MOODY'S MEGA MATH CHALLENGE 2016

The automotive industry represents one of the most active components of American culture with millions of workers traveling by car every day. However, the cost to buy motor vehicles lies outside the price range for many hard working Americans. Therefore, many commuters rely on car sharing to get from place to place as a means of reducing the cost of transportation. As new companies and industries arise, new modes of transportation and new models for car sharing systems are born and provide additional options that in effect, cater to the preferences of peoples from various locations and economic situations.

Companies effectively create models to represent possible situations that hold the potential to benefit those with various needs. Statisticians consider many variables when modeling situations including miles driven per day and the time spent in cars for large populations. From this, modelers find proportions that represent all the different permutations of these variables. Therefore, statisticians can predict the discrepancies within certain populations.

Additionally, statisticians can predict the effectiveness of certain situations and models with respect to certain populations in different locations. In the case of car sharing, there are several options to be considered and as expected, different options may provide greater benefits for different situations. Identifying which option is best requires careful analysis of the location and the needs of the people living there.

Statisticians must also plan for the future. With the emergence of self-driving cars, new possibilities arise with respect to car sharing and other modes of transportation. In addition, advances in renewable energy have generated exciting prospects for more efficient and environmentally friendly motor vehicles. The resulting lower consumption of fossil fuels can generate savings in economic expenses and a lessening in emitted pollutants. In accordance, these new advances hold the potential for the proliferation of the car sharing industry.

In the following sections, we will tackle the challenge of modeling the growing car sharing industry, its range of customers, and the prospects of future technological advancements.

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1 PART 1: DRIVING PATTERNS IN THE UNITED STATES

1.1 ANALYSIS OF THE PROBLEM

Using “low–medium–high” terciles for each dimension implies a 3×3 nine-cell matrix: “low time, low distance” drivers, “low time, medium distance” drivers, “low time, high distance” drivers, “medium time, low distance” drivers, “medium time, medium distance” drivers, and so forth.

Because this two-dimensional distribution is lost through the use of most statistical processes in the data sources available, we must use other cleavage points to decouple the metrics and reconstruct the two-dimensional distributions. One way to do this is to generate random datapoints off of the means of demographic subgroups in the data —groups by age, gender, ethnicity, education, or location.

1.2 ASSUMPTIONS

As we are attempting to reconstruct a dataset based on the sample means of various subgroups in the subject pool, we must state several assumptions before proceeding:

1. The variation among each demographic cleavage is the same—i.e. African Americans exhibit the same *variance* in miles and minutes driven as do white Americans, even if their *mean* miles and minutes driven are unequal. This assumption is justified across all the groups *in a particular cleavage* because there is no apparent reason why a certain demographic section might be more homogeneous than others. This assumption is not justified for groups across *different* cleavages (for example, “men” vs. “college educated”) because some cleavages may be more disjoint than others. However, the existence of such data that would determine the specific variation across cleavages would render this part of the Challenge moot, so to proceed with our model, we will assume all social cleavages are equally variant in regards to the behavior of its members.
2. We drew our sample means from the AAA *American Driving Survey*, which may have surveyed licensed drivers in slightly different proportions than those of the actual population of US licensed drivers. We believe that the survey proportions present in the AAA’s survey are representative to the true proportions of licensed drivers in the US because the AAA published total statistics covering its entire subject pool. As a reputable organization, it would not have calculated such statistics if it knew the total was invalid, as it would be if a sampling bias was present in its demographic subgroups.

1.3 DEFINITION OF THE MODEL

For the purposes of our model, we will consider “low–medium–high” to refer to three terciles in our modeled population. Hence, the “low” category for driving duration will refer to the $\frac{1}{3}$ of drivers who spend the least amount of time behind the wheel, and the “high” category for driving distance will refer to the $\frac{1}{3}$ of drivers who drive the greatest distance. The “high distance, low duration” category will then refer to the intersection of these two sets of drivers, and so forth for our nine time vs. distance categories.

1.4 THE CORRELATION BETWEEN MILES AND MINUTES DRIVEN

When we plot the sample means of each demographic subgroup, a strong linear correlation between miles driven and minutes driven appears.

