RAM Robotic Test Leg

by Christopher Mailer (MLRCHR001)

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1 Design Calculations

1.1 Specifications

- T-Motor U10 Plus 80KV
- Coaxial 5 bar linkage leg design
- Place to attach foot designs
- Components purchased from RS Components
- Use of 2:1 timing belt for lightweight quasi-direct-drive actuation

1.2 T-Motor U10 Plus 80KV

Motor Properties

Pole Pairs	21
Weight	$405\,g$
Internal Resistance	$135\,m\Omega$
Startup Torque	7.5 Nm
Max Rated Speed	$4000\,rpm$
Max Continuous Current	$25\,A$
Max Continuous Power	$1200\,W$

1.3 Timing Belt Selection

1.3.1 Profile and Pitch

The 'T' timing belt profile was chosen as it is a standard profile providing the largest selection of belts and pulleys on RS Components.
The selected timing belt pitch type of T5 was minimum pitch which would support transmission of the 1.2 kW of power produced by each of the motors while still providing a large selection of belt sizes on RS Components. Additional considerations were that a larger pitch such as the T10 pitch would be accompanied by a larger belt width, thus increasing the overall size of the design. A smaller pitch such as the T2.5 on the other hand would be challenging to accurately 3D print the pulley tooth profile.

1.3.2 Material

The predominant T5 timing belt type on RS Components was the polyurethane CONTI SYNCROFLEX with steel tension members. Polyurethane belts have better abrasion resistance than rubber belts.

1.4 Small Pulley Sizing

Based on the dimensions of the motor and pulley mounting interface, the smallest pulley pitch diameter which can be mounted to the motor while still accommodating the mounting bolts is ⌀36.

$$z_1 = \frac{d_1 \cdot \pi}{t} = \frac{(36)(\pi)}{5} = 22.62\, teeth \tag{1}$$

The next available standard number of pulley teeth is 24 resulting in the following tooth pitch

$$d_1 = \frac{z \cdot t}{\pi} = \frac{(24)(5)}{\pi} = 38.20\, mm \tag{2}$$

The centre to centre distance of the small and large pulleys is currently unknown but could be estimated to be approximately 90 mm, thus allowing the number of engaged teeth on the small pulley to be estimated

$$z_e = z_1(0.5 - \frac{d_2 - d_1}{2\pi C}) = (24)(0.5 - \frac{(2)(38.20) - 38.20}{(2)(\pi)(90)}) \approx 11 \tag{3}$$

1.5 Timing Belt Width

The small pulley is the limiting component in the calculating of the timing belt width as it is where maximum tooth shear is experienced.

Belt Specific Tooth Strength at 4000 rpm from datasheet

Specific Torque	M_{spec}	=	1.91	Ncm/cm
Specific Power	P_{spec}	=	3.81	W/cm

Given conditions

Power	P	=	1.2	kW
Torque	M	=	7.5	Nm
Number of Teeth	z_1	=	24	Nm
Teeth in Mesh	z_e	=	11	teeth

1.5.1 Tooth Shear Strength

Calculation of the belt width based on power

$$b = \frac{10^4 \cdot P}{z_1 \cdot z_e \cdot P_{spec}} = \frac{(10^4)(1.2)}{(24)(11)(3.81)} = 11.93\, mm \tag{4}$$

Calculation of the belt width based on torque

$$b = \frac{10^3 \cdot M}{z_1 \cdot z_e \cdot M_{spec}} = \frac{(10^3)(7.5)}{(24)(11)(1.91)} = 14.87\, mm \tag{5}$$

The next standard belt width which exceeds both of these is 16 mm

1.5.2 Tension Member Strength

The maximum allowable tension for a T5 16 mm wide timing belt is 570 N

$$F_u = \frac{2 \cdot 10^3 \cdot M}{d_1} = \frac{(2)(10^3)(7.5)}{38.20} = 393\,N < 570\,N \tag{6}$$

Therefore the selected timing belt will be able to withstand the loads

1.6 Timing Belt Length

The centre to centre distance of the timing belt needs to be larger than 90 mm to provide sufficient clearance between the pulleys, but should also be kept to a minimum to ensure a compact design.

