CS 774A Optimization Techniques

Combinatorial MAB Problem

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Outline

1. Introduction
   - Motivation
   - Exploration - Exploitation Dilemma
2. Combinatorial Multi Arm Bandit
   - CMAB Setting
   - Stochastic CMAB: Prior work
3. Mid term plan of action
4. Two main algorithms studied
   - CUCB algorithm
   - Application specifically for CUCB
   - PMC Bandit
   - Social Influence maximization bandit
   - ESCB Algorithm

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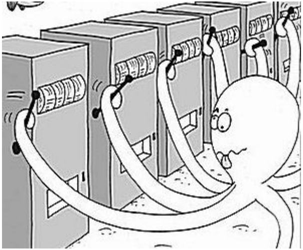
- K arms (or actions)
- Each time t, each arm i pays of a bounded real valued reward $x_i(t)$ say in [0,1].
- Each time t, the learner chooses a single arm $i_t \in \{1, \ldots K\}$ and receives reward $x_{i_t}(t)$. The goal is to maximize the return.

Figure: Multi Arm Slot machine



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Exploration - Exploitation Dilemma

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Examples:

1. Restaurant Selection
   Exploitation Go to your favourite restaurant
   Exploration Try a new restaurant

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- At each round an arm M is selected from finite set $\mathbb{M} \subset \{0,1\}^d$
- Reward recieved is $M^T X(n) = \sum_{i=1}^{i=d} M_i X_i(n)$
- Reward vector is unknown and $\|M\|_1 = m \; \forall M \in \mathbb{M}$
- feedback framework:
    - Semibandit: $X_i(n)$ is revealed $\forall i$ (only if $M_i = 1$)
    - Bandit: Only reward $M^T X(n)$ is revealed

Aim

Based on the feedback received upto round $n-1$, select an arm at round n such that:

- Cumulative reward over a given time horizon consisting of T rounds is maximized
- Regret $R(T)$ is minimized

$$R(T) = \max_{M \in \mathbb{M}} \mathbb{E}[\sum_{n=1}^{T} M^T X(n)] - \mathbb{E}[\sum_{n=1}^{T} M(n)^T X(n)]$$

Challenge: Very large number of arms, i.e., in its combinatorial structure: the size of $\mathbb{M}$ grows as d^m

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- Regret Upper Bounds
  - LLR (Gai 2012) : $\mathcal{O}(\frac{m^3 d \Delta_{max}}{\Delta_{min}^2} log(T))$
  - CUCB (Chen 2013) : $\mathcal{O}(\frac{m^2 d}{\Delta_{min}} log(T))$
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Mid term plan of action

- Study Comb - MAB in stochastic and specific combinatorial setting
- Identifying constraints for $\sqrt{m}$ improvement in bounds in ESCB
- Estimate current bounds under relaxed conditions

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- Oracle does offline computation task, which uses the domain knowledge of the problem instance
- CMAB algorithm takes care of the online learning task, and is oblivious to the domain knowledge of the problem instance
- $\alpha\beta$ Approximate oracle: It takes an input of μ and outputs a super arm $\in S$ such that $\Pr[r_\mu(S) \geq \alpha \text{opt}\mu] \geq \beta$. β is the probability of success for the oracle.

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- **Outcomes of all played base arms are observed**
- Outcome of an arm i has an unknown distribution with unknown mean μ_i

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- Optimal Reward: Optμ $= Max_S\ r_\mu(S)$

Handling Non-Linear Rewards

Two mild assumptions on $r_\mu(S)$

- Monotonicity: If $\mu \leq \mu^{'}$ (pairwise), $r_\mu(S) \leq r_{\mu^{'}}(S)$ for all superarms S

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- Boundedness: There exists a strictly increasing function $f()$ such that for any two expectation vectors μ and $\mu_{'}$,
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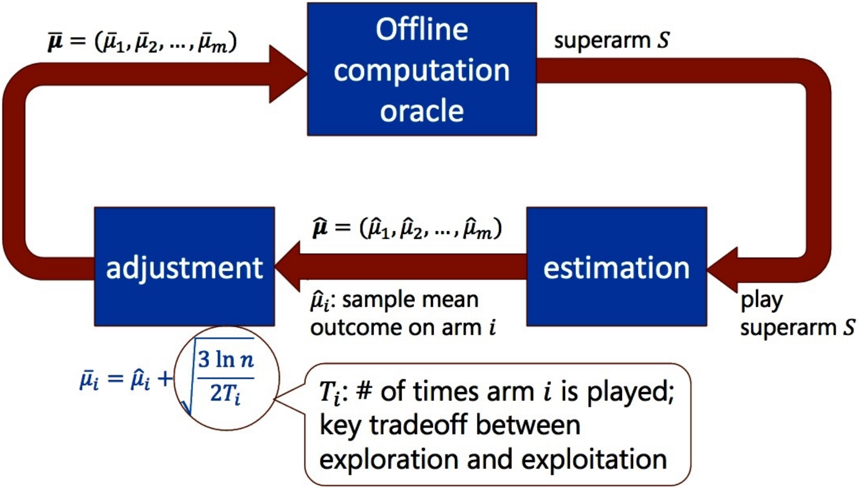
- A large class of reward functions satisfy these conditions (linear and non-linear)

(α, β) Approximation Regret

We compare against the $\alpha\beta$ fraction of the optimal:

$$Regret = n * \alpha * \beta * opt_{\mu} - E(\sum_{i=1}^{n} r_{\mu}(S_t^A)) \tag{1}$$

The Algorithm



Regret bound

Theorem: The $(\alpha\beta)$-approximation regret of CUCB algorithm in *n* rounds using an approximation oracle is at most:

$$\sum_{i\in[m],\Delta^i_{\min}>0}\left(\frac{6\ln n\cdot\Delta^i_{\min}}{(f^{-1}(\Delta^i_{\min}))^2}+\int_{\Delta^i_{\min}}^{\Delta^i_{\max}}\frac{6\ln n}{(f^{-1}(x))^2}\mathrm{d}x\right)+\left(\frac{\pi^2}{3}+1\right)\cdot m\cdot\Delta_{\max}.$$

- Δ^i_{min} is defined as the minimum gap between αopt_μ and reward of a bad super arm containing i
- $\Delta_{min} = Min_i\ \Delta^i_{min}$

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CUCB with non linear rewards

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  - Probabilistic maximum coverage bandit
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PMC Bandit details

- Probabilistic Maximum coverage problem has an input of a weighted bipartite graph G = (L,R,E)
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- PMC Bandit: Probability weights are unknown
- Submodular set function maximization technique shows the existence of oracle and hence we can use methods of CUCB

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- Works as a diffusion process with probability of activation being $p(u, v)$
- Reward is is the total number of activated nodes in the end.
- Each edge is an arm and superedge is a set of outgoing edges from at most k nodes

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- Indices are the natural extension of Upper Confidence Bound (UCB) and KL-UCB algorithms
- ESCB improve over LLR and and CUCB by the multiplicative factor of $\sqrt{m}$

Summary

- Presented Multi Arm Bandit Problem and Combinatorial setting in MAB framework
- Overview of CUCB and ESCB algorithm
- Presented two application where CUCB with non linear rewards can be applied
- Tried to understand and improve bounds, but the direction of application of CUCB looks more promising