MANCHESTER 1824
The University of Manchester

School of Electrical and Electronic Engineering

Embedded Systems Project

DESIGN REPORT #1

Title: Electric Motor Characterisation and Gear Ratios

Group Number: 37

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1. Introduction

This report analyses the measurements taken in the motor characterisation and force laboratory, evaluates their meaning and significance to the characterisation and concludes all the measurements to recommend the final gear ratios for the buggy. This is a crucial part of the embedded systems project as the correct gear ratios must be found for our design to ensure the buggy can move around different terrains, trading off appropriately between torque and speed. A gear box uses efficient gear ratios to provide higher levels of torque for the same mass. However, the speed of the motor is reduced as torque increases so the buggy will not be able to travel around the track as quickly. Furthermore, the microcontroller current is too small to power a motor, so a motor driver is necessary to communicate with the motor and provide the higher currents needed to operate. The microcontroller produces electrical signals and the motor gets mechanical signals that cause it to move. The electrical signal that determines the motor speed is the voltage whilst the current flowing through the motor circuit effects the torque in the input shaft.

2. Motor characterisation

2.1. Introduction

Motor characterisation is an important part of the project as it enables the calculation of the armature resistance, brush voltage and torque constants of the motors.

2.2. Constraints on maximum voltage and current

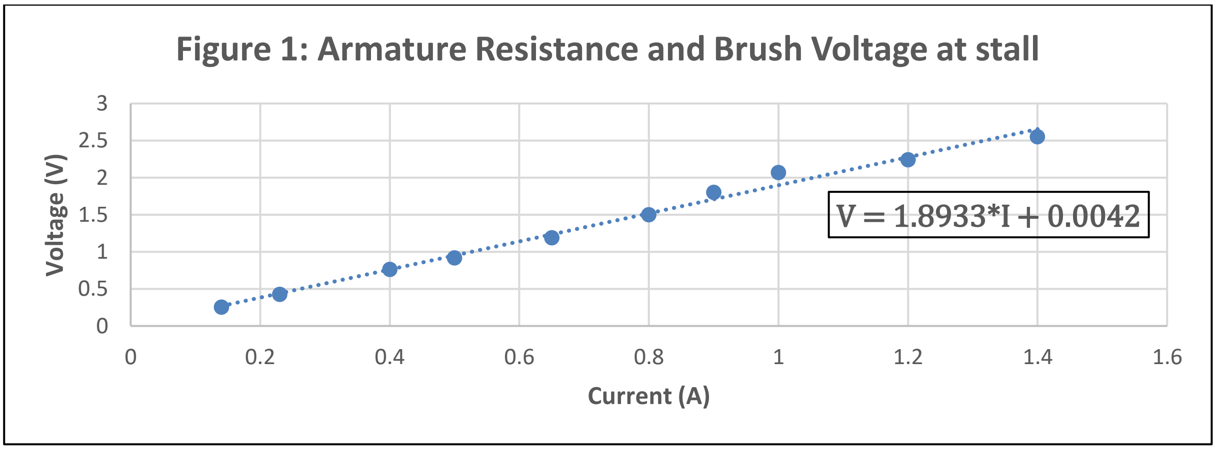
The motor drive board can supply a maximum of 1.4 A [1] restricting the values that can be tested. According to Joule's law ($Q = \frac{V^2}{R} * t$) (Equ. 1) [2] where Q is thermal energy, the higher the voltage, the hotter the motor will be. To protect the motor, voltage was set at 5 V [1].

2.3. Test results and Calculations

2.3.1 Diameter of the shaft

In order to get a more accurate value of the diameter, three measurements were performed, and the average was calculated giving 9.63 mm.

