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2 Math

2.1 Basic Arithmetic

```
1 typedef long long ll;
2 typedef unsigned long long ull;
4 // calculate lg2(a)
5 inline int lg2(ll a)
       return 63 - __builtin_clzll(a);
8 }
10 // calculate the number of 1-bits
11 inline int bitcount(ll a)
12 {
13
       return __builtin_popcountll(a);
14 }
15
16 // calculate ceil(a/b)
|17|/|a|, |b| <= (2^63)-1 (does not dover -2^63)
18 ll ceildiv(ll a, ll b) {
19
      if (b < 0) return ceildiv(-a, -b);</pre>
20
       if (a < 0) return (-a) / b;</pre>
21
       return ((ull)a + (ull)b - 1ull) / b;
22 }
23
24 // calculate floor(a/b)
25 // |a|, |b| <= (2^63)-1 (does not cover -2^63)
26 ll floordiv(ll a, ll b) {
27
       if (b < 0) return floordiv(-a, -b);</pre>
28
       if (a >= 0) return a / b;
29
       return -(ll)(((ull)(-a) + b - 1) / b);
30 }
31
32 // calculate a*b % m
33 // x86-64 only
34 ll large_mod_mul(ll a, ll b, ll m)
36
       return ll((__int128)a*(__int128)b%m);
37 }
38
39 // calculate a*b % m
40 // |m| < 2^62, x86 available
41 // O(Logb)
42 ll large_mod_mul(ll a, ll b, ll m)
43 {
       a \% = m; b \% = m; 11 r = 0, v = a;
44
45
       while (b) {
46
           if (b&1) r = (r + v) % m;
47
           b >>= 1;
48
           v = (v << 1) \% m;
49
       return r;
```

```
51 }
52
53 // calculate n^k % m
54 ll modpow(ll n, ll k, ll m) {
      ll ret = 1;
55
56
       n \% = m;
57
       while (k) {
58
           if (k & 1) ret = large_mod_mul(ret, n, m);
59
           n = large_mod_mul(n, n, m);
60
           k /= 2;
61
       }
62
       return ret;
63 }
65 // calculate gcd(a, b)
66 ll gcd(ll a, ll b) {
       return b == 0 ? a : gcd(b, a % b);
68 }
70 // find a pair (c, d) s.t. ac + bd = qcd(a, b)
71 pair<ll, 1l> extended_gcd(ll a, ll b) {
       if (b == 0) return { 1, 0 };
73
       auto t = extended_gcd(b, a % b);
74
       return { t.second, t.first - t.second * (a / b) };
75 }
76
77 // find x in [0,m) s.t. ax === gcd(a, m) (mod m)
78 ll modinverse(ll a, ll m) {
79
       return (extended_gcd(a, m).first % m + m) % m;
80 }
81
82 // calculate modular inverse for 1 ~ n
83 void calc range modinv(int n, int mod, int ret[]) {
       ret[1] = 1;
85
       for (int i = 2; i <= n; ++i)
86
           ret[i] = (ll)(mod - mod/i) * ret[mod%i] % mod;
87 }
```

2.2 Sieve Methods: Prime, Divisor, Euler phi

```
1 // find prime numbers in 1 ~ n
 2 // ret[x] = false -> x is prime
3 // O(n*loglogn)
 4 void sieve(int n, bool ret[]) {
      for (int i = 2; i * i <= n; ++i)
 6
           if (!ret[i])
               for (int j = i * i; j <= n; j += i)
8
                   ret[i] = true;
9 }
10
11 // calculate number of divisors for 1 ~ n
12 // when you need to calculate sum, change += 1 to += i
13 // O(n*logn)
14 void num_of_divisors(int n, int ret[]) {
```

```
15
       for (int i = 1; i <= n; ++i)
16
            for (int j = i; j \leftarrow n; j \leftarrow i)
17
                ret[i] += 1:
18 }
19
20 // calculate euler totient function for 1 ~ n
21 // phi(n) = number of x s.t. 0 < x < n & acd(n, x) = 1
22 // O(n*LogLogn)
23 void euler phi(int n, int ret[]) {
       for (int i = 1; i <= n; ++i) ret[i] = i;</pre>
25
       for (int i = 2; i <= n; ++i)
26
            if (ret[i] == i)
27
                for (int j = i; j \leftarrow n; j \leftarrow i)
28
                    ret[j] -= ret[j] / i;
29 }
```

2.3 Primality Test

```
1 bool test witness(ull a, ull n, ull s) {
       if (a >= n) a %= n;
       if (a <= 1) return true;
       ull d = n \gg s:
       ull x = modpow(a, d, n);
       if (x == 1 || x == n-1) return true;
       while (s-- > 1) {
           x = large_mod_mul(x, x, n);
9
           if (x == 1) return false;
           if (x == n-1) return true;
10
11
12
       return false;
13 }
14
15 // test whether n is prime
16 // based on miller-rabin test
17 // O(logn*logn)
18 bool is_prime(ull n) {
19
       if (n == 2) return true;
20
       if (n < 2 || n % 2 == 0) return false;</pre>
21
22
       ull d = n \gg 1, s = 1;
23
       for(; (d&1) == 0; s++) d >>= 1;
24
25 #define T(a) test witness(a##ull, n, s)
       if (n < 4759123141ull) return T(2) && T(7) && T(61);</pre>
27
       return T(2) && T(325) && T(9375) && T(28178)
28
           && T(450775) && T(9780504) && T(1795265022);
29 #undef T
30 }
```

2.4 Chinese Remainder Theorem

```
1 // find x s.t. x === a[0] (mod n[0])
2 // === a[1] (mod n[1])
```

```
3 //
4 // assumption: gcd(n[i], n[j]) = 1
5 11 chinese remainder(11* a, 11* n, int size) {
       if (size == 1) return *a;
       11 tmp = modinverse(n[0], n[1]);
       ll tmp2 = (tmp * (a[1] - a[0]) % n[1] + n[1]) % n[1];
       ll ora = a[1];
10
       ll tgcd = gcd(n[0], n[1]);
       a[1] = a[0] + n[0] / tgcd * tmp2;
11
12
       n[1] *= n[0] / tgcd;
13
       ll ret = chinese remainder(a + 1, n + 1, size - 1);
14
       n[1] /= n[0] / tgcd;
15
       a[1] = ora;
16
       return ret;
17 }
```

2.5 Rational Number Class

```
1 struct rational {
       long long p, q;
4
       void red() {
           if (q < 0) {
               p = -p;
               q = -q;
8
9
           11 t = gcd((p >= 0 ? p : -p), q);
10
           p /= t;
11
           q /= t;
12
13
14
       rational(): p(0), q(1) {}
       rational(long long p_): p(p_{-}), q(1) {}
15
16
       rational(long long p_, long long q_): p(p_), q(q_) { red(); }
17
18
       bool operator==(const rational& rhs) const {
19
           return p == rhs.p && q == rhs.q;
20
21
       bool operator!=(const rational& rhs) const {
22
           return p != rhs.p || q != rhs.q;
23
24
       bool operator<(const rational& rhs) const {</pre>
25
           return p * rhs.q < rhs.p * q;</pre>
26
27
       rational operator+(const rational& rhs) const {
28
           ll g = gcd(q, rhs.q);
29
           return rational(p * (rhs.q / g) + rhs.p * (q / g), (q / g) * rhs.q);
30
31
       rational operator-(const rational& rhs) const {
32
           11 g = gcd(q, rhs.q);
33
           return rational(p * (rhs.q / g) - rhs.p * (q / g), (q / g) * rhs.q);
34
35
       rational operator*(const rational& rhs) const {
           return rational(p * rhs.p, q * rhs.q);
```

```
37  }
38  rational operator/(const rational& rhs) const {
39  return rational(p * rhs.q, q * rhs.p);
40  }
41 };
```

2.6 Burnside's Lemma

경우의 수를 세는데, 특정 transform operation(회전, 반사, ..) 해서 같은 경우들은 하나로 친다. 전체 경우의 수는?

