Exercise 5

Problem 12:

(a) Let's start with AR(2):

$$X_t = c + \varphi_1 X_{t-1} + \varphi_2 X_{t-2} + \varepsilon_t$$

$$\rho(1) = \varphi_1 + \varphi_2 = 0{,}5$$

$$\rho(2) = \varphi_1 \cdot \rho(1) + \varphi_2 = 0{,}4$$

$$\varphi_1 = 0{,}5 - \varphi_2$$

$$\varphi_2 = 0{,}4 - \varphi_1 \cdot 0{,}5$$

$$c = 10 - \varphi_1 E[X_{t-1}] - \varphi_2 E[X_{t-2}] =$$

$$= 10 - \varphi_1 \cdot 10 - \varphi_2 10$$

$$\text{var}(\varepsilon_t) = 5$$

$$E[X_{t-1}] = E[X_{t-2}] = 10$$

$$\varphi_1 = 0{,}5 - \varphi_2$$

$$\varphi_2 = 0{,}4 - \varphi_1 \cdot 0{,}5$$

$$c = 10 - \varphi_1 \cdot 10 - \varphi_2 10$$

$$\uparrow$$

$$X_t = c + (0{,}5 - \varphi_2) \cdot X_{t-1} + \varphi_2 \cdot X_{t-2} + \varepsilon_t$$

Now, let's determine ARMA(1,1):

$$X_t = c + \varphi \cdot X_{t-1} + \theta\, \varepsilon_{t-1} + \varepsilon_t$$

$$\rho(1) = \frac{\varphi}{1 + \theta \cdot \rho(1)} = 0,5$$

$$\rho(2) = \frac{\varphi \cdot \rho(1) + \theta}{1 + \theta \cdot \rho(1)} = 0,4$$

$$\varphi = 0,5 \cdot (1 + \theta \cdot \rho(1))$$

$$\theta = \rho(2) - \varphi \cdot \rho(1)$$

$$c = 10 - \varphi\, E[X_{t-1}] - \theta\, E[\varepsilon_{t-1}] =$$

$$= 10 - \varphi \cdot 10 - \theta \cdot 0$$

$$\mathrm{var}(\varepsilon_t) = 5$$

~~$\varphi = 0,5 \cdot (1 + \theta \cdot 0,5)$~~

$$X_t = c + (0,5 \cdot (1 + \theta \cdot 0,5)) \cdot X_{t-1} + (0,4 - (0,5 \cdot (1 + \theta \cdot 0,5)))$$

$$\cdot\, \varepsilon_{t-1} + \varepsilon_t$$

$$c = 10 - (0,5 \cdot (1 + \theta \cdot 0,5)) \cdot 10 - (0,4 - (0,5 \cdot (1 + \theta \cdot 0,5))) \cdot 0$$

(b) $X_t = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$

$\rho(1) = \frac{\theta_1}{1+\theta_2} = 0{,}5$ $\quad \rho(2) = \theta_1 \cdot \rho(1) + \theta_2 / (1+\theta_2) = 0{,}4$

$\theta_1 = 0{,}5 \cdot (1+\theta_2)$

$\theta_2 = (0{,}4 - \theta_1 \cdot \rho(1)) / (1 - \rho(1))$

$\mu = 10 - \theta_1 \cdot E[\varepsilon_{t-1}] - \theta_2 E[\varepsilon_{t-2}] =$

$= 10 - \theta_1 \cdot 0 = \theta_2 \cdot 0 = 10$

$var(\varepsilon_t) = 5$

$X_t = 10 + \varepsilon_t + (0{,}5 \cdot (1+\theta_2)) \cdot \varepsilon_{t-1} +$

$+ ((0{,}4 - (0{,}5 \cdot (1+\theta_2))) / (1 - 0{,}5)) \cdot \varepsilon_{t-2}$

(c) AR(2): $\rho(k) = \varphi_1 \cdot \rho_{(k-1)} + \varphi_2 \cdot \rho_{(k-2)}$

$\rho(3) = \varphi_1 \cdot \rho(2) + \varphi_2 \cdot \rho(1)$

$\rho(4) = \varphi_1 \rho(3) + \varphi_2 \rho(2)$

$\varphi_1 = 0{,}5 - \varphi_2$ $\quad \varphi_2 = 0{,}4 - \varphi_1 \cdot 0{,}5$ $\quad \rho(1) = 0{,}5$ $\quad \rho(2) = 0{,}4$

ARMA(1,1): $\rho(k) = \varphi \cdot \rho(k-1)$

$\rho(3) = \varphi \cdot \rho(2)$

$\rho(4) = \varphi \cdot \rho(3)$

$\varphi = 0{,}5 \cdot (1 + \theta \cdot 0{,}5)$ $\rho(2) = 0{,}4$.