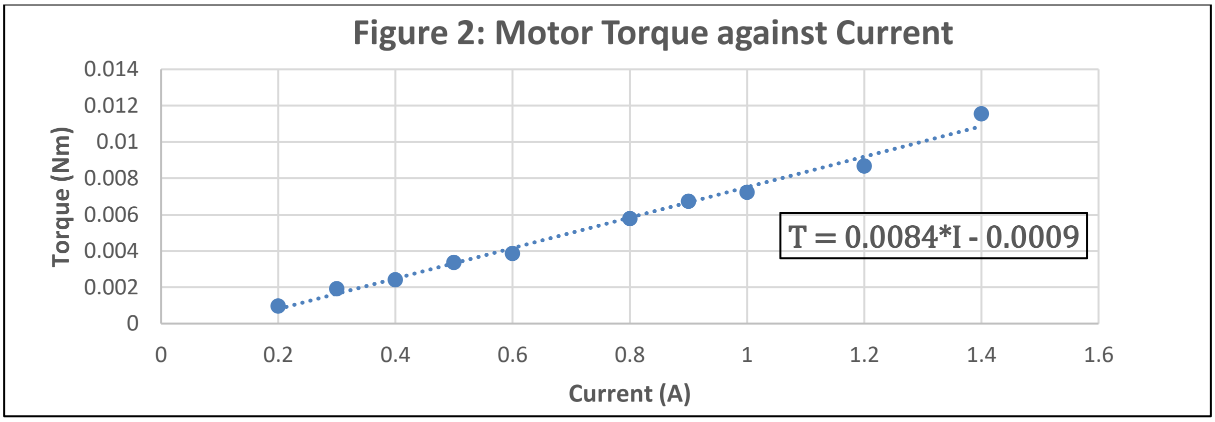
2.3.2 Armature Resistance and Brush voltage with Motor stalled



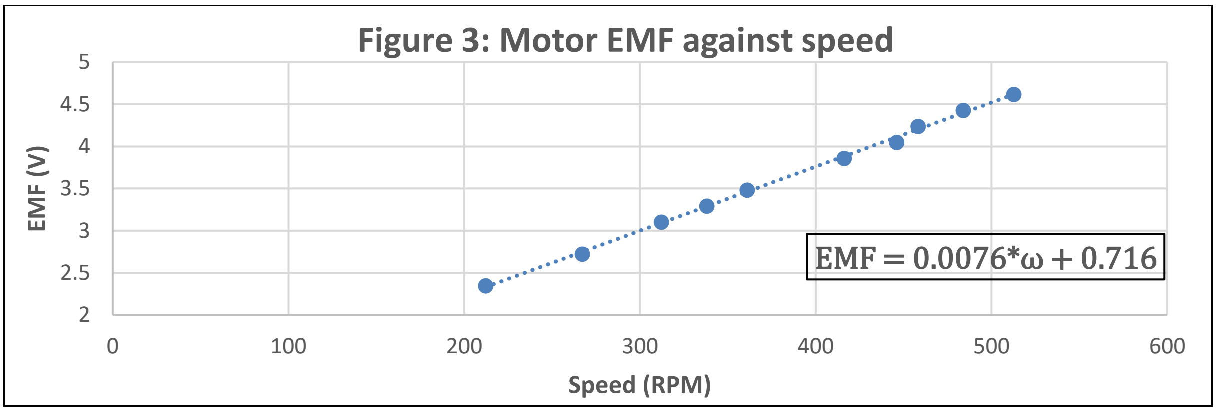
Due to $V = R * I + V_b$ (Equ. 2) [3] , $R = 1.893\ \Omega$ and $V_b = 0.0042\ V$.

2.3.3 K_E and K_T at a constant motor voltage

With voltage set to 5V, the forces F1 and F2 [4] and motor speed were measured. Due to $T = (F1 - F2) * 0.5d$ (Equ. 3) [4] , $EMF = V - V_b - I * R$ (Equ. 4) [5] , torque and motor EMF were found at different currents.



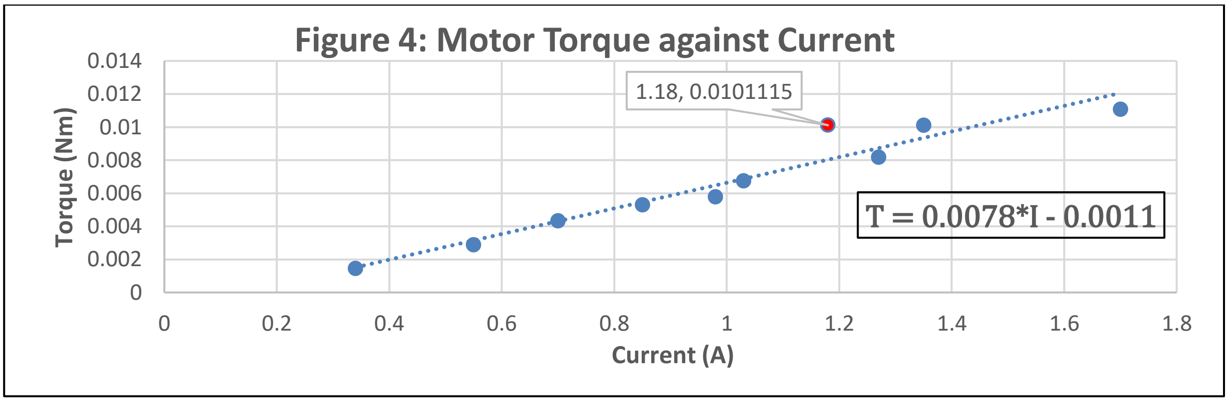
Due to $T = K_T * I - T_{friction}$ (Equ. 5) [6], $K_T = 0.0084$.