- 각 operation마다 이 operation을 했을 때 변하지 않는 경우의 수를 센다 (단, "아무것도 ³⁵ 하지 않는다"라는 operation도 있어야 함!)
- 전체 경우의 수를 더한 후, operation의 수로 나눈다. (답이 맞다면 항상 나누어 떨어져야 한다)

2.7 Kirchoff's Theorem

그래프의 스패닝 트리의 개수를 구하는 정리.

무향 그래프의 Laplacian matrix L를 만든다. 이것은 (정점의 차수 대각 행렬) - (인접행렬) 이다. L에서 행과 열을 하나씩 제거한 것을 L'라 하자. 어느 행/열이든 관계 없다. 그래프의 스패닝 트리의 개수는 det(L')이다.

2.8 Fast Fourier Transform

```
1 void fft(int sign, int n, double *real, double *imag) {
       double theta = sign * 2 * pi / n;
       for (int m = n; m >= 2; m >>= 1, theta *= 2) {
           double wr = 1, wi = 0, c = cos(theta), s = sin(theta);
           for (int i = 0, mh = m >> 1; i < mh; ++i) {
               for (int j = i; j < n; j += m) {
                   int k = j + mh;
                   double xr = real[j] - real[k], xi = imag[j] - imag[k];
                   real[j] += real[k], imag[j] += imag[k];
10
                   real[k] = wr * xr - wi * xi, imag[k] = wr * xi + wi * xr;
11
12
               double wr = wr * c - wi * s, wi = wr * s + wi * c;
13
               wr = wr, wi = wi;
14
15
      }
16
      for (int i = 1, j = 0; i < n; ++i) {
17
           for (int k = n >> 1; k > (j ^= k); k >>= 1);
           if (j < i) swap(real[i], real[j]), swap(imag[i], imag[j]);</pre>
18
19
20 }
21 // Compute Poly(a)*Poly(b), write to r; Indexed from 0
22 // O(n*logn)
```

```
23 int mult(int *a, int n, int *b, int m, int *r) {
       const int maxn = 100;
25
       static double ra[maxn], rb[maxn], ia[maxn], ib[maxn];
26
       int fn = 1;
       while (fn < n + m) fn <<= 1; // n + m: interested length
27
       for (int i = 0; i < n; ++i) ra[i] = a[i], ia[i] = 0;</pre>
28
       for (int i = n; i < fn; ++i) ra[i] = ia[i] = 0;</pre>
29
30
       for (int i = 0; i < m; ++i) rb[i] = b[i], ib[i] = 0;
31
       for (int i = m; i < fn; ++i) rb[i] = ib[i] = 0;
       fft(1, fn, ra, ia);
       fft(1, fn, rb, ib);
34
       for (int i = 0; i < fn; ++i) {
           double real = ra[i] * rb[i] - ia[i] * ib[i];
           double imag = ra[i] * ib[i] + rb[i] * ia[i];
37
           ra[i] = real, ia[i] = imag;
       fft(-1, fn, ra, ia);
       for (int i = 0; i < fn; ++i) r[i] = (int)floor(ra[i] / fn + 0.5);
       return fn:
42 }
```

2.9 Matrix Operations

```
1 const int MATSZ = 100;
 3 inline bool is_zero(double a) { return fabs(a) < 1e-9; }</pre>
 5 // out = A^{(-1)}, returns det(A)
 6 // A becomes invalid after call this
7 // O(n^3)
8 double inverse and det(int n, double A[][MATSZ], double out[][MATSZ]) {
       double det = 1;
10
       for (int i = 0; i < n; i++) {
11
           for (int j = 0; j < n; j++) out[i][j] = 0;
12
           out[i][i] = 1;
13
14
       for (int i = 0; i < n; i++) {
15
           if (is zero(A[i][i])) {
16
               double maxv = 0;
               int maxid = -1;
17
18
               for (int j = i + 1; j < n; j++) {
19
                   auto cur = fabs(A[j][i]);
20
                   if (maxv < cur) {</pre>
21
                        maxv = cur;
22
                        maxid = j;
23
24
               }
               if (maxid == -1 || is_zero(A[maxid][i])) return 0;
25
26
               for (int k = 0; k < n; k++) {
27
                   A[i][k] += A[maxid][k];
28
                   out[i][k] += out[maxid][k];
29
               }
30
           det *= A[i][i];
```

```
32
           double coeff = 1.0 / A[i][i];
33
           for (int j = 0; j < n; j++) A[i][j] *= coeff;</pre>
34
           for (int j = 0; j < n; j++) out[i][j] *= coeff;</pre>
35
           for (int j = 0; j < n; j++) if (j != i) {
36
               double mp = A[j][i];
37
               for (int k = 0; k < n; k++) A[j][k] -= A[i][k] * mp;
38
               for (int k = 0; k < n; k++) out[j][k] -= out[i][k] * mp;
39
           }
       }
40
41
       return det;
42 }
```

Gaussian Elimination

1 const double EPS = 1e-10;

```
2 typedef vector<vector<double>> VVD;
4 // Gauss-Jordan elimination with full pivoting.
5 // solving systems of linear equations (AX=B)
6 // INPUT:
                a[][] = an n*n matrix
                b[][] = an n*m matrix
8 // OUTPUT:
               X = an n*m matrix (stored in b[][])
9 //
                A^{-1} = an n*n matrix (stored in a[][])
10 // O(n^3)
11 bool gauss_jordan(VVD& a, VVD& b) {
12
       const int n = a.size();
13
       const int m = b[0].size();
14
       vector<int> irow(n), icol(n), ipiv(n);
15
16
      for (int i = 0; i < n; i++) {
17
           int pj = -1, pk = -1;
18
           for (int j = 0; j < n; j++) if (!ipiv[j])
19
               for (int k = 0; k < n; k++) if (!ipiv[k])
20
                   if (pj == -1 || fabs(a[j][k]) > fabs(a[pj][pk])) { pj = j; pk =
                     k; }
21
           if (fabs(a[pj][pk]) < EPS) return false; // matrix is singular</pre>
22
           ipiv[pk]++;
23
           swap(a[pj], a[pk]);
24
           swap(b[pj], b[pk]);
25
           irow[i] = pj;
26
           icol[i] = pk;
27
28
           double c = 1.0 / a[pk][pk];
29
           a[pk][pk] = 1.0;
30
           for (int p = 0; p < n; p++) a[pk][p] *= c;
31
           for (int p = 0; p < m; p++) b[pk][p] *= c;
32
           for (int p = 0; p < n; p++) if (p != pk) {
33
               c = a[p][pk];
34
               a[p][pk] = 0;
35
               for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c;
36
               for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c;
37
          }
38
      for (int p = n - 1; p >= 0; p --) if (irow[p] != icol[p]) {
```

```
return true;
43 }
```