Given conditions

Centre to centre distance	C	$\geq$	90	mm
Small pulley no. of teeth	z_1	$=$	24	teeth
Large pulley no. of teeth	z_2	$=$	48	teeth

$$d_i = \frac{z_i \cdot p}{\pi} \tag{7}$$

$$\theta_1 = 2 \cdot \arccos(\frac{d_2 - d_1}{2 \cdot C}) = 2,71\,rad \tag{8}$$

$$\theta_2 = 2\pi - \theta_1 = 3,57\,rad \tag{9}$$

$$L = 2 \cdot C \cdot \sin(\frac{\theta_1}{2}) + \theta_1 \cdot \frac{d_1}{2} + \theta_2 \cdot \frac{d_2}{2} = 364,07\,mm \tag{10}$$

The next available standard timing belt length is 365 mm which requires a centre to centre distance of 90,48 mm.

1.7 Selected Timing Belt

CONTI SYNCROFLEX T5 Timing Belt, 73 teeth, 365 mm length, 16 mm width

1.8 Shaft Loading Condition

The maximum forces experienced by each of the shafts will be during landing when the legs are fully extended and the force will be transmitted directly to the shafts and not shared by the motors as torque.

1.8.1 Leg Impact Force

Approximating the rubber pad on the base of the foot as a spring and assuming all of the gravitational potential energy is transferred to spring energy, the following can be determined:

$$E_p = mgh \tag{11}$$

$$E_s = \frac{1}{2}k\delta^2 \tag{12}$$

$$\delta = \sqrt{\frac{2E_p}{k}} \tag{13}$$

The rubber pad on the base of the foot was approximated to have a spring constant of $30 \times 10^3\, N/m$. The test rails also only allow for a maximum jump height of 1 m. The maximum ground reaction force experienced by the leg during landing is therefore;

$$F_{max} = \sqrt{2mghk} = \sqrt{(2)(3)(9.81)(1)(30 \times 10^3)} = 1329\, N \tag{14}$$

This is a conservative estimate as other means of energy dissipation are ignored. This force will be evenly distributed between each leg linkage producing 665 N on the end of each shaft.

1.8.2 Belt Tension

Assuming belt tensioned to full capacity of $P_1 = 570\, N$

$$P_2 = P_1 - \frac{2M}{d_1} = 570 - \frac{(2)(7.5)}{0.0382} = 141.4\, N \tag{15}$$

The centre to centre distance of timing belt pulleys, and therefore angle of wrap of the belts at this stage in the design is unknown. Instead the belts are conservatively assumed to be parallel, resulting in all of the tension being transmitted to the shaft.

$$P_{res} = P_1 + P_2 = 711.4 \tag{16}$$

1.8.3 Combined Load

The legs will exert maximum force on the shaft during landing when in the fully contracted or extended states. In this case the impact force from the legs will be transmitted to the shaft almost directly upwards. The angle of the belt tensions is acting at 45° to the vertical. The shaft is also expected to transmit 15 Nm of torque after the 2:1 speed reduction from the motor.

$$\begin{aligned} P_x &= 503\,\mathrm{N} \\ P_y &= 503\,\mathrm{N} \\ M &= 15\,\mathrm{Nm} \end{aligned}$$