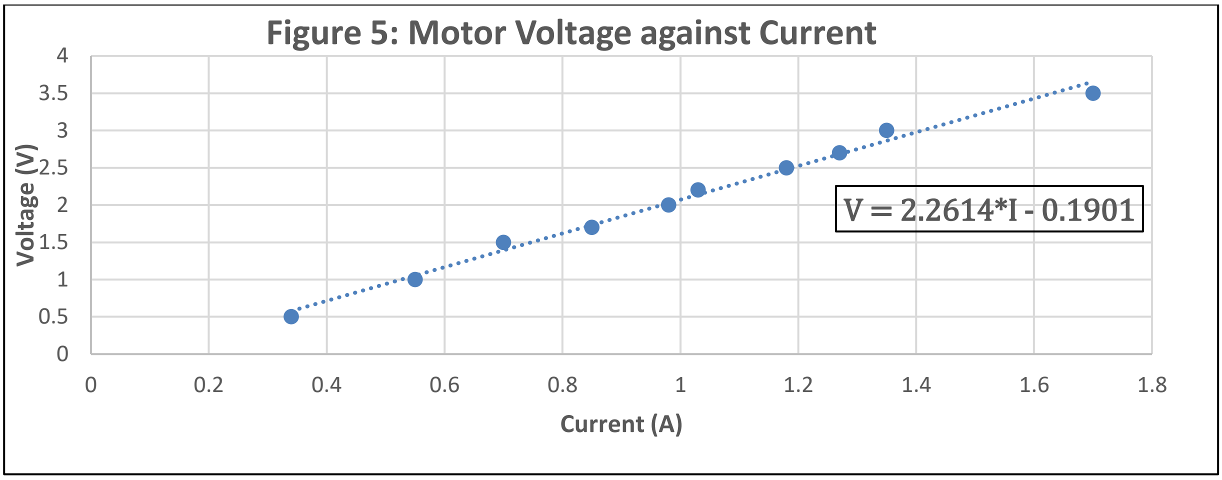
Due to $EMF = K_E * \omega$ (Equ. 6) [7], $K_E = 0.0076$.

2.3.4 K_T and R at stall

By controlling the forces, the motor was stalled and the current was recorded. The process was repeated ten times and the values of force and current under different voltages were measured. According to (Equ. 3), the values of torque were calculated:



Due to (Equ. 5), $K_T = 0.0078$.



Due to $(\mathrm{Equ.}\,2)$, $\mathrm{R} = 2.2614\ \Omega$.

2.4. Comments on the measurement accuracy

2.4.1 Resistance Error

There is a difference in resistance found between Figure 1, $1.893\ \Omega$, and Figure 5, $2.2614\ \Omega$, caused by the heating effect. The measurements presented in Figure 5 have been performed after the motor has already been used for the previous tasks. Therefore, its temperature has already increased. As a result, higher temperature in the motor results in slightly bigger resistance.

2.4.2 Force Error

As shown in Figure 4, the point $(1.18\ \mathrm{A}, 0.0101115\ \mathrm{Nm})$ deviates far from the fitting curve. The main reason for that is the inaccurate measurement of force. There are some errors resulting from not reading the force value accurately. For example, applying bigger forces than required to keep the motor just stalled, or if the perspective of reading the value is changing. In order to increase the accuracy of force measurement, digital equipment could have been used.

3. Load measurements

3.1. Introduction

This section will determine the forces required to move the buggy up the slope and on the flat surface. These forces depend on the mass of the buggy and on the calculation of the friction constant. From the measurements, the constant's error is also derived. From these values of forces, the torque required at the wheel shafts is calculated leading to the selection of the most suitable gearbox.