for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][icol[p]]);</pre>

2.11 Simplex Algorithm

40

41

42

```
1 // Two-phase simplex algorithm for solving linear programs of the form
 2 //
          maximize
                       c^T x
3 //
          subject to Ax <= b
4 //
                        x >= 0
5 // INPUT: A -- an m x n matrix
6 //
             b -- an m-dimensional vector
7 //
             c -- an n-dimensional vector
8 //
             x -- a vector where the optimal solution will be stored
9 // OUTPUT: value of the optimal solution (infinity if unbounded
              above, nan if infeasible)
11 // To use this code, create an LPSolver object with A, b, and c as
12 // arguments. Then, call Solve(x).
13 typedef vector<double> VD;
14 typedef vector<VD> VVD;
15 typedef vector<int> VI;
16 const double EPS = 1e-9;
17
18 struct LPSolver {
19
       int m, n;
20
       VI B, N;
21
       VVD D;
22
23
       LPSolver(const VVD& A, const VD& b, const VD& c):
24
           m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2, VD(n + 2)) 
           for (int i = 0; i < m; i++) for (int j = 0; j < n; j++) D[i][j] = A[i][j]
25
            1;
           for (int i = 0; i < m; i++) { B[i] = n + i; D[i][n] = -1; D[i][n + 1] =
            b[i]; }
27
           for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
28
           N[n] = -1; D[m + 1][n] = 1;
29
       }
30
31
       void pivot(int r, int s) {
32
           double inv = 1.0 / D[r][s];
33
           for (int i = 0; i < m + 2; i++) if (i != r)
34
               for (int j = 0; j < n + 2; j++) if (j != s)
35
                   D[i][j] -= D[r][j] * D[i][s] * inv;
           for (int j = 0; j < n + 2; j++) if (j != s) D[r][j] *= inv;
36
37
           for (int i = 0; i < m + 2; i++) if (i != r) D[i][s] *= -inv;
38
           D[r][s] = inv;
39
           swap(B[r], N[s]);
40
       }
41
42
       bool simplex(int phase) {
43
           int x = phase == 1 ? m + 1 : m;
44
           while (true) {
               int s = -1;
```

```
46
               for (int j = 0; j <= n; j++) {
                                                                                          9 // tree<key_type, value_type(set if null), comparator, ...>
47
                   if (phase == 2 && N[j] == -1) continue;
                                                                                         10 using ordered_set = tree<int, null_type, less<int>, rb_tree_tag,
                   if (s == -1 \mid | D[x][j] < D[x][s] \mid | D[x][j] == D[x][s] && N[j] < 11
                      N[s]) s = j;
49
50
               if (D[x][s] > -EPS) return true;
51
               int r = -1;
52
               for (int i = 0; i < m; i++) {
53
                   if (D[i][s] < EPS) continue;</pre>
                   if (r == -1 \mid | D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][s] \mid |
55
                        (D[i][n + 1] / D[i][s]) == (D[r][n + 1] / D[r][s]) && B[i] < 19
                           B[r]) r = i;
56
               if (r == -1) return false;
57
58
               pivot(r, s);
59
           }
60
       }
61
62
       double solve(VD& x) {
63
                                                                                         28 }
           int r = 0:
64
           for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r][n + 1]) r = i;
65
           if (D[r][n + 1] < -EPS) {</pre>
66
               pivot(r, n);
67
               if (!simplex(1) || D[m + 1][n + 1] < -EPS)
68
                   return -numeric limits<double>::infinity();
69
               for (int i = 0; i < m; i++) if (B[i] == -1) {
70
                   int s = -1;
                   for (int j = 0; j <= n; j++)</pre>
71
                        if (s == -1 || D[i][j] < D[i][s] || D[i][j] == D[i][s] && N[ 5 int query(int p) {
72
                          j] < N[s]) s = j;
73
                   pivot(i, s);
74
               }
75
76
           if (!simplex(2))
77
               return numeric limits<double>::infinity();
78
79
           for (int i = 0; i < m; i++) if (B[i] < n) \times [B[i]] = D[i][n + 1];
80
           return D[m][n + 1];
81
82 };
```

Data Structure

Order statistic tree

```
1 #include <ext/pb_ds/assoc_container.hpp>
2 #include <ext/pb_ds/tree_policy.hpp>
3 #include <ext/pb ds/detail/standard policies.hpp>
4 #include <functional>
5 #include <iostream>
6 using namespace __gnu_pbds;
7 using namespace std;
```

```
tree order statistics node update>;
13 int main()
14 {
15
                                  ordered set X;
16
                                  for (int i = 1; i < 10; i += 2) X.insert(i); // 1 3 5 7 9
17
                                  cout << boolalpha;</pre>
                                  cout << *X.find_by_order(2) << endl; // 5</pre>
                                  cout << *X.find by order(4) << endl; // 9</pre>
                                  cout << (X.end() == X.find_by_order(5)) << endl; // true</pre>
21
22
                                  cout << X.order_of_key(-1) << endl; // 0</pre>
23
                                  cout << X.order_of_key(1) << endl; // 0</pre>
24
                                  cout << X.order_of_key(4) << endl; // 2</pre>
25
                                  X.erase(3);
26
                                  cout << X.order_of_key(4) << endl; // 1</pre>
27
                                  for (int t : X) printf("%d<sub>\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\under</sub>
```

3.2 Fenwick Tree

```
1 const int TSIZE = 100000;
2 int tree[TSIZE + 1];
 4 // Returns the sum from index 1 to p, inclusive
       int ret = 0;
       for (; p > 0; p -= p & -p) ret += tree[p];
       return ret;
9 }
11 // Adds val to element with index pos
12 void add(int p, int val) {
       for (; p <= TSIZE; p += p & -p) tree[p] += val;</pre>
13
14 }
```