MA(2):

$\rho(3) = \theta_1 \cdot \rho(2) + \theta_2 \cdot \rho(1)$

$\rho(4) = \theta_1 \rho(3) + \theta_2 \rho(2)$

$\theta_1 = 0{,}5\,(1 + \theta_2)$

$\theta_2 = (0{,}4 - \theta_1 \cdot \rho(1)) \,/\, (1 - \rho(1))$

$\rho(1) = 0{,}5$ $\rho(2) = 0{,}4$

ts4.R

Acer

2023-05-17

```
library(dplyr)
library(stringr)
library(data.table)
library(ggplot2)
library(mondate)
library(zoo)
library(VGAM)
library(MASS)
library(car)
library(forecast)
library(conflicted)
library(tidyr)
library(scales)
library(reshape2)
library(psych)
library(rmarkdown)
conflict_prefer('select', 'dplyr')
```

```
## [conflicted] Removing existing preference.
## [conflicted] Will prefer dplyr::select over any other package.
```

```
conflict_prefer('filter', 'dplyr')
```

```
## [conflicted] Removing existing preference.
## [conflicted] Will prefer dplyr::filter over any other package.
```

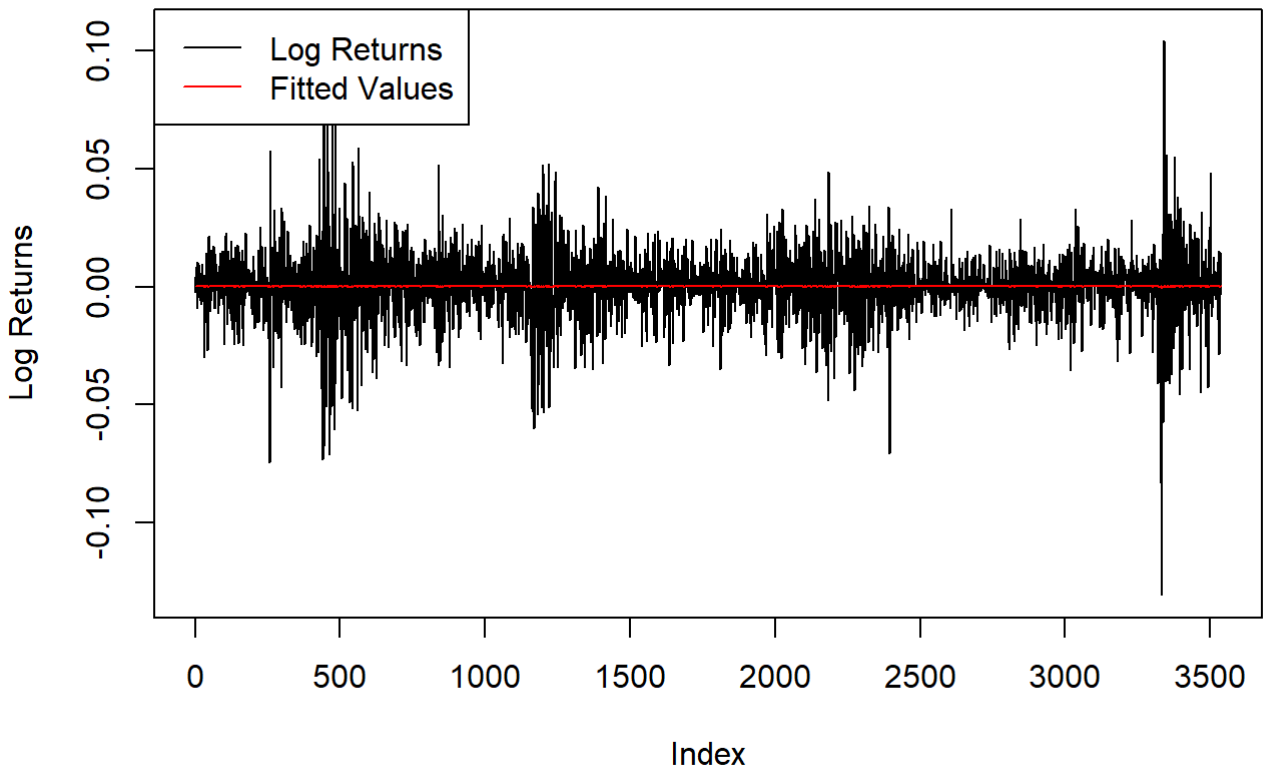
```
conflict_prefer('lag', 'dplyr')
```

```
## [conflicted] Removing existing preference.
## [conflicted] Will prefer dplyr::lag over any other package.
```

```
setwd("C:\\Users\\Acer\\OneDrive - ADA University\\Documents\\ec")
data <- readRDS("dax.rds")
data <- as.data.frame(data)
################################################################################
# a)
model <- arima(data$data, order = c(1, 0, 0))
summary(model)
```

```
##
## Call:
## arima(x = data$data, order = c(1, 0, 0))
##
## Coefficients:
##          ar1  intercept
##       0.0051      2e-04
## s.e.  0.0168      2e-04
##
## sigma^2 estimated as 0.0002029:  log likelihood = 10018.24,  aic = -20030.48
##
## Training set error measures:
##                        ME       RMSE          MAE      MPE     MAPE      MASE
ACF1
## Training set 5.141891e-08 0.01424499 0.009700564 104.7648 110.2888 0.6841773 5.614084
e-05
```