3.2. Estimated Forces required to drive the buggy up the slope

As the mass can only be estimated, a 30% error is added. The best estimate of mass is calculated using the table of mass given [8] + 100g of circuitry/wiring/stripboard:

Best estimate of mass: $m_{est} = 1537\ g\ \pm 30\% = 1537\ g\ \pm 461.1\ g$

$m_{max} = 1998.1\ g$ $\qquad$ $m_{min} = 1075.9\ g$

To find the coefficient of friction, the forces in the horizontal axis must be solved (Fig.7):

$$\sum F = m * a = 0 \text{ (Equ. 7)}$$

$$\sum F = F_x - N = 0 \text{ (Equ. 8)}$$

$$F_x = N = m_{est} * g * c_f \text{ (Equ. 9)}$$

$$c_f = \frac{F_x}{W} \text{ (Equ. 10)}$$

From the measurements in the force lab, the mean value and the standard deviation of c_f can be calculated.

c_f in Flat	Mean	Standard Deviation	Standard Error
Table 1: Friction constant	0.073354847	0.006898639	0.002816357

Hence, $c_f = 0.0734 \pm 0.0028$

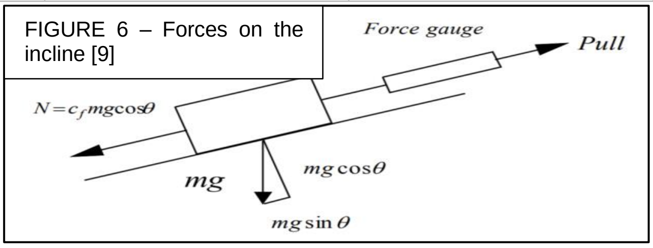


FIGURE 6 – Forces on the incline [9]

For $m = m_{est} = 1537\ g$:

For the friction force on the incline [9]:

$$N = c_f * m_{est} * g * \cos\theta \text{ (Equ. 11)}$$

The buggy needs to overcome this force and the component of weight so for the force from the motor [7]:

$$F = N + m * g * \sin\theta \text{ (Equ. 12)}$$

That is the total force so for each wheel in the maximum incline of 18°:

$$F_1 = \frac{c_f * m_{est} * g * \cos(18°) + m_{est} * g * \sin(18°)}{2} = \frac{0.0734 * 1.537 * 9.8 * \cos(18°) + 1.537 * 9.8 * \sin(18°)}{2} = 2.853 \text{ N}$$

For the propagation of error [10]:

We consider $F_a = \frac{c_f * m_{est} * g * \cos(18°)}{2}$ (Equ. 13) and $F_b = \frac{m_{est} * g * \sin(18°)}{2}$ (Equ. 14)

The error for the first part of the fraction is:

$$\left(\frac{\delta F_a}{F_a}\right)^2 = \left(\frac{\delta c_f}{c_f}\right)^2 + \left(\frac{\delta m_{est}}{m_{est}}\right)^2 \text{ gives } \delta F_a = 0.111 \text{ N} \quad \text{(Equ. 15)}$$

Similarly, for F_b:

$$(\delta F_b)^2 = \left(\frac{g * \sin(18°)}{2}\right)^2 * (\delta m_{est})^2 \text{ gives } \delta F_b = 0.789 \text{ N (Equ. 16)}$$

So $\delta F_1 = \delta F_a + \delta F_b = 0.111 + 0.789 = 0.9$ N (Equ. 17)

Hence $F_1 = 2.853$ N ± 0.9 N

As a result, the maximum force that is needed for each wheel to drive the buggy up the slope is: $F_{1_{max}} = 2.853 + 0.9 = 3.753$ N

3.3. Estimated Forces required to drive the buggy on the flat surface

For $m = m_{est} = 1537$ g:

For the friction force on the flat surface:

$$F = N = c_f * m_{est} * g \text{ (Equ. 7)}$$

That is the total force so for each wheel:

$$F_2 = \frac{c_f * m_{est} * g}{2} = \frac{0.0734 * 1.537 * 9.8}{2} = \frac{1.1056}{2} = 0.553 \text{ N (Equ. 18)}$$