3.3 Segment Tree with Lazy Propagation

```
1 // example implementation of sum tree
 2 const int TSIZE = 131072; // always 2<sup>k</sup> form && n <= TSIZE
 3 int segtree[TSIZE * 2], prop[TSIZE * 2];
 4 void seg_init(int nod, int 1, int r) {
       if (1 == r) segtree[nod] = dat[1];
       else {
           int m = (1 + r) >> 1;
8
           seg init(nod << 1, 1, m);
           seg_init(nod << 1 | 1, m + 1, r);
9
10
           segtree[nod] = segtree[nod << 1] + segtree[nod << 1 | 1];</pre>
11
       }
12 }
13 void seg_relax(int nod, int 1, int r) {
```

```
14
       if (prop[nod] == 0) return;
                                                                                          18
15
       if (1 < r) {</pre>
                                                                                          19
                                                                                                  void init() {
16
                                                                                          20
                                                                                                      // zero-initialize, can be changed freely
            int m = (1 + r) >> 1;
17
            segtree[nod << 1] += (m - 1 + 1) * prop[nod];
                                                                                          21
                                                                                                      memset(&npoll[TSIZE - 1], 0, sizeof(node) * TSIZE);
18
            prop[nod << 1] += prop[nod];</pre>
                                                                                          22
19
            segtree[nod \langle\langle 1 \mid 1 \rangle\rangle += (r - m) * prop[nod];
                                                                                          23
                                                                                                      for (int i = TSIZE - 2; i >= 0; i--) {
            prop[nod << 1 | 1] += prop[nod];</pre>
20
                                                                                          24
                                                                                                           npoll[i].v = 0;
21
                                                                                          25
                                                                                                          npoll[i].l = &npoll[i*2+1];
22
       prop[nod] = 0;
                                                                                                          npoll[i].r = &npoll[i*2+2];
                                                                                          26
23 }
                                                                                          27
24 int seg_query(int nod, int 1, int r, int s, int e) {
                                                                                          28
       if (r < s || e < 1) return 0;
                                                                                          29
25
                                                                                                      head[0] = &npoll[0];
26
       if (s <= 1 && r <= e) return segtree[nod];</pre>
                                                                                          30
                                                                                                      last q = 0;
27
       seg relax(nod, 1, r);
                                                                                          31
                                                                                                      pptr = 2 * TSIZE - 1;
28
       int m = (1 + r) >> 1;
                                                                                          32
                                                                                                      q[0] = 0;
29
       return seg_query(nod << 1, 1, m, s, e) + seg_query(nod << 1 | 1, m + 1, r, s 33
                                                                                                      lqidx = 0;
30 }
                                                                                          35
31 void seg_update(int nod, int l, int r, int s, int e, int val) {
                                                                                          36
                                                                                                  // update val to pos at time t
       if (r < s \mid l \in < 1) return:
                                                                                                  // 0 <= t <= MAX QUERY, 0 <= pos < TSIZE
                                                                                          37
33
       if (s <= 1 && r <= e) {
                                                                                          38
                                                                                                  void update(int pos, int val, int t, int prev) {
34
            segtree[nod] += (r - l + 1) * val;
                                                                                          39
                                                                                                      head[++last q] = &npoll[pptr++];
35
                                                                                          40
            prop[nod] += val;
                                                                                                      node *old = head[q[prev]], *now = head[last_q];
36
           return;
                                                                                          41
                                                                                                      while (lgidx < t) q[lgidx++] = q[prev];</pre>
37
       }
                                                                                          42
                                                                                                      q[t] = last q;
38
       seg_relax(nod, 1, r);
                                                                                          43
39
       int m = (1 + r) >> 1;
                                                                                          44
                                                                                                      int flag = 1 << DEPTH;</pre>
40
       seg update(nod << 1, 1, m, s, e, val);</pre>
                                                                                          45
                                                                                                      for (;;) {
41
                                                                                          46
                                                                                                           now -> v = old -> v + val;
       seg update(nod \langle\langle 1 \mid 1, m + 1, r, s, e, val)\rangle;
42
       segtree[nod] = segtree[nod << 1] + segtree[nod << 1 | 1];</pre>
                                                                                          47
                                                                                                          flag >>= 1;
43 }
                                                                                          48
                                                                                                           if (flag==0) {
44 // usage:
                                                                                          49
                                                                                                               now->l = now->r = nullptr; break;
45 // seg update(1, 0, n - 1, qs, qe, val);
                                                                                          50
46 // seg_query(1, 0, n - 1, qs, qe);
                                                                                          51
                                                                                                          if (flag & pos) {
                                                                                          52
                                                                                                               now->1 = old->1;
                                                                                          53
                                                                                                               now->r = &npoll[pptr++];
   3.4 Persistent Segment Tree
                                                                                          54
                                                                                                               now = now->r, old = old->r;
                                                                                          55
                                                                                                          } else {
                                                                                          56
                                                                                                               now->r = old->r;
1 // persistent segment tree impl: sum tree
                                                                                          57
                                                                                                               now \rightarrow 1 = &npoll[pptr++];
 2 namespace pstree {
                                                                                          58
                                                                                                               now = now->1, old = old->1;
       typedef int val t;
                                                                                          59
                                                                                                          }
       const int DEPTH = 18;
                                                                                          60
                                                                                                      }
       const int TSIZE = 1 << 18;</pre>
                                                                                          61
                                                                                                  }
 6
       const int MAX_QUERY = 262144;
                                                                                          62
                                                                                          63
                                                                                                  val t query(int s, int e, int l, int r, node *n) {
 8
       struct node {
                                                                                          64
                                                                                                      if (s == 1 \&\& e == r) return n \rightarrow v;
9
           val t v;
                                                                                          65
                                                                                                      int m = (1 + r) / 2;
10
            node *1, *r;
                                                                                          66
                                                                                                      if (m \ge e) return query(s, e, l, m, n \ge l);
11
       } npoll[TSIZE * 2 + MAX_QUERY * (DEPTH + 1)];
                                                                                          67
                                                                                                      else if (m < s) return query(s, e, m + 1, r, n->r);
12
                                                                                          68
                                                                                                      else return query(s, m, 1, m, n->1) + query(m + 1, e, m + 1, r, n->r);
13
       int pptr, last_q;
                                                                                          69
                                                                                                  }
14
                                                                                          70
15
       node *head[MAX QUERY + 1];
                                                                                          71
                                                                                                  // query summation of [s, e] at time t
       int q[MAX_QUERY + 1];
16
                                                                                          72
                                                                                                  val t query(int s, int e, int t) {
       int lqidx;
```

```
73
               s = max(0, s); e = min(TSIZE - 1, e);
74
              if (s > e) return 0;
75
               return query(s, e, 0, TSIZE - 1, head[q[t]]);
76
        }
77 }
   3.5 Splay Tree
1 // example : https://www.acmicpc.net/problem/13159
2 struct node {
         node* 1, * r, * p;
         int cnt, min, max, val;
         long long sum;
 6
         bool inv;
         node(int _val) :
 7
 8
              cnt(1), sum(_val), min(_val), max(_val), val(_val), inv(false),
9
              l(nullptr), r(nullptr), p(nullptr) {
10
         }
11 };
12 node* root;
14 void update(node* x) {
15
         x \rightarrow cnt = 1;
         x\rightarrow sum = x\rightarrow min = x\rightarrow max = x\rightarrow val;
16
17
         if (x\rightarrow 1) {
18
              x->cnt += x->l->cnt;
19
              x \rightarrow sum += x \rightarrow 1 \rightarrow sum;
20
              x->min = min(x->min, x->l->min);
21
              x \rightarrow max = max(x \rightarrow max, x \rightarrow 1 \rightarrow max);
22
        }
23
         if (x->r) {
24
              x \rightarrow cnt += x \rightarrow r \rightarrow cnt;
25
              x \rightarrow sum += x \rightarrow r \rightarrow sum;
26
              x-\min = \min(x-\min, x-r-\min);
27
              x \rightarrow max = max(x \rightarrow max, x \rightarrow r \rightarrow max);
28
         }
29 }
30
31 void rotate(node* x) {
32
         node* p = x->p;
33
         node* b = nullptr;
34
         if (x == p->1) {
              p->1 = b = x->r;
35
36
              x \rightarrow r = p;
37
         }
38
         else {
39
              p->r = b = x->1;
40
              x \rightarrow 1 = p;
41
         }
42
         x->p = p->p;
43
         p \rightarrow p = x;
44
         if (b) b \rightarrow p = p;
         x \rightarrow p? (p == x \rightarrow p \rightarrow 1 ? x \rightarrow p \rightarrow 1 : x \rightarrow p \rightarrow r) = x : (root = x);
45
         update(p);
```

```
47
         update(x);
 48 }
 49
 50 // make x into root
 51 void splay(node* x) {
         while (x->p) {
53
              node* p = x->p;
 54
              node* g = p - p;
 55
              if (g) rotate((x == p->1) == (p == g->1) ? p : x);
 56
              rotate(x);
 57
         }
 58 }
 59
 60 void relax_lazy(node* x) {
         if (!x->inv) return;
         swap(x->1, x->r);
 62
 63
         x->inv = false;
         if (x\rightarrow 1) x\rightarrow 1\rightarrow inv = !x\rightarrow 1\rightarrow inv;
 65
         if (x\rightarrow r) x\rightarrow r\rightarrow inv = !x\rightarrow r\rightarrow inv;
 66 }
 67
 68 // find kth node in splay tree
 69 void find_kth(int k) {
         node* x = root;
71
         relax lazy(x);
72
         while (true) {
73
              while (x->1 && x->1->cnt > k) {
74
                   x = x \rightarrow 1;
75
                   relax_lazy(x);
76
77
              if (x\rightarrow 1) k -= x\rightarrow 1\rightarrow cnt;
78
              if (!k--) break;
79
              x = x - r;
 80
              relax_lazy(x);
81
 82
         splay(x);
 83 }
84
 85 // collect [l, r] nodes into one subtree and return its root
 86 node* interval(int l, int r) {
 87
         find kth(l - 1);
         node* x = root;
 88
 89
         root = x - r;
90
         root->p = nullptr;
91
         find_kth(r - l + 1);
 92
         x \rightarrow r = root;
 93
         root -> p = x;
 94
         root = x;
 95
         return root->r->l;
 96 }
97
98 void traverse(node* x) {
99
         relax_lazy(x);
100
         if (x\rightarrow 1) {
101
              traverse(x->1);
```

```
102
103
        // do something
104
        if (x->r) {
105
            traverse(x->r);
106
107 }
108
109 void uptree(node* x) {
110
        if (x->p) {
111
            uptree(x->p);
112
        }
113
        relax_lazy(x);
114 }
```

