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Time Series Analysis

Exercise 5

Submission: The deadline is Wednesday, 17th of May 2023, 10:00am.

Problem 12

(a) Determine an AR(2) process and an ARMA(1, 1) process that each have the first two autocorrelations $\rho(1) = 0.5$ and $\rho(2) = 0.4$ and expected value 10 and variance 5.

(b) Determine an MA(2) process that also has the above properties.

(c) Determine the autocorrelations $\rho(3)$ and $\rho(4)$ for each of the three processes.

Problem 13 The DAX dataset contains log-returns of daily closing prices of the German stock index DAX from January 2007 until Dezember 2020. Note that log-returns r_t are defined as

$$r_t = log(P_t) - log(P_{t-1}),$$

where P_t denotes the closing price at time t.

(a) Fit an AR(1) process to the daily log-returns r_t and interpret the results. Plot the fitted values and the log returns on a single graph and decide how well an AR(1) predicts stock returns.

(b) Fit an AR(1) process to the daily squared log-returns $(r_t)^2$ and interpret the results. Plot the fitted values and the squared log returns on a single graph and decide how well an AR(1) predicts squared stock returns.

(c) Fit an ARCH(1) process to the daily log-returns r_t and plot the fitted values and the log returns on a single graph and decide how well an ARCH(1) predicts stock price volatility. Compare the results with a) and b).

<u>Hint:</u> Use the `garch` function of the `tseries` package with a proper model order to fit an ARCH(1).

Problem 14 Let $z = a + ib$ be a complex number with a its real part and b its imaginary part. Then $|z| := \sqrt{a^2 + b^2}$ is the absolute value. The absolute value corresponds to the length of the corresponding vector in the complex plane.

(a) Show that $z^2 \neq |z|^2$. (This is important for the definition of $L_2^{\mathbb{C}}$ in the lecture.)

From the representation of the corresponding vector in the complex plane, you immediately get what is called the *polar coordinate representation* for $z = a + ib$, which is given by

$$z = |z|(\cos(\omega) + i\sin(\omega))$$

with $\omega \in [0, 2\pi]$ the (anticlockwise) angle between the positive real axis and the vector corresponding to z. (Thus, $\cos(\omega) = a/|z|$ and $\sin(\omega) = b/|z|$.) Finally, using the famous Euler formula

$$e^{ix} = \cos(x) + i\sin(x) \quad \forall x \in \mathbb{R}$$

we can simplify the polar coordinate representation to $z = |z|e^{i\omega}$. Note, from here it is clear that the complex conjugate can be written as $\bar{z} = |z|(\cos(\omega) - i\sin(\omega)) = |z|e^{-i\omega}$

(b) Use the Euler formula to show that

$$\cos(\omega) = \frac{e^{i\omega} + e^{-i\omega}}{2} \text{ and } \sin(\omega) = \frac{e^{i\omega} - e^{-i\omega}}{2i}$$

(c) Use the previous to show that $cos(\lambda + \mu) = \cos(\lambda)\cos(\mu) - \sin(\lambda)\sin(\mu)$. (This has been used in the lecture and in question 4 of exercise set 1.)

(d) Show that

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{is\omega} d\omega = \begin{cases} 1 & , s = 0 \\ 0 & , s \neq 0 \end{cases}.$$