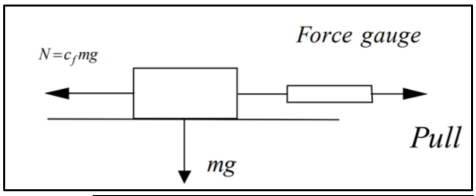


FIGURE 7 – Forces on a flat surface [9]

For the propagation of error [10]:

Error for the fraction:

$$\left(\frac{\delta F_2}{F_2}\right)^2 = \left(\frac{\delta c_f}{c_f}\right)^2 + \left(\frac{\delta m_{est}}{m_{est}}\right)^2 \quad (\text{Equ. } 20)$$

Which gives $\delta F_2 = 0.167\ \text{N}$

Hence $F_2 = 0.553\ \text{N} \pm 0.167\ \text{N}$

The maximum force needed for each wheel to drive the buggy on a flat surface is: $F_{2_{max}} = 0.553 + 0.167 = 0.72\ \text{N}$

3.4. Required Torque at the wheel shafts

<u>On the ramp</u>

For the torque for each wheel [9]: $T = F_{1_{max}} * r \quad (\text{Equ. } 21)$

where r is the radius of the wheel, $r = \frac{d}{2} = \frac{76.1\text{mm}}{2} = 38.05\ \text{mm} = 0.03805\ \text{m}$.

$$\therefore \quad T = 3.753 * 0.03805 = 0.1428\ \text{Nm}$$

<u>On the flat surface</u>

For the torque for each wheel [9] using $(\text{Equ. } 21)$ for $F_{2_{max}}$:

$$\therefore \quad T = F_{2_{max}} * r = 0.72 * 0.038805 = 0.0274\ \text{Nm}$$

4. Gear ratio selection

4.1. Introduction

This section is crucial for the progress of the report and the overall project. The calculations for each available gearbox are made with the aim of getting the required torque calculated in section 3.4.

The aim is to get the highest mechanical power, which is a trade-off between speed and torque. This can be achieved by finding the operating point (I and V), where the wheels gain maximum mechanical power with high speed and torque.

On one hand, high speed is needed to drive the buggy as quickly as possible, however high torque must also be considered so that the necessary force to drive the buggy up the slope will be provided.

4.2. Current and voltage required for the incline without gearbox

From Figure 2 and $(\text{Equ. } 5)$, the value of current needed for the required torque of $0.1428\ \text{Nm}$ is $I = 17.11\ \text{A}$. From Figure 1 and $(\text{Equ. } 2)$ this would result in a voltage value of $32.39\ \text{V}$. These values are obviously not achievable.

4.3. Available motor torque with given motor and drive board

The motors will be operated at $5\ \text{V}$ and maximum current $(1.4\ \text{A})$, hence $T_{in} = 0.011556\ \text{Nm}\ (\text{Equ. } 5)$ and $\omega_{in} = 212.264\ \frac{\text{rad}}{\text{s}}\ (\text{Equ. } 6)$. Motor torque of $0.011556\ \text{Nm}$ is not enough to drive the buggy up the maximum incline with the use of motors only, so a suitable gearbox must be chosen.

4.4. Calculations of torque and speed for the gearboxes

The calculations of torque and speed for each gearbox are performed.

For **gearbox 1:**

Pinion, N_1	Intermediate N_{2A}	Intermediate N_{2B}	Final Drive, N_3
16 tooth	48 tooth	12 tooth	48 tooth

<u>First stage</u>

$T_1 = T_{in} * \left(\frac{N_{2A}}{N_1}\right)$ [11] $\qquad T_1 = 0.011556 * \left(\frac{48}{16}\right) = 0.03467\ \mathrm{Nm}$

Assuming 85% efficiency [11]: $T_{1_{eff}} = T_1 * 0.85 = 0.02947\ \mathrm{Nm}$

<u>Second stage</u>

$T_{out} = T_{1_{eff}} * \left(\frac{N_3}{N_{2B}}\right)$ [11] $\qquad T_{out} = 0.02947 * \left(\frac{48}{12}\right) = 0.11788\ \mathrm{Nm}$

Assuming 85% efficiency [11]: $T_{out_{eff}} = T_{out} * 0.85 = 0.1002\ \mathrm{Nm}$

$\omega_{out} = \omega_{in} * \left(\frac{N_{2B}}{N_{2A}} * \frac{N_1}{N_3}\right) = 212.264 * \left(\frac{12}{48} * \frac{16}{48}\right) = 17.69\ \frac{\mathrm{rad}}{\mathrm{s}}$ or $168.93\ \mathrm{RPM}$