3.6 Link/Cut Tree

4 DP

4.1 Convex Hull Optimization

4.1.1 requirement

```
O(n^2) \rightarrow O(n \log n)
조건 1) DP 점화식 꼴 D[i] = \min_{j < i} (D[j] + b[j] * a[i]) 조건 2) b[j] \le b[j+1]
```

특수조건) $a[i] \le a[i+1]$ 도 만족하는 경우, 마지막 쿼리의 위치를 저장해두면 이분검색이 필요없어지기 때문에 amortized O(n) 에 해결할 수 있음

4.1.2 Source Code

```
1 //O(n^3) -> O(n^2)
2
3 #define sz 100001
4 long long s[sz];
5 long long dp[2][sz];
6 //deque {index, x pos }
7 int dqi[sz];
8 long long dqm[sz];
9 //pointer to deque
10 int ql,qr;
11 //dp[i][j] = max(dp[i][k] + s[j]*s[k] - s[k]^2)
12 //let y = dp[i][j], x = s[j] -> y = max(s[k]*x + dp[i][k] - s[k]^2);
13
14 //push new value to deque
15 //i = index, x = current x pos
```

```
16 void setq(int i, int x)
17 {
18
       //a1,b1 = prv line, a2,b2 = new line
       int a1, a2 = s[i];
19
       long long b1, b2 = dp[0][i] - s[i] * s[i], r;
20
       //renew deque
22
       while (qr>=ql)
23
24
           //last line enqueued
           a1 = s[dqi[qr]];
25
26
           b1 = dp[0][dqi[qr]] - s[dqi[qr]] * s[dqi[qr]];
27
           //tie breaking to newer one
28
           if (a1 == a2)
29
30
               dqi[qr] = i;
31
               return;
32
33
           // x intersection between last line and new line
34
           r = (b1 - b2) / (a2 - a1);
35
           if ((b1 - b2) % (a2 - a1)) r++;
36
           //last line is not needed
37
           if (r <= dqm[qr])
38
39
               qr--;
40
41
           else break;
42
       if (r < 0) r = 0;
43
       //push back new line
45
       if (dqm[qr] < s[n - 1] && r <= s[n - 1])
46
       {
47
           dqi[++qr] = i;
           dqm[qr] = r;
       //discard old lines
51
       while (qr-ql && dqm[ql+1] <= x)
52
53
           q1++;
54
55 }
56
57 int main()
58 {
59
       for (int j = 0; j < k; j++)
60
61
           ql = 0;
62
           qr = 1;
63
           dqi[0] = dqm[0] = 0;
64
           for (int i = 1; i < n; i++)
65
66
               //get line used by current x pos
67
               setq(i, s[i]);
68
               //line index to use
69
               int g = dqi[ql];
70
               //set dp value
```

4.2 Divide & Conquer Optimization

```
O(kn^2) 	o O(kn\log n) 조건 1) DP 점화식 꼴 D[t][i] = \min_{j < i} (D[t-1][j] + C[j][i]) 조건 2) A[t][i] 는 D[t][i]의 답이 되는 최소의 j라 할 때, 아래의 부등식을 만족해야 함 A[t][i] \le A[t][i+1] 조건 2-1) 비용C가 다음의 사각부등식을 만족하는 경우도 조건 2)를 만족하게 됨 C[a][c] + C[b][d] \le C[a][d] + C[b][c] \; (a \le b \le c \le d)
```

4.3 Knuth Optimization

 $A[i][j-1] \le A[i][j] \le A[i+1][j]$

```
O(n^3) \to O(n^2) 조건 1) DP 점화식 꼴 D[i][j] = \min_{i < k < j} (D[i][k] + D[k][j]) + C[i][j] 조건 2) 사각 부등식 C[a][c] + C[b][d] \le C[a][d] + C[b][c] \ (a \le b \le c \le d) 조건 3) 단조성 C[b][c] \le C[a][d] \ (a \le b \le c \le d) 결론) 조건 2, 3을 만족한다면 A[i][j]를 D[i][j]의 답이 되는 최소의 k라 할 때, 아래의 부등 식을 만족하게 됨
```

3중 루프를 돌릴 때 위 조건을 이용하면 최종적으로 시간복잡도가 $O(n^2)$ 이 됨

5 Graph

5.1 SCC (Tarjan)

```
1 const int MAXN = 100;
 2 vector<int> graph[MAXN];
 3 int up[MAXN], visit[MAXN], vtime;
 4 vector<int> stk;
5 int scc_idx[MAXN], scc_cnt;
 7 void dfs(int nod) {
       up[nod] = visit[nod] = ++vtime;
       stk.push_back(nod);
10
       for (int next : graph[nod]) {
11
           if (visit[next] == 0) {
12
               dfs(next);
13
               up[nod] = min(up[nod], up[next]);
14
15
           else if (scc_idx[next] == 0)
               up[nod] = min(up[nod], visit[next]);
17
18
       if (up[nod] == visit[nod]) {
19
           ++scc cnt;
20
           int t;
21
           do {
22
               t = stk.back();
23
               stk.pop_back();
               scc_idx[t] = scc_cnt;
25
           } while (!stk.empty() && t != nod);
26
27 }
29 // find SCCs in given directed graph
30 // O(V+E)
31 void get_scc() {
       vtime = 0;
       memset(visit, 0, sizeof(visit));
33
       scc cnt = 0;
       memset(scc_idx, 0, sizeof(scc_idx));
35
36
       for (int i = 0; i < n; ++i)
37
           if (visit[i] == 0) dfs(i);
38 }
```

5.2 SCC (Kosaraju)

```
1 const int MAXN = 100;
2 vector<int> graph[MAXN], grev[MAXN];
3 int visit[MAXN], vcnt;
4 int scc_idx[MAXN], scc_cnt;
5 vector<int> emit;
6
7 void dfs(int nod, vector<int> graph[]) {
```

```
visit[nod] = vcnt;
9
       for (int next : graph[nod]) {
10
           if (visit[next] == vcnt) continue;
           dfs(next, graph);
11
12
13
       emit.push back(nod);
14 }
15
16 // find SCCs in given graph
17 // O(V+E)
18 void get scc() {
19
       scc cnt = 0;
20
       vcnt = 1;
21
       emit.clear();
22
       memset(visit, 0, sizeof(visit));
23
24
       for (int i = 0; i < n; i++) {
           if (visit[i] == vcnt) continue;
25
26
           dfs(i, graph);
27
      }
28
29
       ++vcnt;
30
       for (auto st : vector<int>(emit.rbegin(), emit.rend())) {
31
           if (visit[st] == vcnt) continue;
32
           emit.clear();
33
           dfs(st, grev);
34
           ++scc cnt;
35
           for (auto node : emit)
36
               scc_idx[node] = scc_cnt;
37
      }
38 }
```