1.9 Inner Shaft BMD

Side View

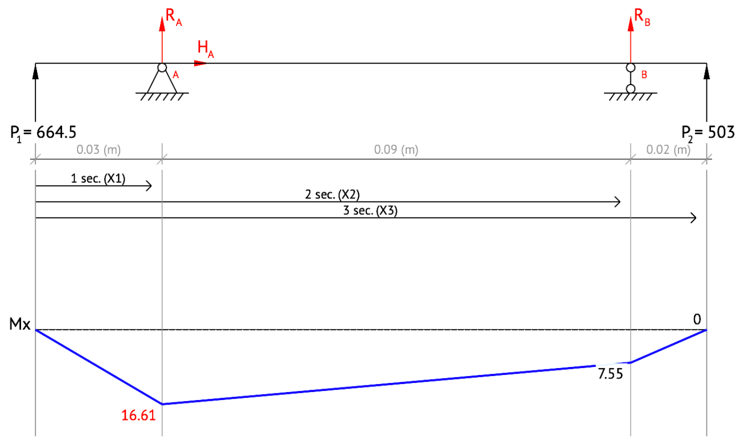

Figure 1: Inner shaft side view bending moment diagram BeamGuru

$R_{Ay} = -762.53\,N$ $\qquad$ $R_{By} = -404.97\,N$ $\qquad$ $M_{xz} = 16.61$

Top View

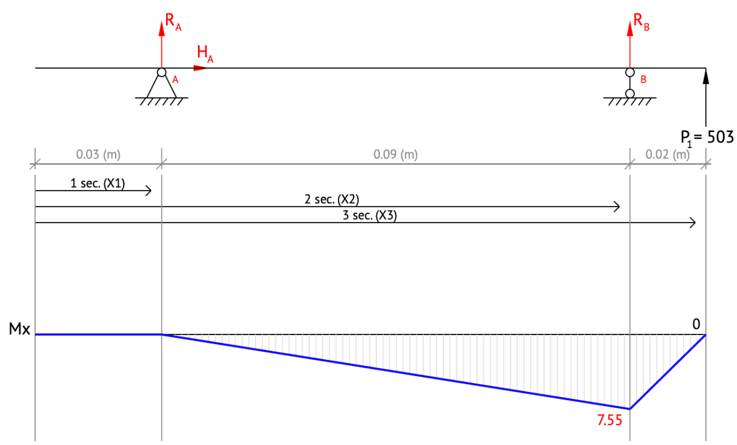

Figure 2: Inner shaft top view bending moment diagram BeamGuru

$R_{Ax} = 81.57\,N$ $\qquad$ $R_{Bx} = -584.57\,N$ $\qquad$ $M_{yz} = 0$

Resultant

Critical section at leftmost bearing

$$M = \sqrt{{M_{xz}}^2 + {M_{yz}}^2} = 16.61\,Nm$$

$$T = 15\,Nm$$

1.10 Outer Shaft BMD

Side View

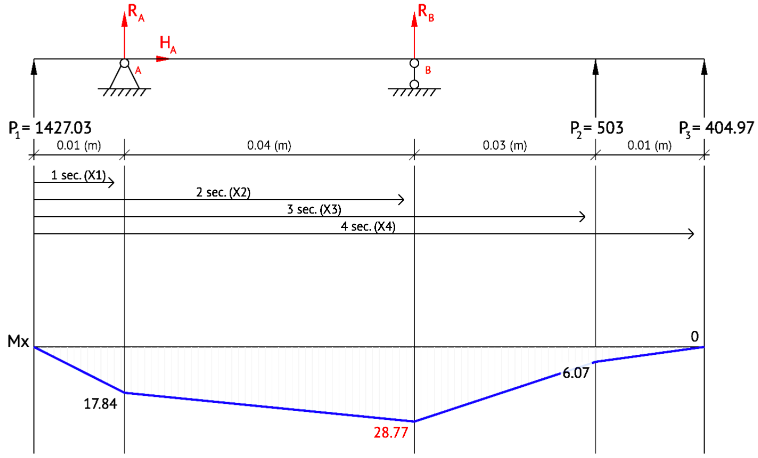

Figure 3: Outer shaft side view bending moment diagram BeamGuru

$$R_{Ay} = -1153.63\,N \qquad R_{By} = -1181.37\,N \qquad M_{xz} = 28.77$$

Top View

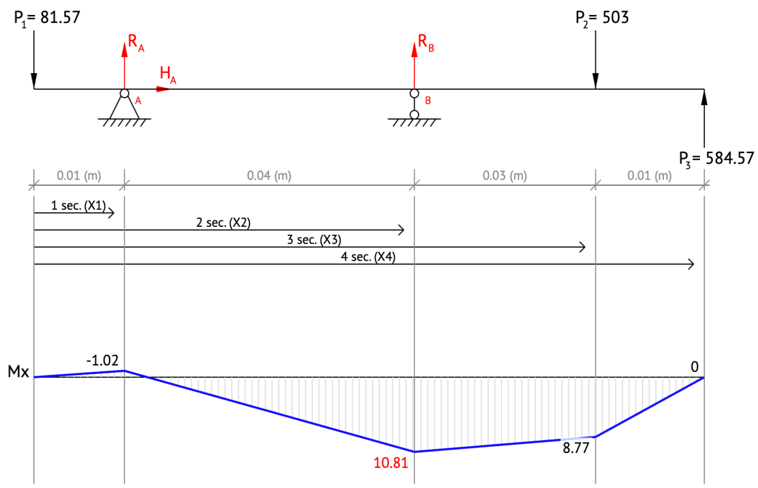

Figure 4: Outer shaft top view bending moment diagram BeamGuru

$$R_{Ax} = 377.26\,N \qquad R_{Bx} = -377.26\,N \qquad M_{yz} = -1.02$$

Resultant

Critical section at rightmost bearing

$$M = \sqrt{{M_{xz}}^2 + {M_{yz}}^2} = 30.73\,Nm$$

$$T = 15\,Nm$$

1.11 Shaft Sizing

$$\sigma = \frac{Mr}{I} = \frac{32M}{\pi {d_o}^3(1 - \frac{d_i}{d_0}^4)} \tag{17}$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi {d_o}^3(1 - \frac{d_i}{d_0}^4)} \tag{18}$$

1.11.1 Material

The shaft will be made from 304 Stainless Steel, as this was available in the workshop, thus the Maximum Distortion Energy Theory will be a sufficient failure theory.

$S_u = 215\,MPa$

$${\sigma_b}^2 + 3\tau^2 \leq (\frac{S_u}{RF})^2 \tag{19}$$

The following calculations were done in the spreadhseet *Shaft and Belt Calculations*

1.11.2 Inner Shaft Size