For **gearbox 2:**

Pinion, N_1	Intermediate N_{2A}	Intermediate N_{2B}	Final Drive, N_3
16 tooth	50 tooth	10 tooth	48 tooth

<u>First stage</u>

$T_1 = T_{in} * \left(\frac{N_{2A}}{N_1}\right)$ [11] $\qquad T_1 = 0.011556 * \left(\frac{50}{16}\right) = 0.03611\ \mathrm{Nm}$

Assuming 85% efficiency [11]: $T_{1_{eff}} = T_1 * 0.85 = 0.0307\ \mathrm{Nm}$

<u>Second stage</u>

$$T_{out} = T_{1_{eff}} * \left(\frac{N_3}{N_{2B}}\right) = 0.0307 * \left(\frac{48}{10}\right) = 0.14736\ \mathrm{Nm}$$

Assuming 85% efficiency [11]: $T_{out_{eff}} = T_{out} * 0.85 = 0.1253\ \mathrm{Nm}$

$\omega_{out} = \omega_{in} * \left(\frac{N_{2B}}{N_{2A}} * \frac{N_1}{N_3}\right) = 212.264 * \left(\frac{10}{50} * \frac{16}{48}\right) = 14.15\ \frac{\mathrm{rad}}{\mathrm{s}}$ or $135.12\ \mathrm{RPM}$

For **gearbox 3:**

Pinion, N_1	Intermediate N_{2A}	Intermediate N_{2B}	Final Drive, N_3
16 tooth	50 tooth	10 tooth	60 tooth

<u>First stage</u>

$T_1 = T_{in} * \left(\frac{N_{2A}}{N_1}\right)$ $\qquad T_1 = 0.011556 * \left(\frac{50}{16}\right) = 0.0361\ \mathrm{Nm}$

Assuming 85% efficiency [11]: $T_{1_{eff}} = T_1 * 0.85 = 0.0307\ \mathrm{Nm}$

<u>Second stage</u>

$$T_{out} = T_{1_{eff}} * \left(\frac{N_3}{N_{2B}}\right) = 0.0307 * \left(\frac{60}{10}\right) = 0.1842\ \mathrm{Nm}$$

Assuming 85% efficiency [11]: $T_{out_{eff}} = T_{out} * 0.85 = 0.1566\ \mathrm{Nm}$

$$\omega_{out} = \omega_{in} * \left(\frac{N_{2B}}{N_{2A}} * \frac{N_1}{N_3}\right) = 212.264 * \left(\frac{10}{50} * \frac{16}{60}\right) = 11.32\ \frac{\mathrm{rad}}{\mathrm{s}}\ \text{ or } 108.1\ \mathrm{RPM}$$

4.5. Selection of gearbox – Required position of the intermediate shaft as an (x,y) coordinate

Gearbox 3 is the only one which produces the required torque. Hence, **gearbox 3 is the selection for the buggy** to go up the maximum incline of 18°.

The coordinate system and the position of the origin are shown on Figure 8.

Assuming input and intermediate shaft align horizontally and MOD=0.5 mm [12]:

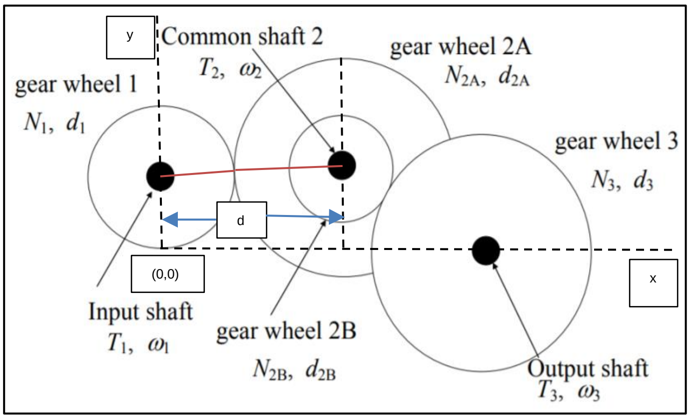


FIGURE 8 – Gearbox Dimensions [11]

For gearwheel 1: $\mathrm{PCD}(1) = N_1 * \mathrm{MOD} = 16 * 0.5\ \mathrm{mm} = 8\ \mathrm{mm}$

For gearwheel 2A: $PCD(2A) = N_{2A} * \mathrm{MOD} = 50 * 0.5\ \mathrm{mm} = 25\ \mathrm{mm}$

$$\mathrm{x\ co-ordinate} = \mathrm{d} = \frac{\mathrm{PCD(1)+PCD(2A)}}{2} + 0.1\ \mathrm{mm} = 16.6\ \mathrm{mm}$$

For the y coordinate, from Figure 8: $y = \mathrm{PCD}(1) = 8\ \mathrm{mm}$

Hence, the position of the intermediate shaft is (16.6, 8).