5.3 2-SAT

 $(b_x \lor b_y) \land (\neg b_x \lor b_z) \land (b_z \lor \neg b_x) \land \cdots$ 같은 form을 2-CNF라고 함. 주어진 2-CNF 식을 36 참으로 하는 $\{b_1, b_2, \cdots\}$ 가 존재하는지, 존재한다면 그 값은 무엇인지 구하는 문제를 2-SAT 38 이라 함.

boolean variable b_i 마다 b_i 를 나타내는 정점, $\neg b_i$ 를 나타내는 정점 2개를 만듦. 각 clause 41 $b_i \lor b_j$ 마다 $\neg b_i \to b_j$, $\neg b_j \to b_i$ 이렇게 edge를 이어줌. 그렇게 만든 그래프에서 SCC를 다 42 구함. 어떤 SCC 안에 b_i 와 $\neg b_i$ 가 같이 포함되어있다면 해가 존재하지 않음. 아니라면 해가 존재함.

해가 존재할 때 구체적인 해를 구하는 방법. 위에서 SCC를 구하면서 SCC DAG를 만들어 $^{46}_{47}$ 준다. 거기서 위상정렬을 한 후, 앞에서부터 SCC를 하나씩 봐준다. 현재 보고있는 SCC에 $^{48}_{57}$ 속해있는데 얘가 $^{-1}_{7}$ 보다 먼저 등장했다면 $^{18}_{57}$ 들 false, 반대의 경우라면 $^{18}_{57}$ 을 잡이 assign되었다면 pass.

5.4 BCC, Cut vertex, Bridge

```
1 const int MAXN = 100;
 2 vector<pair<int, int>> graph[MAXN]; // { next vertex id, edge id }
 3 int up[MAXN], visit[MAXN], vtime;
 4 vector<pair<int, int>> stk;
 6 int is_cut[MAXN];
                               // v is cut vertex if is cut[v] > 0
                               // list of edge ids
 7 vector<int> bridge;
 8 vector<int> bcc_idx[MAXN]; // list of bccids for vertex i
 9 int bcc cnt;
10
11 void dfs(int nod, int par edge) {
12
       up[nod] = visit[nod] = ++vtime;
13
       int child = 0;
       for (const auto& e : graph[nod]) {
15
           int next = e.first, edge_id = e.second;
16
           if (edge_id == par_edge) continue;
17
           if (visit[next] == 0) {
18
               stk.push_back({ nod, next });
19
               ++child;
               dfs(next, edge_id);
20
21
               if (up[next] == visit[next]) bridge.push_back(edge_id);
               if (up[next] >= visit[nod]) {
22
23
                   ++bcc_cnt;
24
                   do {
25
                       auto last = stk.back();
26
                       stk.pop_back();
27
                       bcc_idx[last.second].push_back(bcc_cnt);
28
                       if (last == pair<int, int>{ nod, next }) break;
29
                   } while (!stk.empty());
30
                   bcc_idx[nod].push_back(bcc_cnt);
31
                   is_cut[nod]++;
32
33
               up[nod] = min(up[nod], up[next]);
34
35
           else
               up[nod] = min(up[nod], visit[next]);
       if (par edge == -1 && is cut[nod] == 1)
39
           is_cut[nod] = 0;
40 }
42 // find BCCs & cut vertexs & bridges in undirected graph
43 // O(V+E)
44 void get_bcc() {
       vtime = 0;
       memset(visit, 0, sizeof(visit));
       memset(is_cut, 0, sizeof(is_cut));
       bridge.clear();
       for (int i = 0; i < n; ++i) bcc idx[i].clear();</pre>
       bcc_cnt = 0;
51
       for (int i = 0; i < n; ++i) {
52
           if (visit[i] == 0)
53
               dfs(i, -1);
54
       }
55 }
```

5.5 Shortest Path Faster Algorithm

```
1 // shortest path faster algorithm
2 // average for random graph : O(E) , worst : O(VE)
4 \text{ const int MAXN} = 20001;
5 const int INF = 100000000;
6 int n, m;
7 vector<pair<int, int>> graph[MAXN];
8 bool inqueue[MAXN];
9 int dist[MAXN];
10
11 void spfa(int st) {
12
       for (int i = 0; i < n; ++i) {
13
           dist[i] = INF;
14
15
       dist[st] = 0;
16
17
       queue<int> q;
18
       q.push(st);
19
       inqueue[st] = true;
20
       while (!q.empty()) {
21
           int u = q.front();
22
           q.pop();
23
           inqueue[u] = false;
24
           for (auto& e : graph[u]) {
25
               if (dist[u] + e.second < dist[e.first]) {</pre>
26
                    dist[e.first] = dist[u] + e.second;
27
                    if (!inqueue[e.first]) {
28
                        q.push(e.first);
29
                        inqueue[e.first] = true;
30
31
32
           }
33
34 }
```

5.6 Lowest Common Ancestor

```
1 const int MAXN = 100;
2 const int MAXLN = 9;
3 vector<int> tree[MAXN];
4 int depth[MAXN];
5 int par[MAXLN][MAXN];
6
7 void dfs(int nod, int parent) {
8
       for (int next : tree[nod]) {
9
           if (next == parent) continue;
10
           depth[next] = depth[nod] + 1;
11
           par[0][next] = nod;
12
           dfs(next, nod);
13
      }
14 }
15
```

```
16 void prepare lca() {
17
       const int root = 0;
18
       dfs(root, -1);
       par[0][root] = root;
19
       for (int i = 1; i < MAXLN; ++i)</pre>
20
21
           for (int j = 0; j < n; ++j)
22
               par[i][j] = par[i - 1][par[i - 1][j]];
23 }
25 // find lowest common ancestor in tree between u & v
26 // assumption : must call 'prepare lca' once before call this
27 // O(LogV)
28 int lca(int u, int v) {
       if (depth[u] < depth[v]) swap(u, v);</pre>
30
       if (depth[u] > depth[v]) {
31
           for (int i = MAXLN - 1; i >= 0; --i)
32
               if (depth[u] - (1 << i) >= depth[v])
33
                   u = par[i][u];
34
35
       if (u == v) return u;
36
       for (int i = MAXLN - 1; i >= 0; --i) {
37
           if (par[i][u] != par[i][v]) {
38
               u = par[i][u];
39
               v = par[i][v];
           }
40
41
42
       return par[0][u];
```