The Inner Shaft diameter selection was an iterative process driven by the Outer Shaft diameters and the available bearings on RS Components. A diameter of $\varnothing 12\,mm$ was selected as this provided a reserve factor of 1.7 and ensured a reasonable selection of roller bearings. A larger inner shaft would have resulted in a significantly larger outer shaft, greatly increasing the weight of the design.

$$D_o = 12\,mm$$

$$RF = 1,7$$

1.11.3 Outer Shaft Size

The outer diameter of the shaft was selected to be $\varnothing 20\,mm$ based on bearing availability on RS Components. With the goal of keeping size and mass to a minimum, the smallest roller bearings were selected which required the Outer Shaft to have an inner diameter of $\varnothing 16\,mm$. This produced a reserve factor of 3. To create a shoulder for the bearings, the remaining portion of the shaft has an inner diameter of $\varnothing 14\,mm$.

$$D_o = 20\,mm$$

$$D_i = 16\,mm$$

$$RF = 3,0$$

2 Manufacture

2.1 3D Printing

The following parts are 3D printed

Part Name	Qty
Inner Timing Belt Gear	1
Outer Timing Belt Gear	1
Motor Mounting	2
Support Brace	1
Encoder Shaft	2
Leg Spacer	3

2.1.1 Printer Settings

The Cura Ender 3 presets were used with the following modifications.

Printer	Ender 3 Pro
Material	PLA
Printing Temp.	200°
Printing Speed	50 mm/s
Layer Height	0.2 mm
Infill Density	60%
Infill Pattern	Cubic
Adhesion Type	Raft

2.1.2 Considerations

The main challenge with 3D printed parts was shrinkage. To compensate holes were printed 4% larger and reamed afterwards to remove seams. The shrinkage also resulted in the timing belt initially not being properly tensioned. New output pulleys were printed with 1 mm larger outer diameters which corrected the problem.

