Contents

1 Setting

1.1 vimrc

```
set ts=4 sts=4 sw=4
set ai si nu
```

2 Math

2.1 Basic Arithmetic

```
typedef long long ll;
typedef unsigned long long ull;

// calculate ceil(a/b)
// |a|, |b| <= (2^63)-1 (does not dover -2^63)
ll ceildiv(ll a, ll b) {
    if (b < 0) return ceildiv(-a, -b);
    if (a < 0) return (-a) / b;
    return ((ull)a + (ull)b - 1ull) / b;
}

// calculate floor(a/b)
// |a|, |b| <= (2^63)-1 (does not cover -2^63)
ll floordiv(ll a, ll b) {
    if (b < 0) return floordiv(-a, -b);
    if (a >= 0) return a / b;
    return -(ll)(((ull)(-a) + b - 1) / b);
}

// calculate a*b % m
// x86-64 only
ll large_mod_mul(ll a, ll b, ll m)
{
```

```
    return ll((__int128)a*(__int128)b%m);
}

// calculate a*b % m
// |m| < 2^62, x86 available
// O(logb)
ll large_mod_mul(ll a, ll b, ll m)
{
    a %= m; b %= m; ll r = 0, v = a;
    while (b) {
        if (b&1) r = (r + v) % m;
        b >>= 1;
        v = (v << 1) % m;
    }
    return r;
}

// calculate n^k % m
ll modpow(ll n, ll k, ll m) {
    ll ret = 1;
    n %= m;
    while (k) {
        if (k & 1) ret = large_mod_mul(ret, n, m);
        n = large_mod_mul(n, n, m);
        k /= 2;
    }
    return ret;
}

// calculate gcd(a, b)
ll gcd(ll a, ll b) {
    return b == 0 ? a : gcd(b, a % b);
}

// find a pair (c, d) s.t. ac + bd = gcd(a, b)
pair<ll, ll> extended_gcd(ll a, ll b) {
    if (b == 0) return { 1, 0 };
    auto t = extended_gcd(b, a % b);
    return { t.second, t.first - t.second * (a / b) };
}

// find x in [0,m) s.t. ax === gcd(a, m) (mod m)
ll modinverse(ll a, ll m) {
    return (extended_gcd(a, m).first % m + m) % m;
}

// calculate modular inverse for 1 ~ n
void calc_range_modinv(int n, int mod, int ret[]) {
    ret[1] = 1;
    for (int i = 2; i <= n; ++i)
        ret[i] = (ll)(mod - mod/i) * ret[mod%i] % mod;
}
```

2.2 Sieve Methods : Prime, Divisor, Euler phi

```
// find prime numbers in 1 ~ n
// ret[x] = false  ->  x is prime
// O(n*loglogn)
void sieve(int n, bool ret[]) {
    for (int i = 2; i * i <= n; ++i)
        if (!ret[i])
            for (int j = i * i; j <= n; j += i)
                ret[i] = true;
}

// calculate number of divisors for 1 ~ n
// when you need to calculate sum, change += 1 to += i
// O(n*logn)
void num_of_divisors(int n, int ret[]) {
    for (int i = 1; i <= n; ++i)
        for (int j = i; j <= n; j += i)
            ret[j] += 1;
}

// calculate euler totient function for 1 ~ n
// phi(n) = number of x s.t. 0 < x < n && gcd(n, x) = 1
// O(n*loglogn)
void euler_phi(int n, int ret[]) {
    for (int i = 1; i <= n; ++i) ret[i] = i;
    for (int i = 2; i <= n; ++i)
        if (ret[i] == i)
            for (int j = i; j <= n; j += i)
                ret[j] -= ret[j] / i;
}
```

2.3 Primality Test

```
bool test_witness(ull a, ull n, ull s) {
    if (a >= n) a %= n;
    if (a <= 1) return true;
    ull d = n >> s;
    ull x = modpow(a, d, n);
    if (x == 1 || x == n-1) return true;
    while (s-- > 1) {
        x = large_mod_mul(x, x, n);
        x = x * x % n;
        if (x == 1) return false;
        if (x == n-1) return true;
    }
    return false;
}

// test whether n is prime
// based on miller-rabin test
// O(logn*logn)
bool is_prime(ull n) {
    if (n == 2) return true;
    if (n < 2 || n % 2 == 0) return false;

    ull d = n >> 1, s = 1;
```

```
    for(; (d&1) == 0; s++) d >>= 1;

#define T(a) test_witness(a##ull, n, s)
    if (n < 4759123141ull) return T(2) && T(7) && T(61);
    return T(2) && T(325) && T(9375) && T(28178)
        && T(450775) && T(9780504) && T(1795265022);
#undef T
}
```