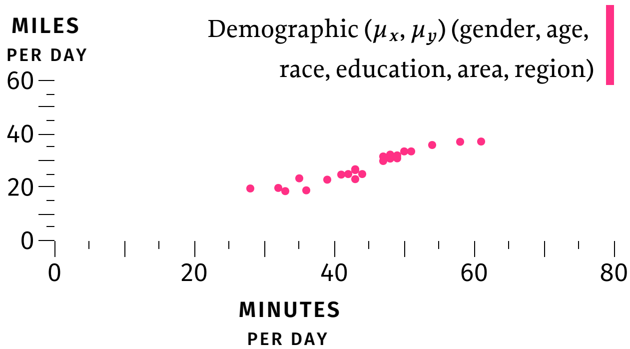


Figure 1: Driving behavior by demographic group.

This should not be unexpected, as spending more time behind the wheel should generally result in a greater distance traveled (traffic jams notwithstanding).

The regression line comes out to be

$$\hat{y} = 0.6536195419x - 0.2189987302\,,$$

with an r^2 equaling 0.9952216228.

The upshot of such a strong linearity is that the nine cells of our driver behavior matrix will not be equal—the top-left and bottom-right cells will have very few drivers (few drivers are able to travel a long distance in a short time, and few are willing to drive for a long time to travel a short distance). Similarly, cells aligned on the axis of our regression will have many drivers, since they will match typical driving patterns—long time, long distance; short time, short distance.

1.5 WEIGHTING THE SUBGROUPS

Because each demographic subgroup is not the same size, we must weight our means appropriately to reflect the proportion of individuals belonging to that subgroup. Note the proportion total of each cleavage is by definition equal to 1.

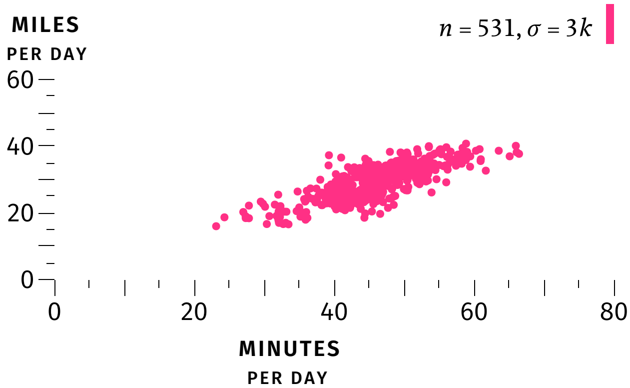
GROUP	PROPORTION
Gender	
Men	0.4956
Women	0.5044
Race	
Whites	0.7255
Blacks	0.1256
Hispanics	0.0874
Other	0.0434
Age	
16–19	0.0648
20–29	0.1320
30–49	0.2627
50–64	0.2757
65–74	0.1341
75+	0.1308
Education	
Some high school	0.0618
High school graduate	0.2814
Some college	0.2332
College graduate	0.2474
Graduate school	0.1320
Setting	
Urban	0.6327
Rural	0.3673
Section	
Northeastern	0.1877
Midwestern	0.2440
Southern	0.3902
Western	0.1781

Table 1: Demographic proportions

Note: cleavage totals may not equal 1 due to rounding.

1.6 MODELING THE DRIVER POPULATION

Clearly, members of each demographic group are not homogeneous. 16–19 year old drivers do not all drive the mean number of miles each day. We have to then blur each demographic mean into a two-dimensional normal probability distribution to produce a more realistic model of the driving population.



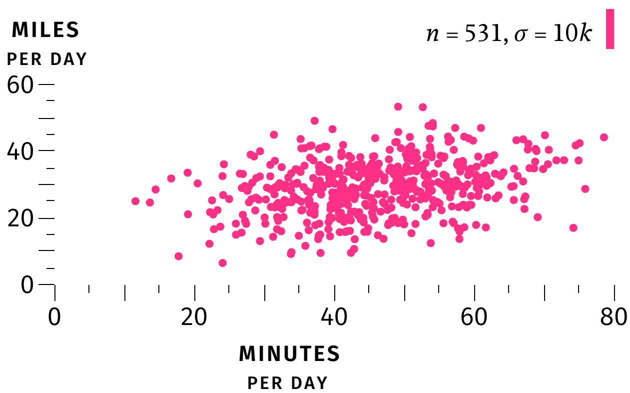


Figure 2: Reconstructed driver populations
Randomly generated "drivers", from input standard deviations σ = 3 min, σ = 10 min.

That then begs the question, what standard deviation do we use? Data on the variance of driving habits is unreachable or nonexistent, which forces us to create a function that would return a model of the driving population for any possible input σ.