2.1.3 Additional Machining

- Holes need to be drilled out to remove seams

2.2 Laser Cutting

The following parts are laser cut

Part Name	Material	Thickness	Qty
Mounting Plate	5182 Aluminium	3 mm	1
Inner Short Leg Link	5083 Aluminium	10 mm	1
Outer Short Leg Link	5083 Aluminium	10 mm	1
Extra Long Leg Link	5083 Aluminium	10 mm	1
Long Leg Link	5083 Aluminium	10 mm	1

2.2.1 Considerations

Laser cutting thick material produces a rough edge and the cut does not remain perpendicular to the material surface. Thus, holes should be undersized before laser cutting and then later reamed to the correct size on a drill press to ensure a smooth finish and perpendicularity.

2.2.2 Additional Machining

- The Mounting Plate requires sanding to remove rough edges
- The leg links require the bearing seats to be machined. This is a counterbore to a depth of 5 mm. A cutting tool on a drill press achieves this.
- The holes for the shoulder bolts need to be drilled and tapped for the M5 shoulder bolt
- The holes for the inner and outer output shafts need to be reamed to the correct sizes

2.3 Machining

The following parts were machined by the UCT workshop

Part Name	**Material**	**Qty**
Flange	6082 Aluminium	1
Motor Shaft (Short)	6082 Aluminium	1
Motor Shaft (Long)	6082 Aluminium	1
Inner Shaft	304 Stainless Steel	1
Outer Shaft	304 Stainless Steel	1
Timing Belt Pulley 24T	NA	2

2.3.1 Considerations

The machining of the Timing Belt Pulleys 24T is simple and can be done with a drill press to reduce time and cost.

2.3.2 Additional Machining

- All of the shafts are slightly oversized for the bearings. They therefore need to be carefully sanded down until a locational fit with the inner race is achieved.

3 Future Modifications

3.1 Gear Ratio

Very little analysis was done on the optimum gear ratio for the leg. If it is later found that a different gear ratio is required, then the following parameters will need to be changed to implement this.

1. The new inner and outer shaft pulley number of teeth and outer diameter will need to be modified to produce the required reduction ratio. These pulleys will in turn need to be 3D printed again.
2. A new centre to centre distance between the motors and output shafts will need to be calculated based on the new pulley diameters and the available timing belt lengths.
3. The mounting plate SolidWorks file will need to be updated with the new centre to centre distance and laser cut.

3.2 Leg Dimensions

Baleka's legs were used as a basis for the proportions of the leg links, however very little analysis was done on the optimum lengths. As future design changes to the legs is inevitable, the SolidWorks part dimensions were laid out to in such a way to make changes to the pivot to pivot lengths simple to modify.