2.4 Chinese Remainder Theorem

```
// find x s.t.  x === a[0] (mod n[0])
//                === a[1] (mod n[1])
//                ...
// assumption: gcd(n[i], n[j]) = 1
ll chinese_remainder(ll* a, ll* n, int size) {
    if (size == 1) return *a;
    ll tmp = modinverse(n[0], n[1]);
    ll tmp2 = (tmp * (a[1] - a[0]) % n[1] + n[1]) % n[1];
    ll ora = a[1];
    ll tgcd = gcd(n[0], n[1]);
    a[1] = a[0] + n[0] / tgcd * tmp2;
    n[1] *= n[0] / tgcd;
    ll ret = chinese_remainder(a + 1, n + 1, size - 1);
    n[1] /= n[0] / tgcd;
    a[1] = ora;
    return ret;
}
```

2.5 Burnside's Lemma

경우의 수를 세는데, 특정 transform operation(회전, 반사, ..) 해서 같은 경우들은 하나로 친다. 전체 경우의 수는?

- 각 operation마다 이 operation을 했을 때 변하지 않는 경우의 수를 센다 (단, "아무것도 하지 않는다" 라는 operation도 있어야 함!)

- 전체 경우의 수를 더한 후, operation의 수로 나눈다. (답이 맞다면 항상 나누어 떨어져야 한다)

2.6 Kirchoff's Theorem

그래프의 스패닝 트리의 개수를 구하는 정리.

무향 그래프의 Laplacian matrix L를 만든다. 이것은 (정점의 차수 대각 행렬) - (인접행렬) 이다. L에서 행과 열을 하나씩 제거한 것을 L'라 하자. 어느 행/열이든 관계 없다. 그래프의 스패닝 트리의 개수는 $det(L')$이다.

2.7 Fast Fourier Transform

```
void fft(int sign, int n, double *real, double *imag) {
    double theta = sign * 2 * pi / n;
    for (int m = n; m >= 2; m >>= 1, theta *= 2) {
        double wr = 1, wi = 0, c = cos(theta), s = sin(theta);
        for (int i = 0, mh = m >> 1; i < mh; ++i) {
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                double xr = real[j] - real[k], xi = imag[j] - imag[k];
                real[j] += real[k], imag[j] += imag[k];
                real[k] = wr * xr - wi * xi, imag[k] = wr * xi + wi * xr;
            }
            double _wr = wr * c - wi * s, _wi = wr * s + wi * c;
            wr = _wr, wi = _wi;
        }
    }
    for (int i = 1, j = 0; i < n; ++i) {
        for (int k = n >> 1; k > (j ^= k); k >>= 1);
        if (j < i) swap(real[i], real[j]), swap(imag[i], imag[j]);
    }
}
// Compute Poly(a)*Poly(b), write to r; Indexed from 0
// O(n*logn)
int mult(int *a, int n, int *b, int m, int *r) {
    const int maxn = 100;
    static double ra[maxn], rb[maxn], ia[maxn], ib[maxn];
    int fn = 1;
    while (fn < n + m) fn <<= 1; // n + m: interested length
    for (int i = 0; i < n; ++i) ra[i] = a[i], ia[i] = 0;
    for (int i = n; i < fn; ++i) ra[i] = ia[i] = 0;
    for (int i = 0; i < m; ++i) rb[i] = b[i], ib[i] = 0;
    for (int i = m; i < fn; ++i) rb[i] = ib[i] = 0;
    fft(1, fn, ra, ia);
    fft(1, fn, rb, ib);
    for (int i = 0; i < fn; ++i) {
        double real = ra[i] * rb[i] - ia[i] * ib[i];
        double imag = ra[i] * ib[i] + rb[i] * ia[i];
        ra[i] = real, ia[i] = imag;
    }
    fft(-1, fn, ra, ia);
    for (int i = 0; i < fn; ++i) r[i] = (int)floor(ra[i] / fn + 0.5);
    return fn;
}
```

2.8 Matrix Operations

2.9 Gaussian Elimination

2.10 Simplex Algorithm

3 Data Structure

3.1 Order statistic tree

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>
#include <functional>
#include <iostream>
using namespace __gnu_pbds;
using namespace std;