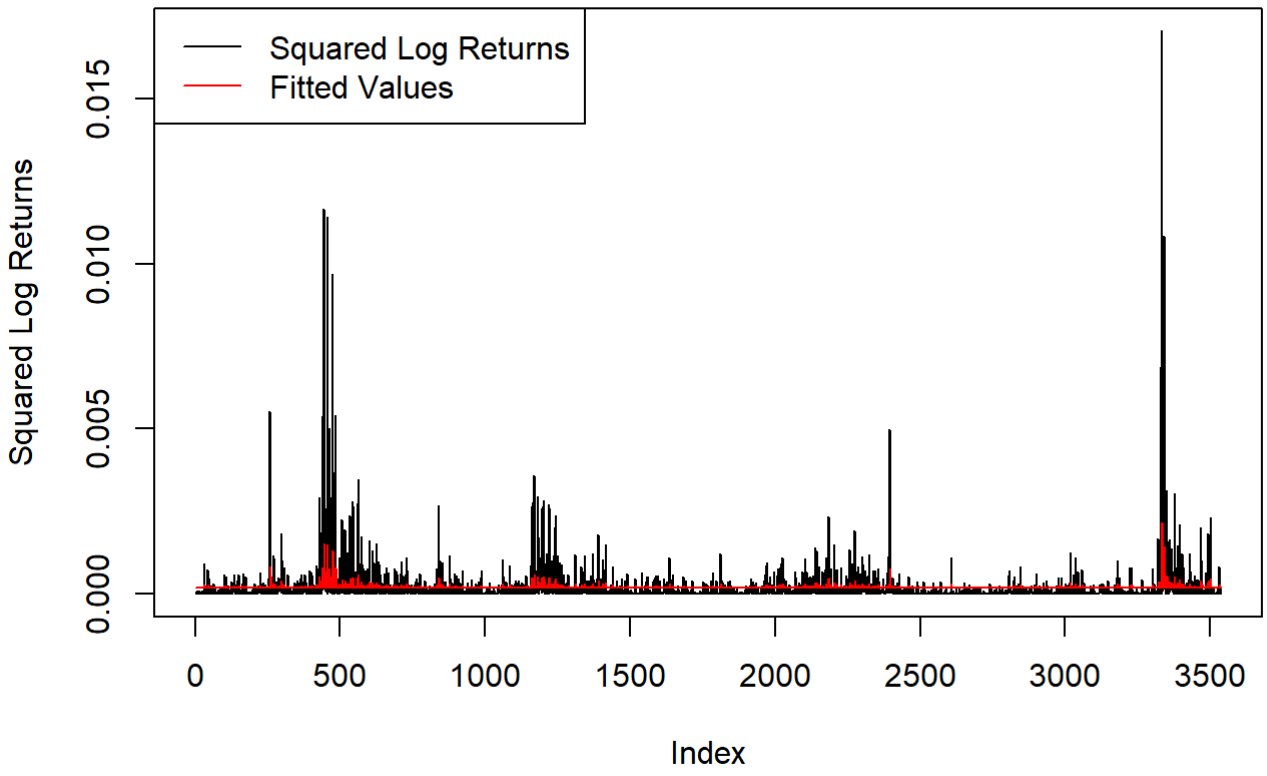
```
plot(data$data, type = "l", ylab = "Log Returns")
lines(fitted(model), col = "red")
legend("topleft", legend = c("Log Returns", "Fitted Values"), col = c("black", "red"), l
ty = 1)
```



```
################################################################################
# b)
model <- arima(data$data^2, order = c(1, 0, 0))
summary(model)
```

```
##
## Call:
## arima(x = data$data^2, order = c(1, 0, 0))
##
## Coefficients:
##          ar1  intercept
##       0.1159      2e-04
## s.e.  0.0167      0e+00
##
## sigma^2 estimated as 4.052e-07:  log likelihood = 21011.59,  aic = -42017.18
##
## Training set error measures:
##                         ME         RMSE          MAE      MPE    MAPE      MASE
ACF1
## Training set -9.119537e-09 0.0006365475 0.0002426972 -1442400 1442423 0.8415227 -0.02
279896
```

```
plot(data$data^2, type = "l", ylab = "Squared Log Returns")
lines(fitted(model), col = "red")
legend("topleft", legend = c("Squared Log Returns", "Fitted Values"), col = c("black",
"red"), lty = 1)
```