4 Leg Kinematics

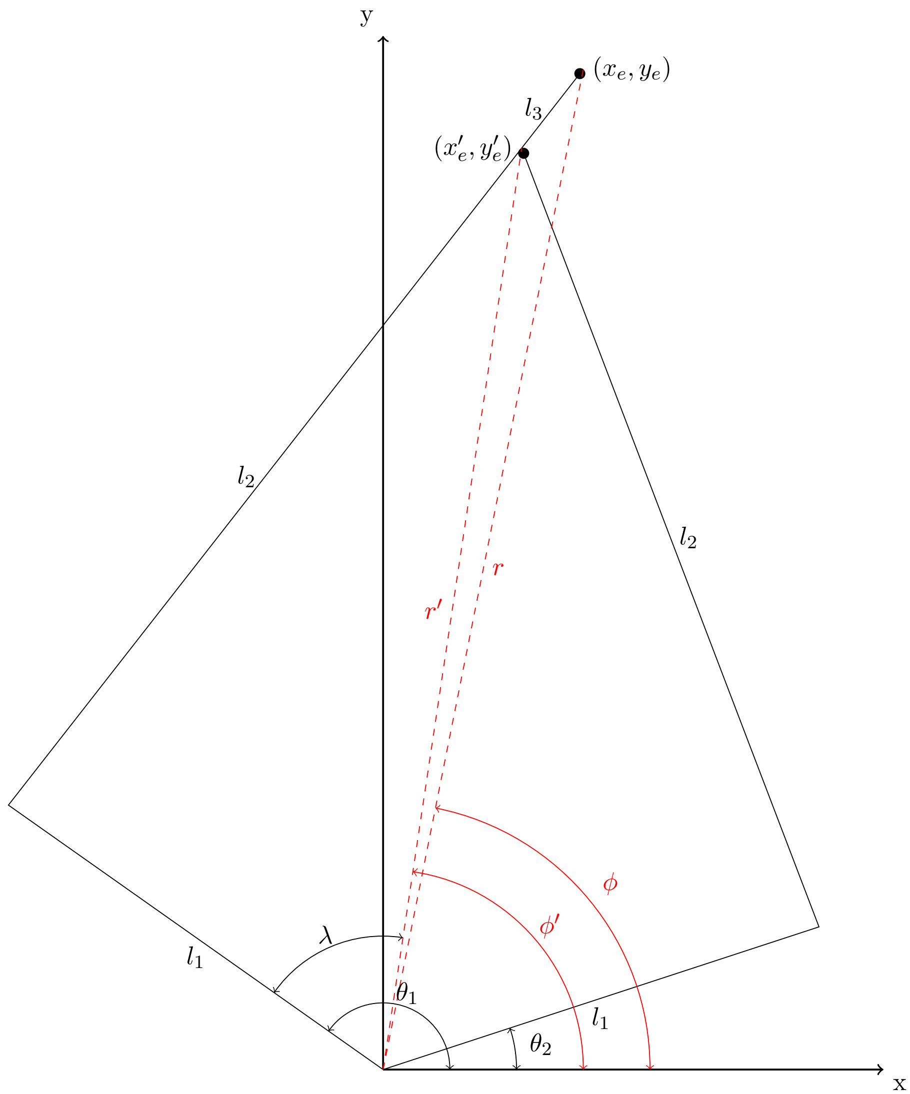

Figure 5: Kinematic diagram of leg

The red dashed lines represent "virtual legs" in the polar coordinate space. This representation makes computing leg compliance significantly simpler.

$$Let\ \lambda = \frac{\theta_1 - \theta_2}{2} \tag{20}$$

4.1 Forward Kinematics (Excluding l_3)

4.1.1 Cartesian Coordinates

$$x'_e = \left(l_1 \cos\left(\frac{\theta_1 - \theta_2}{2}\right) + \sqrt{{l_2}^2 - {l_1}^2 \sin^2\left(\frac{\theta_1 - \theta_2}{2}\right)}\right) \cos\left(\frac{\theta_1 + \theta_2}{2}\right) \tag{21}$$

$$y'_e = \left(l_1 \cos\left(\frac{\theta_1 - \theta_2}{2}\right) + \sqrt{{l_2}^2 - {l_1}^2 \sin^2\left(\frac{\theta_1 - \theta_2}{2}\right)}\right) \sin\left(\frac{\theta_1 + \theta_2}{2}\right) \tag{22}$$

For computing the Jacobian

$$\frac{\partial x'_e}{\partial \theta_1} = \cos\left(\frac{\theta_1+\theta_2}{2}\right)\left(-\frac{1}{2}l_1 \sin(\lambda) - \frac{{l_1}^2 \sin(\lambda)\cos(\lambda)}{2\sqrt{{l_2}^2 - {l_1}^2 \sin^2(\lambda)}}\right) - \frac{1}{2}\sin\left(\frac{\theta_1+\theta_2}{2}\right)\left(l_1\cos(\lambda) + \sqrt{{l_2}^2 - {l_1}^2 \sin^2(\lambda)}\right) \tag{23}$$

$$\frac{\partial x'_e}{\partial \theta_2} = \cos\left(\frac{\theta_1+\theta_2}{2}\right)\left(\frac{1}{2}l_1 \sin(\lambda) + \frac{{l_1}^2 \sin(\lambda)\cos(\lambda)}{2\sqrt{{l_2}^2 - {l_1}^2 \sin^2(\lambda)}}\right) - \frac{1}{2}\sin\left(\frac{\theta_1+\theta_2}{2}\right)\left(l_1\cos(\lambda) + \sqrt{{l_2}^2 - {l_1}^2 \sin^2(\lambda)}\right) \tag{24}$$