4.6. Speed of the buggy on the ramp and on the flat surface

Making the calculations for gearbox 3 the output torque and the correspondent speed for each value of current is calculated.

Table 2: Output Torque and Speed in different currents

I(A)	$T_{out_{eff}}$(Nm)	ω_{out}(rad/s)	I(A)	$T_{out_{eff}}$(Nm)	ω_{out}(rad/s)
0.2	0.013045641	27.35288889	0.8	0.078273844	19.25308444
0.3	0.026091281	25.81777778	0.9	0.091319484	18.03616
0.4	0.032614102	24.44455111	1	0.097842305	16.65176889
0.5	0.045659742	23.79143111	1.2	0.117410766	14.25141333
0.6	0.052182563	22.20049778	1.4	0.156547688	11.32074667

From Table 2, operating the motor at 0.4 A gives the required torque of $0.0274\ \mathrm{Nm}$ for the flat surface and high speed. From the table $\omega_{out} = 24.4446 \ \frac{rad}{s}$, so the speed for each wheel [9] will be: $v = \omega * r = 24.4446 * 0.03805 = 0.9301 \ \frac{m}{s}$ or $3.3484 \ \frac{km}{h}$. The total speed of the buggy (combining two wheels) is: $V = 2 * v = 1.8602 \ \frac{m}{s}$ or $6.6968 \frac{km}{h}$.

<u>On the ramp:</u>

From the previous calculations $\omega_{out} = 11.32 \ \frac{rad}{s}$, so the speed for each wheel will be: $v = \omega * r = 11.32 * 0.03805 = 0.4307 \ \frac{m}{s}$ or $1.5505 \ \frac{km}{h}$. The total speed of the buggy (combining two wheels) is: $V = 2 * v = 0.8615 \ \frac{m}{s}$ or $3.1014 \ \frac{km}{h}$.

5. Summary

In conclusion, this report disposes the measurements acquired in the Motor and Force Lab to make sure that the buggy can run on different terrains. According to the graphs, the armature resistance is $1.893\,\Omega$ and the brush voltage is $0.0042\,V$. The torque constant (K_T) is 0.0084 and the EMF constant (K_E) is 0.0076 for a rotating motor while the torque constant (K_T) is 0.0078 at a standstill. As it is shown, the values of the constants lie close to each other as expected. Using torque and EMF constants and assuming that the mass of the buggy is $1537\ g\ \pm 30\%$, the maximum force needed for each wheel is $3.753\ N$ on the slope and $0.72\ N$ on the flat surface. In addition, the required torque for each wheel is $0.1428\ Nm$ on the ramp and $0.0274\ Nm$ on the flat surface. Using torque calculations and if each gearbox has 85% efficiency, only **gearbox 3** is suitable for the torque needed for the maximum incline. Hence the motors will be operated at 5 V and 1.4 A on the ramp giving an expected maximum speed of $0.8615\ \frac{m}{s}$ and at 5 V and 0.4 A on the flat giving an expected maximum speed of $1.8602\ \frac{m}{s}$.

6. References

[1] ESP Technical Handbook p.32 – “Maximum Current”

[2] <u>https://brilliant.org/wiki/heat-dissipated-by-resistors/</u>

[3] ESP Technical Handbook p.33

[4] ESP Technical Handbook p.34

[5] ESP Technical Handbook p.24

[6] ESP Technical Handbook p.36

[7] ESP Technical Handbook p.22

[8] ESP Technical Handbook p.40 – Table of “Mass of buggy components”

[9] ESP Technical Handbook p.25-26

[10] Measurements and Analytical Software notes – Part 1: Measurements & Errors p.54,57,70

[11] ESP Technical Handbook p.27-29

[12] ESP Technical Handbook p.30