We also have the problem of relating σ_{miles} to σ_{minutes}. We will assume the two-dimensional normal distribution about each mean is "circular", but to prevent this from being susceptible to changes in scale between the two axes (which could arbitrarily stretch or squash the distributions into ellipses), we must affix the dimensions of the distribution to the slope of the regression line of the original points so that it is circular with respect to our data. Because the slope of the regression k equals 0.6536195419, that is the amount we must scale σ_y . (We could have also scaled σ_x instead of σ_y , by the reciprocal of k, but our input σ would then be in units of miles instead of minutes.)

Dividing this simulated population into nine behavioral groups is then a matter of dividing the datapoints into three quantiles in each dimension, yielding nine regions. Table 2 gives the positions of the 33rd and 67th percentiles along both quantities in the $\sigma = 3k$ model.

QUANTITY	33RD	67TH
min	41.6225	51.3922
miles	26.3570	33.5156

Table 2: Tercile positions, σ = 3k.

Sorting the "drivers" into the nine regions is then a matter of counting the number of points in each region. The following graph shows the proportion of drivers in each matrix cell, as a function of σ. The cells are labeled (h, k), where h is the horizontal index and k is the vertical index. ((0, 0) is the bottom left.)

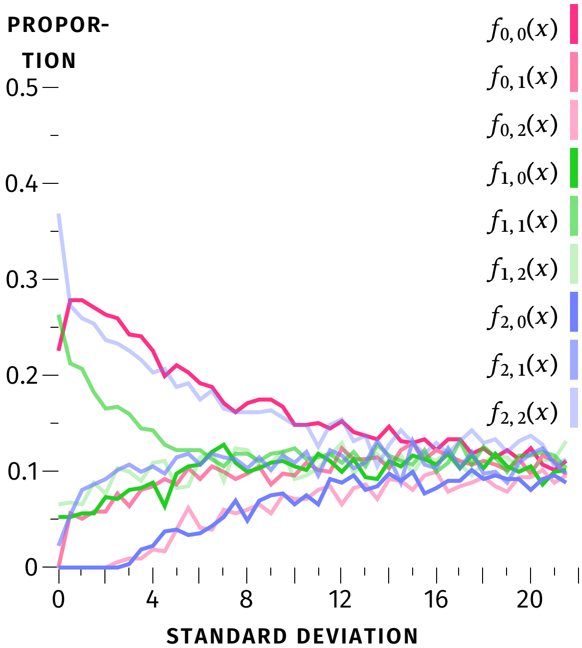


Figure 3: Cell proportions as a function of σ
It should be apparent that the three cells aligned with the axis of regression ((0, 0), (1, 1), (2, 2)) contain the greatest percentages of drivers. The two cells opposite the axis of regression ((0, 2), (2, 0)) contain the least. And the four cells peripheral to the axis contain an intermediate percentage of drivers.

As σ increases, the cell proportions converge until each contains a roughly equal percentage of drivers. This is expected, because as the subgroup distribution radius increases, the linear stress of the entire population gets drowned out, spreading more drivers away from the axis of regression and into the two extreme cells. Table 3 gives the percentages of drivers in each category for selected values of σ.

DRIVER TYPE	σ = 2	σ = 4	σ = 8
low mileage, low time	26.99%	22.47%	16.84%
low mileage, mid time	6.27%	8.67%	10.33%
low mileage, high time	0.08%	2.18%	6.16%
mid mileage, low time	6.31%	9.27%	10.69%
mid distance, mid time	17.94%	14.06%	11.67%
mid distance, high time	9.09%	10.01%	10.98%
high distance, low time	0.03%	1.58%	5.80%
high distance, mid time	9.13%	10.61%	11.34%
high distance, high time	24.17%	21.14%	16.19%

Table 3: Car usage patterns
This table shows the percentages of drivers who would fit into each car usage pattern under selected values of σ.

2 PART 2: IMPLEMENTING AND OPERATING A CAR SHARING SYSTEM

2.1 ANALYSIS OF THE PROBLEM

To plan a viable car sharing system, we need to be able to model public participation in the system, given cost and factors such as density of car stations, willingness to travel to pick up a car ("first leg costs"), willingness to travel to the destination ("last leg costs"), the cost of gasoline, the cost of parking, and the required car investment for the car sharing company. These costs fall into two types: **monetary costs** (measured in dollars) and **convenience costs** (measured in probabilities). These two types of costs can then be compared on the basis of income—the marginal value of money decreases as disposable income increases, in other words, the richer a city's population is, the more willing they are to trade money for convenience.