$$\frac{\partial y'_e}{\partial \theta_1} = \sin\left(\frac{\theta_1+\theta_2}{2}\right)\left(-\frac{1}{2}l_1 \sin(\lambda) - \frac{{l_1}^2 \sin(\lambda)\cos(\lambda)}{2\sqrt{{l_2}^2 - {l_1}^2 \sin^2(\lambda)}}\right) + \frac{1}{2}\cos\left(\frac{\theta_1+\theta_2}{2}\right)\left(l_1\cos(\lambda) + \sqrt{{l_2}^2 - {l_1}^2 \sin^2(\lambda)}\right) \tag{25}$$

$$\frac{\partial y'_e}{\partial \theta_2} = \sin\left(\frac{\theta_1+\theta_2}{2}\right)\left(\frac{1}{2}l_1 \sin(\lambda) + \frac{{l_1}^2 \sin(\lambda)\cos(\lambda)}{2\sqrt{{l_2}^2 - {l_1}^2 \sin^2(\lambda)}}\right) + \frac{1}{2}\cos\left(\frac{\theta_1+\theta_2}{2}\right)\left(l_1\cos(\lambda) + \sqrt{{l_2}^2 - {l_1}^2 \sin^2(\lambda)}\right) \tag{26}$$

$$\mathbf{J} = \begin{bmatrix} \frac{\partial x'_e}{\partial \theta_1} & \frac{\partial x'_e}{\partial \theta_2} \\ \frac{\partial y'_e}{\partial \theta_1} & \frac{\partial y'_e}{\partial \theta_2} \end{bmatrix}$$

4.1.2 Polar Coordinates

$$\phi' = \frac{\theta_1 + \theta_2}{2} \tag{27}$$

$$r' = l_1 \cos(\frac{\theta_1 - \theta_2}{2}) + \sqrt{{l_2}^2 - {l_1}^2 \sin^2(\frac{\theta_1 - \theta_2}{2})} \tag{28}$$

For computing the Jacobian

$$\frac{\partial \phi'_e}{\partial \theta_1} = \frac{\partial \phi'_e}{\partial \theta_2} = \frac{1}{2} \tag{29}$$

$$\frac{\partial r'_e}{\partial \theta_1} = -\frac{1}{2}l_1 \sin(\lambda) - \frac{{l_1}^2 \sin(\lambda)\cos(\lambda)}{2\sqrt{{l_2}^2 - {l_1}^2 \sin^2(\lambda)}} \tag{30}$$

$$\frac{\partial r'_e}{\partial \theta_2} = \frac{1}{2}l_1 \sin(\lambda) + \frac{{l_1}^2 \sin(\lambda)\cos(\lambda)}{2\sqrt{{l_2}^2 - {l_1}^2 \sin^2(\lambda)}} \tag{31}$$

$$\mathbf{J} = \begin{bmatrix} \frac{\partial \phi'_e}{\partial \theta_1} & \frac{\partial \phi'_e}{\partial \theta_2} \\ \frac{\partial r'_e}{\partial \theta_1} & \frac{\partial r'_e}{\partial \theta_2} \end{bmatrix}$$

4.2 Forward Kinematics (Including l_3)

4.2.1 Cartesian Coordinates

$$x_e = \left(l_1 \cos\left(\tfrac{\theta_1-\theta_2}{2}\right) + \sqrt{{l_2}^2 - {l_1}^2 \sin^2\left(\tfrac{\theta_1-\theta_2}{2}\right)} \right) \cos\left(\tfrac{\theta_1+\theta_2}{2}\right) + l_3 \cos(\arcsin(\frac{l_1 \sin(\frac{\theta_1-\theta_2}{2})}{l_2})) \tag{32}$$

$$y_e = \left(l_1 \cos\left(\tfrac{\theta_1-\theta_2}{2}\right) + \sqrt{{l_2}^2 - {l_1}^2 \sin^2\left(\tfrac{\theta_1-\theta_2}{2}\right)} \right) \sin\left(\tfrac{\theta_1+\theta_2}{2}\right) + \frac{l_3 l_1 \sin(\frac{\theta_1-\theta_2}{2})}{l_2} \tag{33}$$