// tree<key_type, value_type(set if null), comparator, ...>
using ordered_set = tree<int, null_type, less<int>, rb_tree_tag,
    tree_order_statistics_node_update>;

int main()
{
    ordered_set X;
    for (int i = 1; i < 10; i += 2) X.insert(i); // 1 3 5 7 9
    cout << boolalpha;
    cout << *X.find_by_order(2) << endl; // 5
    cout << *X.find_by_order(4) << endl; // 9
    cout << (X.end() == X.find_by_order(5)) << endl; // true

    cout << X.order_of_key(-1) << endl; // 0
    cout << X.order_of_key(1) << endl; // 0
    cout << X.order_of_key(4) << endl; // 2
    X.erase(3);
    cout << X.order_of_key(4) << endl; // 1
    for (int t : X) printf("%d ", t); // 1 5 7 9
}
```

3.2 Fenwick Tree

```
const int TSIZE = 100000;
int tree[TSIZE + 1];

// Returns the sum from index 1 to p, inclusive
int query(int p) {
    int ret = 0;
    for (; p > 0; p -= p & -p) ret += tree[p];
    return ret;
}

// Adds val to element with index pos
void add(int p, int val) {
    for (; p <= TSIZE; p += p & -p) tree[p] += val;
}
```

3.3 Segment Tree with Lazy Propagation

3.4 Persistent Segment Tree

3.5 Link/Cut Tree

4 DP

4.1 Convex Hull Optimization

$O(n^2) \to O(n \log n)$

조건 1) DP 점화식 꼴

$D[i] = \min_{j<i}(D[j] + b[j] * a[i])$

조건 2) $b[j] \le b[j+1]$

특수조건) $a[i] \le a[i+1]$ 도 만족하는 경우, 마지막 쿼리의 위치를 저장해두면 이분검색이 필요없어지기 때문에 amortized $O(n)$ 에 해결할 수 있음

4.2 Divide & Conquer Optimization

$O(kn^2) \to O(kn \log n)$

조건 1) DP 점화식 꼴

$D[t][i] = \min_{j<i}(D[t-1][j] + C[j][i])$

조건 2) $A[t][i]$는 $D[t][i]$의 답이 되는 최소의 j라 할 때, 아래의 부등식을 만족해야 함

$A[t][i] \le A[t][i+1]$

조건 2-1) 비용C가 다음의 사각부등식을 만족하는 경우도 조건 2)를 만족하게 됨

$C[a][c] + C[b][d] \le C[a][d] + C[b][c] \;\; (a \le b \le c \le d)$

4.3 Knuth Optimization

$O(n^3) \to O(n^2)$

조건 1) DP 점화식 꼴

$D[i][j] = \min_{i<k<j}(D[i][k] + D[k][j]) + C[i][j]$

조건 2) 사각 부등식

$C[a][c] + C[b][d] \le C[a][d] + C[b][c] \;\; (a \le b \le c \le d)$

조건 3) 단조성

$C[b][c] \le C[a][d] \;\; (a \le b \le c \le d)$

결론) 조건 2, 3을 만족한다면 $A[i][j]$를 $D[i][j]$의 답이 되는 최소의 k라 할 때, 아래의 부등식을 만족하게 됨

$A[i][j-1] \le A[i][j] \le A[i+1][j]$

3중 루프를 돌릴 때 위 조건을 이용하면 최종적으로 시간복잡도가 $O(n^2)$ 이 됨

5 Graph

5.1 SCC (Tarjan)

5.2 SCC (Kosaraju)

```
const int MAXN = 100;
vector<int> graph[MAXN], grev[MAXN];
int visit[MAXN], vcnt;
int scc_idx[MAXN], scc_cnt;
vector<int> emit;

void dfs(int nod, vector<int> graph[]) {
    visit[nod] = vcnt;
    for (int next : graph[nod]) {
        if (visit[next] == vcnt) continue;
        dfs(next, graph);
    }
    emit.push_back(nod);
}

// find SCCs in given graph
// O(V+E)
void get_scc() {
    scc_cnt = 0;
    vcnt = 1;
    emit.clear();
    memset(visit, 0, sizeof(visit));

    for (int i = 0; i < n; i++) {
        if (visit[i] == vcnt) continue;
        dfs(i, graph);
    }

    ++vcnt;
    for (auto st : vector<int>(emit.rbegin(), emit.rend())) {
        if (visit[st] == vcnt) continue;
        emit.clear();
        dfs(st, grev);
        ++scc_cnt;
        for (auto node : emit)
            scc_idx[node] = scc_cnt;
    }
}
```

5.3 2-SAT

5.4 BCC, Cut vertex, Bridge

5.5 Heavy-Light Decomposition

5.6 Bipartite Matching (Hopcroft-Karp)

5.7 Maximum Flow (Edmonds-Karp)

5.8 Maximum Flow (Dinic)

5.9 Min-cost Maximum Flow

6 Geometry

6.1 Basic Operations

6.2 Compare angles

6.3 Convex Hull

6.4 Polygon Cut

6.5 Pick's theorem

격자점으로 구성된 simple polygon이 주어짐. i는 polygon 내부의 격자점 수, b는 polygon 선분 위 격자점 수, A는 polygon의 넓이라고 할 때, 다음과 같은 식이 성립한다.

$A = i + \frac{b}{2} - 1$

7 String

7.1 KMP