2.2 DESIGN OF THE MODEL

To begin, let's model a typical car usage routine. Most Americans are incredibly rut-bound—we wake up, transport ourselves to work, work an eight hour day, transport ourselves home, go to sleep, and repeat the next day:

$$\text{home} \xrightarrow{\textit{car}} \text{work} \xrightarrow{\textit{car}} \text{home}\,.$$

Since we are using car self-ownership as our baseline, we can assign the convenience cost C of driving a car as being equal to 1—driving is perfectly convenient:

$$C_{\text{total}} = {C_{\text{car}}}^2 = 1\,.$$

Note that we multiply convenience costs to find the total—if you must do two things to accomplish an activity, it doesn't matter how easy one of the things is, if the other thing is incredibly difficult—you won't perform the activity.

2.3 SYSTEM 1

Now let's consider the first of our possible car sharing systems: customers rent a car for a certain period of time, and return it to the same station when they are done with the car. For most Americans, the story would involve walking on "foot" to the nearest car station (we will call any form of non automotive travel, i.e. biking, "on foot"). One would then drive the car to their workplace (leaving the car in a parking lot during the workday) and drive it back home in the afternoon. One would return the car to the original car station, then make one's way back home on foot. The travel graph ("H" for home, "S" for station, and "W" for workplace) would then look something like this:

$$\text{H} \xrightarrow{\textit{foot}} \text{S}_1 \xrightarrow{\textit{car}} \text{W} \xrightarrow{\textit{car}} \text{S}_1 \xrightarrow{\textit{foot}} \text{H}\,.$$

The convenience cost C_1 could then be expressed as

$$C_1 = {C_{\text{car}}}^2 \cdot {C_f}^2 = {C_f}^2\,,$$

since the convenience cost of driving a car is still 1. The quantity C_f, the convenience cost of having to walk to or from the car station, can also be broken down further, because C_f is proportional to the distance a person has to walk to reach the station. Hence,

$$C_f = \frac{P_f}{x_{SC}} = \frac{P_f\sqrt{D}}{s}\,,$$

where P_f is a person's **propensity for foot travel**, and x_{SC} is the distance they must travel to reach the nearest car station. (In a slightly more nuanced model, x_{SC} would probably be ${x_{SC}}^2$ or some other concave-up function, as Western humans are generally very willing to travel very short distances on foot, but extremely adamant against traveling longer distances on their own power). If we consider that a car sharing company can only afford to build a certain number of car stations to serve a given number of people, then x_{SC} can be expressed as $\frac{s}{\sqrt{D}}$, where D is the density of the city the car sharing company has deployed car sharing in, and s is some constant determined by the company's own ability to invest in building car stations. D is raised to a power of ½, because to halve the spacing between stations, four times as many stations must be built. (By the same line of geometric reasoning, $\sqrt{D}$ approaches D as the city becomes less square and more eccentric.) The total convenience cost of system 1 can then be expressed as

$$C_1 = C_f{}^2 = \frac{P_f{}^2 D}{s^2} .$$

2.4 SYSTEM 2

Let's consider system 2: on-demand car sharing. On demand car sharing is simple, and its convenience costs are very similar to that of car ownership, since the car is brought to your door, and the company picks up the car for you when you are finished.

H ~***wait***~> –***car***–> W ~***wait***~> –***car***–> H .

The only convenience cost would be the time that one has to wait for the company to bring the car.

$$C_2 = C_{\text{car}}{}^2 \cdot C_w{}^2 = C_w{}^2 .$$

Note that while it is theoretically possible to relate C_w to to the city wage level as an time-opportunity cost, this is probably not worth it because this cost is so tenuous (people happily waste their own time without hesitation) and so dependent on the car sharing company's own organization and logistical skill, that for the purposes of our model, we will consider

$$C_2 \approx C_0 = 1 ,$$

where C_0 is the baseline convenience cost of car ownership, by definition equal to 1.

2.5 SYSTEM 3

System 3, one-way stationed car sharing, involves customers picking up and returning cars to car stations, much like in system 1, with the difference being that customers can return a car to a different car station than where they originally got it. Although the travel graph for this system may degenerate into that of system 1 (discussed later), the proper travel graph of system 3 looks something like this:

H —***foot***—> S_1 —***car***—> S_2 —***foot***—> W —***foot***—> S_2 —***car***—> S_1 —***foot***—> H .