For computing the Jacobian

$$\frac{\partial x_e}{\partial \theta_1} = \cos\left(\tfrac{\theta_1+\theta_2}{2}\right)\left(-\tfrac{1}{2} l_1 \sin(\lambda) - \frac{{l_1}^2 \sin(\lambda)\cos(\lambda)}{2\sqrt{{l_2}^2 - {l_1}^2 \sin^2(\lambda)}}\right) - \tfrac{1}{2}\sin\left(\tfrac{\theta_1+\theta_2}{2}\right)\left(l_1\cos(\lambda) + \sqrt{{l_2}^2 - {l_1}^2 \sin^2(\lambda)}\right) - \frac{{l_1}^2 l_3 \sin(\lambda)\cos(\lambda)}{2{l_2}^2\sqrt{1 - \frac{{l_1}^2 \sin^2(\lambda)}{{l_2}^2}}} \tag{34}$$

$$\frac{\partial x_e}{\partial \theta_2} = \cos\left(\tfrac{\theta_1+\theta_2}{2}\right)\left(\tfrac{1}{2} l_1 \sin(\lambda) + \frac{{l_1}^2 \sin(\lambda)\cos(\lambda)}{2\sqrt{{l_2}^2 - {l_1}^2 \sin^2(\lambda)}}\right) - \tfrac{1}{2}\sin\left(\tfrac{\theta_1+\theta_2}{2}\right)\left(l_1\cos(\lambda) + \sqrt{{l_2}^2 - {l_1}^2 \sin^2(\lambda)}\right) + \frac{{l_1}^2 l_3 \sin(\lambda)\cos(\lambda)}{2{l_2}^2\sqrt{1 - \frac{{l_1}^2 \sin^2(\lambda)}{{l_2}^2}}} \tag{35}$$

$$\frac{\partial y_e}{\partial \theta_1} = \sin\left(\tfrac{\theta_1+\theta_2}{2}\right)\left(-\tfrac{1}{2} l_1 \sin(\lambda) - \frac{{l_1}^2 \sin(\lambda)\cos(\lambda)}{2\sqrt{{l_2}^2 - {l_1}^2 \sin^2(\lambda)}}\right) + \tfrac{1}{2}\cos\left(\tfrac{\theta_1+\theta_2}{2}\right)\left(l_1\cos(\lambda) + \sqrt{{l_2}^2 - {l_1}^2 \sin^2(\lambda)}\right) + \frac{l_3 l_1 \cos(\lambda)}{2 l_2} \tag{36}$$

$$\frac{\partial y_e}{\partial \theta_2} = \sin\left(\tfrac{\theta_1+\theta_2}{2}\right)\left(\tfrac{1}{2} l_1 \sin(\lambda) + \frac{{l_1}^2 \sin(\lambda)\cos(\lambda)}{2\sqrt{{l_2}^2 - {l_1}^2 \sin^2(\lambda)}}\right) + \tfrac{1}{2}\cos\left(\tfrac{\theta_1+\theta_2}{2}\right)\left(l_1\cos(\lambda) + \sqrt{{l_2}^2 - {l_1}^2 \sin^2(\lambda)}\right) - \frac{l_3 l_1 \cos(\lambda)}{2 l_2} \tag{37}$$

$$\mathbf{J} = \begin{bmatrix} \frac{\partial x_e}{\partial \theta_1} & \frac{\partial x_e}{\partial \theta_2} \\ \frac{\partial y_e}{\partial \theta_1} & \frac{\partial y_e}{\partial \theta_2} \end{bmatrix}$$

5 ODrive

5.1 Notes

- It is significantly easier to use a python script to edit the settings on the ODrive than using the ODrive Tool shell. A lot of time was initially wasted navigating through heavily nested settings in the shell.

5.2 Configuration

The file *configuration.py* will setup the ODrive to control the T-Motor U10 Plus 80KV with feedback from the E6B2-CWZ6C Encoder. The script saves the settings to memory on the ODrive, so it only needs to be run once at the beginning.