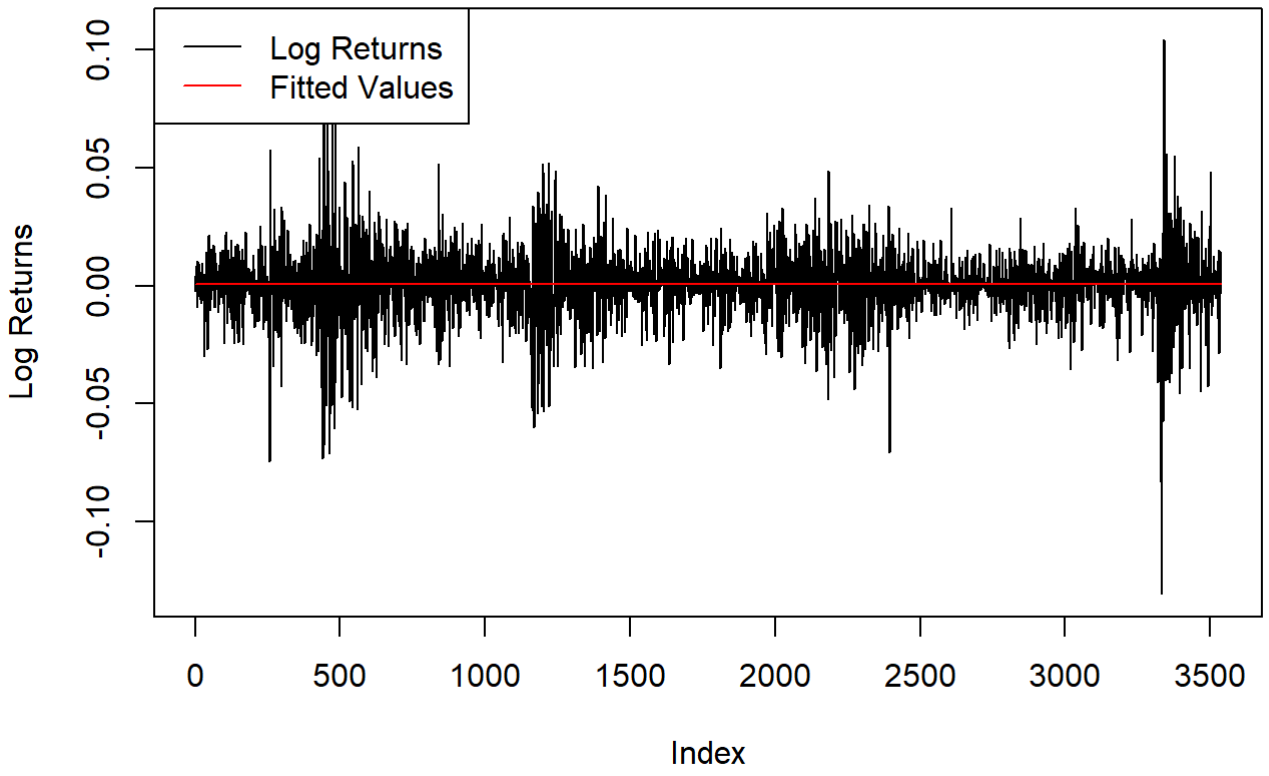



```
################################################################################
# c)
library(rugarch)

# Fit the ARCH(1) model
model <- ugarchspec(variance.model = list(model = "sGARCH", garchOrder = c(1, 0)), mean.
model = list(armaOrder = c(0, 0)))
fit <- ugarchfit(data = data$data, spec = model)

# Generate forecasted conditional variances
forecast <- ugarchforecast(fit, n.ahead = length(data$data))

# Plot the log returns and fitted values
plot(data$data, type = "l", ylab = "Log Returns")
lines(forecast@forecast$seriesFor, col = "red")
legend("topleft", legend = c("Log Returns", "Fitted Values"), col = c("black", "red"), l
ty = 1)
```


Log Returns
Fitted Values
Log Returns
0.10
0.05
0.00
-0.05
-0.10
0
500
1000
1500
2000
2500
3000
3500
Index