In other words, a customer walks to a car station, drives the car to the station nearest to their workplace, returns the car, retrieves the car in the afternoon, drives to the station nearest to their residence, and walks home. The convenience cost C_3 is then

$$C_3 = C_{\text{car}}{}^2 \cdot C_f{}^4 = C_f{}^4 = \frac{P_f{}^4 D^2}{s^4} .$$

Note that we are assuming that the distance from the second car station to the workplace is equal to the distance from the first car station to the customer's house, which given a uniform city, it will average out to be, for all of a city's residents. If a city is nonuniform, this would not hold if a car company strategizes by building more car stations near the residential sectors or the commercial core. However there is no reason the car company would ever do this, since the maximum of the product of two terms (the $C_f{}^2$ from the home and the $C_f{}^2$ from the workplace) occurs when the two terms are equal, and it is this maximum where customer convenience is best (a higher C is better).

We should also note that this travel pattern will only occur if the monetary cost of keeping a car in a parking lot, K, plus the monetary cost of car rent, M_R (zero for daily or mileage rent) exceeds the convenience cost of walking from the second car station to the workplace and back, plus the monetary cost of getting to work on foot, M_f, discussed in the monetary costs section (t = work hours):

$$tK + M_R - M_f > C_f{}^2 ,$$

otherwise the customer will simply pay the parking fee and leave the car in an office parking lot instead of checking it back in at a car station.

2.6 SYSTEM 4

Finally, in system 4, fractional car ownership (similar to a timeshare), the only additional convenience cost over that of regular car ownership would be the trouble of organizing and planning a car sharing schedule between the various part-owners, C_L.

$$C_4 = C_{\text{car}}{}^2 \cdot C_L = C_L \, .$$

2.7 MONETARY COSTS

Each of our possible car sharing systems includes of course, a monetary cost—the cost of car sharing that is paid for directly with dollars. The primary monetary cost in all of these systems would be gas and car maintenance (whether or not it is initially paid for by the customer or the company is irrelevant—the company will pass on the additional cost to the customer to remain operational, and therefore profitable). These costs will be bundled together by the company entrepreneur as M_R , the price of car rent.

All of the components of M_R are dependent on the number of miles, d, the cars are driven, times the per-mile cost of gasoline, $\frac{G}{\varepsilon}$, and the per-mile cost of car maintenance, M_M,

$$M_R = d\left(\frac{G}{\varepsilon} + M_M\right) ,$$

M_M is roughly constant across different cities, although minor variations due to climate and insurance may occur (these small incongruencies have been ignored in our model). The per-mile cost of gasoline is equal to the price of gasoline, divided by the car's fuel efficiency. In 2015, the average American car had a fuel efficiency, ε, of 25.27 miles per gallon (*University of Michigan*). The price of gasoline is highly variable across location and is given in Table 4.

CITY	GASOLINE PRICE US DOLLARS/GALLON
Poughkeepsie, NY	2.64
Knoxville, TN	1.90
Riverside, CA	2.92
Richmond, VA	1.94

Table 3: Gasoline prices in subject cities
The price of gasoline (as of February 28, 2016) in the four subject cities in our model. Gas prices are highly variant—gas is almost 53.7% more expensive in Riverside than Knoxville.

In system 1, round-trip stationed car sharing, the total distance traveled per day, per customer, would simply be twice the distance from the residential car station to the workplace, x_{SW}.

$$d = 2x_{SW}$$

The per-customer cost of rent would then be

$$M_R = 2x_{SW}\left(\frac{G}{\varepsilon} + M_M\right) .$$

However, there are also several costs which are borne directly by the customer—the cost of parking (K = the parking rate), and the cost of transporting himself or herself to the residential car station, M_f (i.e. bike maintenance costs, shoe-rubber costs, etc.).

$$M_1 = M_R + tK + M_f$$

Note that even if rent is charged by hour, t (work hours) is not multiplied into M_R, because M_R is determined by the overall revenue the car sharing company needs to stay operational, not *how* that revenue is charged. However, this does have implications for customer behavior, as we saw in the convenience analysis of system 3.

If rent is charged by mile, this system also incurs an opportunity cost on the car sharing company, since a rented car sitting in a parking lot could be rented out to someone else and driven for more revenue. But again, this is a question of *how* rent is charged, and it is unclear how much demand there would be for mid-day car rentals anyway.

System 2, on-demand car sharing, incurs a slightly greater operational cost because of the cost of labor (car jockeys to reposition cars), and the slightly greater total distance the cars must travel.