5.7 Heavy-Light Decomposition

```
1 // heavy-light decomposition
2 //
3 // hld h;
4 // insert edges to tree[0~n-1];
5 // h.init(n);
6 // h.decompose(root);
7 // h.hldquery(u, v); // edges from u to v
8 struct hld {
9
       static const int MAXLN = 18;
10
       static const int MAXN = 1 << (MAXLN - 1);</pre>
       vector<int> tree[MAXN];
11
       int subsize[MAXN], depth[MAXN], pa[MAXLN][MAXN];
12
13
14
       int chead[MAXN], cidx[MAXN];
15
       int lchain;
16
       int flatpos[MAXN + 1], fptr;
17
18
       void dfs(int u, int par) {
19
           pa[0][u] = par;
20
           subsize[u] = 1;
21
           for (int v : tree[u]) {
22
               if (v == pa[0][u]) continue;
               depth[v] = depth[u] + 1;
23
```

```
24
               dfs(v, u);
                                                                                        79
                                                                                                        }
25
                                                                                        80
               subsize[u] += subsize[v];
26
                                                                                        81
                                                                                                    return pa[0][u];
27
      }
                                                                                        82
                                                                                               }
28
                                                                                        83
29
       void init(int size)
                                                                                        84
                                                                                               // TODO: implement query functions
30
                                                                                        85
                                                                                               inline int query(int s, int e) {
31
           lchain = fptr = 0;
                                                                                        86
                                                                                                    return 0;
32
           dfs(0, -1);
                                                                                        87
                                                                                               }
33
           memset(chead, -1, sizeof(chead));
                                                                                        88
34
                                                                                        89
                                                                                               int subquery(int u, int v, int t) {
           for (int i = 1; i < MAXLN; i++) {</pre>
35
                                                                                        90
                                                                                                    int uchain, vchain = cidx[v];
36
               for (int j = 0; j < size; j++) {</pre>
                                                                                        91
                                                                                                    int ret = 0;
37
                   if (pa[i - 1][j] != -1) {
                                                                                        92
                                                                                                    for (;;) {
38
                       pa[i][j] = pa[i - 1][pa[i - 1][j]];
                                                                                        93
                                                                                                        uchain = cidx[u];
                                                                                        94
39
                                                                                                        if (uchain == vchain) {
40
               }
                                                                                        95
                                                                                                            ret += query(flatpos[v], flatpos[u]);
41
                                                                                        96
           }
                                                                                                            break;
42
      }
                                                                                        97
                                                                                                        }
43
                                                                                        98
44
       void decompose(int u) {
                                                                                        99
                                                                                                        ret += query(flatpos[chead[uchain]], flatpos[u]);
45
           if (chead[lchain] == -1) chead[lchain] = u;
                                                                                       100
                                                                                                        u = pa[0][chead[uchain]];
46
           cidx[u] = lchain;
                                                                                       101
47
           flatpos[u] = ++fptr;
                                                                                       102
                                                                                                    return ret;
48
                                                                                       103
                                                                                               }
49
           int maxchd = -1;
                                                                                       104
           for (int v : tree[u]) {
50
                                                                                       105
                                                                                               inline int hldquery(int u, int v) {
51
               if (v == pa[0][u]) continue;
                                                                                       106
                                                                                                    int p = lca(u, v);
52
               if (maxchd == -1 || subsize[maxchd] < subsize[v]) maxchd = v;</pre>
                                                                                       107
                                                                                                    return subquery(u, p) + subquery(v, p) - query(flatpos[p], flatpos[p]);
53
                                                                                       108
54
           if (maxchd != -1) decompose(maxchd);
                                                                                       109 };
55
56
           for (int v : tree[u]) {
                                                                                                 Bipartite Matching (Hopcroft-Karp)
57
               if (v == pa[0][u] || v == maxchd) continue;
58
               ++lchain; decompose(v);
59
           }
                                                                                         1 // in: n, m, graph
60
      }
                                                                                         2 // out: match, matched
61
                                                                                         3 // vertex cover: (reached[0][left_node] == 0) || (reached[1][right_node] == 1)
62
       int lca(int u, int v) {
                                                                                         4 // O(E*sqrt(V))
63
           if (depth[u] < depth[v]) swap(u, v);</pre>
                                                                                         5 struct BipartiteMatching {
64
                                                                                         6
                                                                                               int n, m;
65
           int logu;
                                                                                         7
                                                                                               vector<vector<int>> graph;
66
           for (logu = 1; 1 << logu <= depth[u]; logu++);</pre>
                                                                                               vector<int> matched, match, edgeview, level;
67
           logu--;
                                                                                               vector<int> reached[2];
                                                                                         9
68
                                                                                        10
                                                                                               BipartiteMatching(int n, int m) : n(n), m(m), graph(n), matched(m, -1),
69
           int diff = depth[u] - depth[v];
                                                                                                 match(n, -1) {}
70
           for (int i = logu; i >= 0; --i) {
                                                                                        11
71
               if ((diff >> i) & 1) u = pa[i][u];
                                                                                        12
                                                                                               bool assignLevel() {
72
                                                                                        13
                                                                                                    bool reachable = false;
73
           if (u == v) return u;
                                                                                        14
                                                                                                    level.assign(n, -1);
74
                                                                                        15
                                                                                                    reached[0].assign(n, 0);
75
           for (int i = logu; i >= 0; --i) {
                                                                                        16
                                                                                                    reached[1].assign(m, 0);
               if (pa[i][u] != pa[i][v]) {
76
                                                                                        17
                                                                                                    queue<int> q;
77
                   u = pa[i][u];
                                                                                        18
                                                                                                    for (int i = 0; i < n; i++) {
78
                   v = pa[i][v];
                                                                                        19
                                                                                                        if (match[i] == -1) {
```

```
20
                   level[i] = 0;
21
                   reached[0][i] = 1;
22
                   q.push(i);
23
               }
24
25
           while (!q.empty()) {
26
               auto cur = q.front(); q.pop();
27
               for (auto adj : graph[cur]) {
28
                   reached[1][adj] = 1;
29
                   auto next = matched[adj];
30
                   if (next == -1) {
31
                       reachable = true;
32
33
                   else if (level[next] == -1) {
34
                       level[next] = level[cur] + 1;
35
                       reached[0][next] = 1;
36
                       q.push(next);
37
38
               }
39
40
           return reachable;
      }
41
42
43
       int findpath(int nod) {
44
           for (int &i = edgeview[nod]; i < graph[nod].size(); i++) {</pre>
45
               int adj = graph[nod][i];
46
               int next = matched[adj];
47
               if (next >= 0 && level[next] != level[nod] + 1) continue;
               if (next == -1 || findpath(next)) {
48
49
                   match[nod] = adj;
50
                   matched[adj] = nod;
51
                   return 1;
52
               }
53
           }
54
           return 0;
55
      }
56
57
       int solve() {
58
           int ans = 0;
59
           while (assignLevel()) {
60
               edgeview.assign(n, 0);
61
               for (int i = 0; i < n; i++)
62
                   if (match[i] == -1)
63
                       ans += findpath(i);
64
65
           return ans;
66
67 };
         Maximum Flow (Dinic)
1 // usaae:
2 // MaxFlowDinic::init(n);
3 // MaxFlowDinic::add_edge(0, 1, 100, 100); // for bidirectional edge
```

```
4 // MaxFlowDinic::add_edge(1, 2, 100); // directional edge
 5 // result = MaxFlowDinic::solve(0, 2); // source -> sink
 6 // graph[i][edgeIndex].res -> residual
 8 // in order to find out the minimum cut, use `l'.
 9 // if l[i] == 0, i is unrechable.
10 //
11 // O(V*V*E)
12 // with unit capacities, O(\min(V^{(2/3)}, E^{(1/2)}) * E)
13 struct MaxFlowDinic {
14
       typedef int flow t;
       struct Edge {
15
16
           int next;
17
           int inv; /* inverse edge index */
18
           flow_t res; /* residual */
19
       };
20
       int n;
21
       vector<vector<Edge>> graph;
22
       vector<int> q, 1, start;
23
24
       void init(int _n) {
25
           n = _n;
26
           graph.resize(n);
27
           for (int i = 0; i < n; i++) graph[i].clear();</pre>
28
29
       void add_edge(int s, int e, flow_t cap, flow_t caprev = 0) {
30
           Edge forward{ e, graph[e].size(), cap };
31
           Edge reverse{ s, graph[s].size(), caprev };
