

”Basic DE’s: Separation of Variables”

$$dy/dx = 2y + 1 \quad (1)$$

$$\frac{dy}{2y+1} = dx \quad (2)$$

$$\int_a^b \frac{1}{2y+1} dy = \int_a^b dx \quad (3)$$

$$(4)$$

$$u := 2y + 1, f(u) := \frac{1}{2y+1}$$

$$\therefore \frac{dy}{du} \int_a^b \frac{1}{u} \frac{du}{dy} dy = \int \frac{1}{u} \frac{du}{dy} \frac{dy}{du} dy = \int \frac{1}{u} dy = \int_a^b \frac{1}{2y+1} dy = \int_a^b dx \quad (5)$$

$$\frac{dy}{du} \int_a^b \frac{1}{u} \frac{du}{dy} dy = \int_a^b dx \quad (6)$$

$$(7)$$

$$\therefore \int (f \circ g) g' = \int f$$

$$\therefore \int f(g(y)) g'(y) dy = \int f(u) u' dy = \int f(u) \frac{du}{dy} dy = \int f(u) du$$

$$\frac{dy}{du} \int_a^b \frac{1}{u} \frac{du}{dy} dy = \frac{dy}{du} \int_a^b f(u) du \quad (8)$$

$$= \frac{dy}{du} \int_a^b \frac{1}{u} du \quad (9)$$

$$(10)$$

$$\therefore \frac{dy}{du} \int_a^b \frac{1}{u} \frac{du}{dy} dy = \frac{dy}{du} \int_a^b \frac{1}{u} du = \int_a^b dx \quad (11)$$

$$\frac{dy}{du} \int_a^b \frac{1}{u} du = \int_a^b dx \quad (12)$$

$$\therefore u = 2y + 1 \therefore \frac{du}{dy} = 2$$

$$\therefore \frac{dy}{du} = \left(\frac{du}{dy}\right)^{-1} = 2^{-1} = \frac{1}{2}$$

$$\frac{1}{2} \int_a^b \frac{1}{u} du = \frac{dy}{du} \int_a^b \frac{1}{u} du = \int_a^b dx \quad (13)$$

$$\frac{1}{2} \int_a^b \frac{1}{u} du = \int_a^b dx \quad (14)$$

$$\frac{1}{2} \ln |u| + c_1 = x + c_2 \quad (15)$$

$$\frac{1}{2} \ln |u| = x + c_2 - c_1 \quad (16)$$

$$\ln |u| = 2x + 2(c_2 - c_1) \quad (17)$$

$$\ln |u| = 2x + c_3 \quad (18)$$

$$\ln |2y + 1| = 2x + c_3 \because u = 2y + 1 \quad (19)$$

$$\therefore e^{\ln |2y+1|} = e^{2x+c_3} \quad (20)$$

$$\therefore 2y + 1 = e^{2x} e^{c_3} \quad (21)$$

$$2y = e^{2x} e^{c_3} - 1 \quad (22)$$

$$2y = e^{2x} e^{c_3} - 1 \quad (23)$$

$$2y = C e^{2x} - 1 \quad (24)$$

$$y = \frac{C e^{2x} - 1}{2} \quad \blacksquare \quad (25)$$