The total distance traveled is then equal to twice the distance from the customer to the workplace, $2x_D$, plus the *four times* the average distance from the stations to the customer, x_{SC} (a jockey from a station must deliver the car to, and retrieve it from the customer each time it is rented). However, the "consumer costs" $tK + M_f$ have vanished.

$$M_2 = 2(x_D + 2x_{SC})(\frac{G}{\varepsilon} + M_M)\,,$$

System 3, one-way stationed car sharing, is again very much like system 1, except the one-way distance is from a station to another station, x_{SS}, instead of from the customer to the workplace.

$$M_3 = 2x_{SS}(\frac{G}{\varepsilon} + M_M) + M_f\,.$$

The cost of parking has disappeared, although the "on-foot" costs remain. While the on-foot distance traveled has effectively doubled in this system, we will treat M_f as constant since it is unclear how strongly M_f is affected by distance walked.

In system 4, fractional car ownership, the distance traveled per customer (or in this case, part-owner) is simply $2x_D$.

$$M_4 = 2x_D(\frac{G}{\varepsilon} + M_M)\,.$$

There may also be an amorphous cost savings in system 4, due to the fact that the middleman (the car sharing company) has been cut out, though the size (or even sign) of this cost savings is unclear because of volume-buying considerations and the experience the car sharing company would bring in terms of car logistics and maintenance.

Throughout this monetary analysis, we have kept distinct the differences between x_{SW}, x_D, and x_{SS}, although on average, all of these distances will be equivalent (remember, the cities are two dimensional, and the nearest station may not be partway between the customer and his/her destination), hence we can treat them as,

$$x_{SW} \approx x_D \approx x_{SS}\,.$$

As in the convenience analysis, x_{SC} is inversely proportional to the square root of the density of the city, however this quantity in unimportant for vehicular travel, since we assume that for a car sharing service to be useful, the distance from customers to the stations must be much smaller than the distances the customers want to travel; i.e.

$$x_{SC} \ll x_D\,.$$

x_D itself is also roughly proportional to the square root of the density of the city.

2.8 DETERMINATION OF BEHAVIORAL CONSTANTS

As we are attempting to model human behavior, our math relies on several behavioral constants that reflect human behavior, such as the propensity for foot travel, P_f. Like any parameter that attempts to quantify human behavior, this value can only be determined by surveys or behavioral scientific studies. Similarly, a quantity like s, a firm's ability to invest in building car stations, can only be known by entrepreneurs familiar with the car business. However, while we cannot determine an absolute estimate of costs, and therefore, of participation in various car sharing models, we can perform a relative analysis, ranking the systems in estimated car sharing participation for each of our four subject cities.

2.9 KNOXVILLE, TN (SYSTEM 3)

Knoxville, TN would best respond to a one-way model where customers would pick up and drop off cars at existing stations (system 3) because it is the cheapest option due to the fact that it eliminates the need for parking (tK). Although K in Knoxville is the lowest of any subject city (0.45¢/hr), it is still more expensive than zero. M_R, for its part, is equivalent between systems 1 and 2 since $x_{SW} \approx x_{SS}$. Although system 1 has a better convenience cost, we can eliminate system 1 for the same reason we eliminate system 2: Systems 2 and 3 are distinguished by their exchange of convenience and monetary costs—system 2 has high monetary costs and low convenience costs while system 3 has low monetary costs and high convenience costs, since this system requires the customer to travel to each station before picking up their cars. Because Knoxville has the lowest disposable income of the four cities ($2,392.61 per month), money has a greater marginal value to Knoxvillians. Customers will be more willing to sacrifice convenience in exchange for spending less money. Therefore system 3 is the best fit.

2.10 POUGHKEEPSIE, NY (SYSTEM 2)

Poughkeepsie, NY would best respond to a "jockey" system in which cars are dropped off to the customers on demand by the company and subsequently picked up by the company (system 2). This option is most convenient for customers as it would eliminate the need for customers to travel to stations to pick up their cars. The "jockey" system would be much more expensive than system 3 in terms of dollars as the company would have to pay for drivers, and the slightly increased mileage on the cars ($x_{SW} \approx x_D \approx x_{SS} < x_D + 2x_{SC}$). There will also be an additional expense for the gas usage because gas prices are high in Poughkeepsie. However, because Poughkeepsie also has the highest disposable income of the four cities ($4,000 per month), money has a lower marginal value to its residents, and so customers will be much more willing to spend additional money in exchange for convenience. It's unlikely that system 1 will be much less expensive than system 2, because parking in Poughkeepsie is extremely high—$1.50/hr.

Both Knoxville and Poughkeepsie are probably poor areas to run a car sharing system of any type in, which we will discuss later.

