets with many relevant features like this one, where L1's feature selection might lead to underfitting.
- OLS regression performance is very close to Ridge with low regularization, especially when using PowerTransformer. OLS shows strong R2 and MSE values showing that without regularization, Ridge tends to behave similarly to OLS in terms of model complexity and performance.
- MinMaxScaler shows the worst performance with Lasso regression, particularly at higher alpha values, with significantly bad results in R2 and MSE. This may indicate that MinMaxScaler doesn't transform the data effectively for Lasso, leading to underfitting at even moderate levels of regularization
- PowerTransformer proves to be the most effective scaler for both Ridge and Lasso, providing stability and better model performance across various alpha values, especially when you compare to StandardScaler and MinMaxScaler.

1.3 6. Derivations:

[423]:
```
from PIL import Image, ImageOps
from IPython.display import display

img = Image.open("derivations/task1_1.jpg")
img_corrected = ImageOps.exif_transpose(img)
display(img_corrected)
```

OLS:

matrix of features

$$\hat{y} = X\beta$$

Pred vector

vector of coefficients

$$J(\beta) = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = (y - X\beta)^T (y - X\beta)$$

objective

$$\frac{\partial J(\beta)}{\partial \beta} = -2X^T(y - X\beta) = 0$$

Normal Eq.
Set to zero

$$\boxed{\beta = (X^TX)^{-1}X^Ty}$$

Solve for β

Ridge: L2

$$J(\beta) = (y - X\beta)^T(y - X\beta) + \lambda\beta^T\beta$$

add L2 penalty

Identity matrix

$$\boxed{\beta = (X^TX + \lambda I)^{-1}X^Ty}$$

Regularization parameter

* Form of regularized linear regression.

Lasso: L1

$$J(\beta) = (y - X\beta)^T(y - X\beta) + \lambda \sum_{j=1}^{p} |\beta_j|$$

* No closed form solution and is solved via optimization.