```
typedef vector<int> seq_t;

void calculate_pi(vector<int>& pi, const seq_t& str) {
    pi[0] = -1;
    int j = -1;
    for (int i = 1; i < str.size(); i++) {
        while (j >= 0 && str[i] != str[j + 1]) j = pi[j];
        if (str[i] == str[j + 1])
            pi[i] = ++j;
        else
            pi[i] = -1;
    }
}

// returns all positions matched
// O(|text|+|pattern|)
vector<int> kmp(seq_t& text, seq_t& pattern) {
    vector<int> pi(pattern.size());
    vector<int> ans;
    if (pattern.size() == 0) return ans;
    calculate_pi(pi, pattern);
    int j = -1;
    for (int i = 0; i < text.size(); i++) {
        while (j >= 0 && text[i] != pattern[j + 1]) j = pi[j];
        if (text[i] == pattern[j + 1]) {
            j++;
            if (j + 1 == pattern.size()) {
                ans.push_back(i - j);
                j = pi[j];
            }
        }
    }
    return ans;
}
```

7.2 Aho-Corasick

```
#include <algorithm>
#include <vector>
#include <queue>
using namespace std;

struct AhoCorasick
{
    const int alphabet;
    struct node {
        node() {}
        explicit node(int alphabet) : next(alphabet) {}
        vector<int> next, report;
        int back = 0, output_link = 0;
    };
    int maxid = 0;
    vector<node> dfa;
    explicit AhoCorasick(int alphabet) : alphabet(alphabet), dfa(1, node(
      alphabet)) { }
    template<typename InIt, typename Fn> void add(int id, InIt first, InIt
      last, Fn func) {
        int cur = 0;
        for ( ; first != last; ++first) {
            auto s = func(*first);
            if (auto next = dfa[cur].next[s]) cur = next;
            else {
                cur = dfa[cur].next[s] = (int)dfa.size();
                dfa.emplace_back(alphabet);
            }
        }
        dfa[cur].report.push_back(id);
        maxid = max(maxid, id);
    }
    void build() {
        queue<int> q;
        vector<char> visit(dfa.size());
        visit[0] = 1;
        q.push(0);
        while(!q.empty()) {
            auto cur = q.front(); q.pop();
            dfa[cur].output_link = dfa[cur].back;
            if (dfa[dfa[cur].back].report.empty())
                dfa[cur].output_link = dfa[dfa[cur].back].output_link;
            for (int s = 0; s < alphabet; s++) {
                auto &next = dfa[cur].next[s];
                if (next == 0) next = dfa[dfa[cur].back].next[s];
                if (visit[next]) continue;
                if (cur) dfa[next].back = dfa[dfa[cur].back].next[s];
                visit[next] = 1;
                q.push(next);
            }
        }
    }
    template<typename InIt, typename Fn> vector<int> countMatch(InIt first,
      InIt last, Fn func) {
        int cur = 0;
        vector<int> ret(maxid+1);
        for (; first != last; ++first) {
            cur = dfa[cur].next[func(*first)];
            for (int p = cur; p; p = dfa[p].output_link)
                for (auto id : dfa[p].report) ret[id]++;
        }
        return ret;
    }
};
```

7.3 Suffix Array with LCP

7.4 Suffix Tree

7.5 Manacher's Algorithm

8 Miscellaneous

8.1 Fast I/O

```
namespace fio {
    const int BSIZE = 524288;
    char buffer[BSIZE];
    int p = BSIZE;
    inline char readChar() {
        if(p == BSIZE) {
            fread(buffer, 1, BSIZE, stdin);
            p = 0;
        }
        return buffer[p++];
    }
    int readInt() {
        char c = readChar();
        while ((c < '0' || c > '9') && c != '-') {
            c = readChar();
        }
        int ret = 0; bool neg = c == '-';
        if (neg) c = readChar();
        while (c >= '0' && c <= '9') {
            ret = ret * 10 + c - '0';
            c = readChar();
        }
        return neg ? -ret : ret;
    }
}
```

8.2 Magic Numbers