2.11 RIVERSIDE, CA AND RICHMOND, VA

While Riverside, CA and Richmond, VA have very similar disposable incomes ($2766.67 and $2775.75 respectively), similar population densities (3315/sq. mi, 3,625/sq. mi, respectively), and somewhat similar parking prices ($1.00/hr, and $0.75/hr, respectively), the gas prices differ by about a dollar between these cities. Gas in Riverside is \$2.92 per gallon while gas in Richmond is \$1.94 per gallon. Clearly, system 2 is a bad fit for Riverside, since with a high G, the small number of additional miles driven in that system incurs a greater monetary cost, and it is probably not a good fit for Richmond either. Both cities have moderate-to-low disposable incomes which makes the high convenience cost/low monetary cost options more appropriate.

Because of the high cost of gasoline in Riverside, we believe that system 4, where people take fractional ownership in a car may be most appropriate. Our line of reasoning is that because gasoline would cut so deeply into residents' incomes, they will be less able to sustain the higher marginal cost of the renting systems. They may, however be more able to pool resources and incur the greater initial cost of option 4 in exchange for more day-to-day financial breathing room, as well as the amorphous cost savings from cutting out the car company middleman. The initial cost will be mitigated by the fact that multiple people will be sharing the cost, while all receiving the same benefits as if they each bought their own car. The adoption of the system then rests on a comparison between its organizational convenience cost and the large dollar savings from partial ownership.

We believe that either system 1 or system 3 would appropriate for people in Richmond. Because of the lower gas prices and comparable disposable income of the two cities, Richmond residents would feel less pressure to avoid the high marginal costs of renting over partial ownership. For the same reasons as discussed for Knoxville and Poughkeepsie, system 3 will work for residents of Richmond, who have a low–moderate amount of disposable income. However because disposable income in Richmond is some \$400 higher than in Knoxville (and an additional \$400 is very significant in the lower income brackets), Richmond residents may demonstrate a greater affinity for system 1 than do Knoxville residents.

2.12 BEST CITY FOR CAR SHARING COMPANIES

Just as it is possible to rank the systems for the cities, it is also possible to rank the cities for the car companies. Some of the reasons why a company would choose one city over another include differences in population density and disposable income. Since we assume most people are using these services for short distance travels, higher population density correlates to greater popularity. Also, since people with greater disposable incomes are more likely to buy their own car, a more moderate disposable income would correlate to greater popularity as well (though this could also nudge potential customers into partial-ownership arrangements). While there are apparent flaws in each of the systems and cities, several stand out above the others. In Poughkeepsie, disposable incomes are much higher than in any of the other four cities. Also, the population density here is the lowest of all the cities (1,520 people per square mile in Poughkeepsie, 1,816 people per square mile in Knoxville, 3,315 people per square mile in Riverside, and 3,625 people per square mile in Richmond). These are two factors that weigh heavily against Poughkeepsie being the most opportune city for a company to set up a rental car service. In Knoxville, the population density is the second lowest of the four cities, being nearly half of Riverside and Richmond. For this same reason, it would not be as good of a choice for a company to set up a rental car service there. Richmond and Riverside have very similar qualities and both seem opportune for a company to set up a rental car service due to high population densities and moderate disposable incomes. However, since Richmond has a slightly higher population density, a rental car service would be slightly more popular here than in Riverside. In addition, Richmond residents would be more able and willing to pay the higher marginal cost of a car rental over a private car sharing arrangement due to the much lower cost of gasoline in Richmond. (Just 4¢ higher than in Knoxville!)

3 PART 3: DISCUSSION OF EMERGING TECHNOLOGIES

3.1 DRIVERLESS AND ELECTRIC CARS

The expansion of car sharing enterprises (systems 1–3) offers a unique opportunity to deploy emerging automotive technologies such as electric cars and driverless cars on a mass scale.

We believe that car sharing enterprises will be quick to deploy self-driving car technologies because with the implementation of driverless cars, costs would decrease due to decreased labor needs. Driverless cars also offer an opportunity to decrease the distance each car must travel, and hence the cost of gas and maintenance, to service the same number of customers, as self-driving car software can reduce wasteful navigation errors.

Car sharing enterprises will also be much more likely to deploy electric and fuel-efficient car technologies, since these technologies have the characteristics of having high initial but low marginal costs (they eliminate the need to pay for gas). Firms are much more able and willing than individuals to make large initial investments that pay off over the course of operations, and through their greater volume of use.

