The copula-GARCH model combines two important concepts in statistical modeling: copulas and GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models. It is a powerful framework for analyzing the joint distribution of multiple time series while accounting for their conditional heteroskedasticity.

Copulas: A copula is a function that links marginal distributions to form a joint distribution. It allows us to model the dependence structure between random variables separately from their individual marginal distributions. Copulas are particularly useful when the variables have different marginal distributions or when we are interested in modeling their dependence structure independently.

GARCH Models: GARCH models are widely used in financial econometrics to capture time-varying volatility and conditional heteroskedasticity in time series data. They incorporate autoregressive and moving average components to model the conditional mean, along with a volatility component that captures the dynamics of the conditional variance. GARCH models are known for their ability to capture the persistence and clustering of volatility observed in financial data.

The copula-GARCH model combines these two concepts by using a copula to model the dependence structure between the standardized residuals of GARCH models. Here's an outline of the steps involved:

1. Fit univariate GARCH models: For each time series in the analysis, fit a univariate GARCH model to capture the conditional mean and volatility dynamics. This involves estimating the autoregressive and moving average parameters as well as the parameters related to the volatility process.
2. Extract standardized residuals: Calculate the standardized residuals from the GARCH models. These residuals represent the observed values adjusted for the estimated volatility.
3. Apply copula transformation: Apply a copula transformation, such as the probability integral transformation (PIT), to the standardized residuals. This transforms the residuals to be uniformly distributed.
4. Select and fit a copula model: Choose an appropriate copula family (e.g., Gaussian, t, Clayton, Archimedean, etc.) based on the characteristics of the data and the desired dependence structure. Fit the copula model to the transformed residuals, estimating the copula parameters.
5. Simulate joint distributions: Using the estimated copula parameters and the transformed residuals, simulate joint distributions of the variables. This allows for generating scenarios and understanding the joint behavior of the time series.

The copula-GARCH model enables capturing the conditional dependence structure between time series while accounting for their conditional heteroskedasticity. It is commonly used in financial risk management, portfolio optimization, and modeling complex multivariate systems where the variables have different distributions and varying volatility patterns.