The upshot is that M_R will collapse significantly due to the replacement of the $\frac{G}{\varepsilon}$ term with a much smaller E term—the price of electricity in a given city. Monetary cost terms such as tK and M_f suddenly become much more influential in the total monetary cost of car rental. We could predict an environment where systems 1 and 4—round-trip car rental, and partial private ownership—become much less attractive since they involve a parking cost. System 2 conversely becomes more viable, since the labor cost of rearranging the cars disappears and its distance penalty is greatly reduced. We will not speculate on the development of behavioral pattern recognition software that would allow companies to preemptively make cars available to regular renters, but this could eliminate the (already mostly irrelevant) C_w term in the system 2 convenience cost.

3.2 AVAILABILITY AND PRACTICALITY

In a world where system 2 becomes very viable, and systems 1 and 4 obsolete, we could see parking become obsolete as the service would become a direct on demand taxi service. Customers would no longer have to find parking spaces or pay for parking tolls; rather, self-driving cars would pick up and drop off customers at their homes and workplaces on demand. In the short run, this could briefly revive systems 1 and 4 by reducing parking prices, but supply would contract in response. All of this would do nothing to affect the fact that without the $\frac{G}{\varepsilon}$ term, parking now makes up a much larger fraction of a car sharing system's cost.

3.3 OVERALL

Because the savings from electric and self driving cars would decrease prices, system 2 and 1—the more convenient systems—would become more attractive over system 3.

1. Poughkeepsie, NY (High disposable income), nothing would change—system 2 is still optimal, though participation will increase due to the lower prices (demand is negatively sloped). Due to high gas and parking costs there is a high incentive for companies to deploy both emerging technologies here.
2. Riverside, CA (Medium disposable income, medium availability for parking, high gas prices) There is a very high incentive for a company to deploy new technologies in Riverside because of the high cost of gasoline. This makes the car sharing services less expensive relative to private partial ownership (which most likely still uses a gasoline car). Either systems 2 or 1 would then be viable in Riverside.
3. Richmond, VA (Medium disposable income, medium availability for parking, low gas prices). The decreased monetary prices and corresponding lower opportunity cost of increased convenience will give system 1 an edge over system 3 here. System 1 then becomes the optimal system in Richmond. Due to low gas prices and mid–low parking prices however, there is no particularly high incentive for a company to deploy the technology here.
4. Knoxville, TN (Low disposable income, high availability for parking) It is ambiguous how emerging technologies would affect car sharing in Knoxville, TN. Knoxville has a very low disposable income level, which keeps option 3 viable, and there are conflicting incentives for the companies to deploy—both gasoline, electricity, and parking are very inexpensive in Knoxville.

While solutions requiring parking availability would not be required through the introduction of self driving cars, the parking for the cars after fulfilling their purpose would still exist, causing the baseline costs of running the service to not decline entirely. Also, as most self driving cars are proposed to run off electricity, costs for charging the cars would be introduced in the wake of gas charges dropping. On top of the need to charge the cars, the charging of the cars will take additional time that will take away from how often a car could be rented out. In order to be able to serve more customers in the same time frame, a larger fleet of cars would need to be purchased.

Richmond has a slightly higher population density, it also carries a smaller cost per kilowatt hour, the main cost in charging an electric vehicle (Self driving or not). Virginia's cost per kWh is \$0.105 while California's cost per kWh is \$0.152. This furthers Richmond as the prime location for the starting location for this service, in an age of electric and self driving cars. Tennessee has a very low kWh at \$0.098, meaning electric cars are more feasible in Tennessee, boosting the likelihood of using such vehicles there. New York has a large kWh of \$0.181, further reducing the likelihood of using self driving cars in Poughkeepsie. Based on the Tesla, a self driving capable car that is fully electric, it takes roughly 0.32 kWh to charge an electric car per mile driven (With the best charging station setup, most likely bought by a company specializing in the use of them). In addition, it takes roughly one minute to charge per mile driven, meaning that two electric cars must be used for what each gasoline powered car could service. The cost to charge an electric vehicle is found through the equation $\mathrm{Cost}_{\mathbf{Charge}} = D_{\mathbf{total}}(.32\ \mathrm{kWh/Mile})(\mathrm{Cost}_{\mathrm{kWh}})$. In addition, the time for the charge is give by the formula $\mathrm{Time}_{\mathbf{Charge}} = D_{\mathbf{total}}$ (in minutes).

4 CONCLUSION

Richmond would be an ideal city to deploy a car sharing system of any type. It has the highest population density out of all four subject cities, adequate disposable income to demand car sharing services, and low gas and electricity prices.

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(Used in order to gain population information on each city)