32
           graph[s].push back(forward);
33
           graph[e].push_back(reverse);
34
35
       bool assign_level(int source, int sink) {
36
           int t = 0;
37
           memset(&1[0], 0, sizeof(1[0]) * 1.size());
38
           1[source] = 1;
39
           q[t++] = source;
40
           for (int h = 0; h < t && !1[sink]; h++) {</pre>
41
               int cur = q[h];
42
               for (const auto& e : graph[cur]) {
43
                   if (l[e.next] || e.res == 0) continue;
44
                   l[e.next] = l[cur] + 1;
45
                   q[t++] = e.next;
46
               }
47
48
           return 1[sink] != 0;
49
50
       flow_t block_flow(int cur, int sink, flow_t current) {
51
           if (cur == sink) return current;
52
           for (int& i = start[cur]; i < graph[cur].size(); i++) {</pre>
53
               auto& e = graph[cur][i];
54
               if (e.res == 0 || l[e.next] != l[cur] + 1) continue;
55
               if (flow t res = block flow(e.next, sink, min(e.res, current))) {
56
                   e.res -= res;
57
                    graph[e.next][e.inv].res += res;
58
                   return res;
```

```
59
                                                                                      32
                                                                                                  if (s == e) return;
60
                                                                                      33
                                                                                                  edge forward={e, cost, cap, cap, graph[e].size()};
61
                                                                                      34
                                                                                                  edge backward={s, -cost, 0, 0, graph[s].size()};
           return 0;
62
                                                                                      35
                                                                                                  if (cost < 0 || ranbefore) needNormalize = true;</pre>
63
      flow_t solve(int source, int sink) {
                                                                                      36
                                                                                                  graph[s].emplace_back(forward);
64
           q.resize(n);
                                                                                      37
                                                                                                  graph[e].emplace back(backward);
                                                                                      38
65
          1.resize(n);
66
           start.resize(n);
                                                                                      39
                                                                                             bool normalize(int s) {
           flow t ans = 0;
                                                                                      40
                                                                                                  auto infinite cost = numeric limits<cost t>::max();
67
68
           while (assign_level(source, sink)) {
                                                                                      41
                                                                                                  vector<cost_t> dist(n, infinite_cost);
                                                                                      42
69
               memset(&start[0], 0, sizeof(start[0]) * n);
                                                                                                  dist[s] = 0;
               while (flow t flow = block flow(source, sink, numeric limits<flow t</pre>
70
                                                                                                  queue<int> q;
                >::max()))
                                                                                                  vector<int> v(n), relax_count(n);
71
                   ans += flow:
                                                                                      45
                                                                                                  v[s] = 1; q.push(s);
72
                                                                                      46
                                                                                                  while(!q.empty()) {
73
                                                                                      47
                                                                                                      int cur = q.front();
           return ans;
74
                                                                                      48
                                                                                                      v[cur] = 0; q.pop();
75 };
                                                                                      49
                                                                                                      if (++relax_count[cur] >= n) return false;
                                                                                      50
                                                                                                      for (const auto &e : graph[cur]) {
                                                                                                          if (iszerocap(e.residual capacity)) continue;
                                                                                      51
          Min-cost Maximum Flow
                                                                                      52
                                                                                                          auto next = e.target;
                                                                                                          auto ncost = dist[cur] + e.cost;
                                                                                      53
                                                                                      54
                                                                                                          if (dist[next] > ncost) {
1 // precondition: there is no negative cycle.
                                                                                      55
                                                                                                              dist[next] = ncost;
2 // usage:
                                                                                                              if (v[next]) continue;
                                                                                      56
3 // MinCostFlow mcf(n);
                                                                                      57
                                                                                                              v[next] = 1; q.push(next);
4 // for(each edges) mcf.addEdge(from, to, cost, capacity);
                                                                                      58
                                                                                                         }
5 // mcf.solve(source, sink); // min cost max flow
                                                                                      59
                                                                                                      }
6 // mcf.solve(source, sink, 0); // min cost flow
                                                                                      60
7 // mcf.solve(source, sink, goal_flow); // min cost flow with total_flow >=
                                                                                      61
                                                                                                  for (int i = 0; i < n; i++) pi[i] = dist[i];</pre>
    goal flow if possible
                                                                                      62
                                                                                                  return true;
8 struct MinCostFlow
                                                                                      63
                                                                                             }
9 {
                                                                                      64
10
       typedef int cap_t;
                                                                                      65
                                                                                             pair<cost_t, cap_t> AugmentShortest(int s, int e, cap_t flow_limit) {
11
       typedef int cost t;
                                                                                      66
                                                                                                  auto infinite cost = numeric limits<cost t>::max();
12
                                                                                      67
                                                                                                  auto infinite flow = numeric limits<cap t>::max();
13
       bool iszerocap(cap_t cap) { return cap == 0; }
                                                                                      68
                                                                                                  typedef pair<cost_t, int> pq_t;
14
                                                                                      69
                                                                                                  priority_queue<pq_t, vector<pq_t>, greater<pq_t>> pq;
15
      struct edge {
                                                                                                  vector<pair<cost_t, cap_t>> dist(n, make_pair(infinite_cost, 0));
                                                                                      70
16
           int target;
                                                                                      71
                                                                                                  vector<int> from(n, -1), v(n);
17
           cost t cost;
                                                                                      72
18
           cap t residual capacity;
                                                                                      73
                                                                                                  if (needNormalize || (ranbefore && lastStart != s))
19
           cap_t orig_capacity;
                                                                                      74
                                                                                                      normalize(s):
20
           size_t revid;
                                                                                      75
                                                                                                  ranbefore = true;
21
      };
                                                                                      76
                                                                                                  lastStart = s;
22
                                                                                      77
23
       int n;
                                                                                      78
                                                                                                  dist[s] = pair<cost_t, cap_t>(0, infinite_flow);
24
       vector<vector<edge>> graph;
                                                                                      79
                                                                                                  pq.emplace(dist[s].first, s);
25
       vector<cost t> pi;
                                                                                      80
                                                                                                  while(!pq.empty()) {
26
       bool needNormalize, ranbefore;
                                                                                      81
                                                                                                      auto cur = pq.top().second; pq.pop();
27
       int lastStart;
                                                                                                      if (v[cur]) continue;
                                                                                      82
28
                                                                                      83
                                                                                                      v[cur] = 1;
29
      MinCostFlow(int n): graph(n), n(n), pi(n, 0), needNormalize(false),
                                                                                                      if (cur == e) continue;
        ranbefore(false) {}
                                                                                      85
                                                                                                      for (const auto &e : graph[cur]) {
30
       void addEdge(int s, int e, cost_t cost, cap_t cap)
                                                                                                          auto next = e.target;
31
```

```
87
                    if (v[next]) continue;
                                                                                         6 // for (each edge) mc.addEdge(a,b,weight);
 88
                    if (iszerocap(e.residual_capacity)) continue;
                                                                                         7 // mincut = mc.solve();
 89
                    auto ncost = dist[cur].first + e.cost - pi[next] + pi[cur];
                                                                                         8 // mc.cut = {0,1}^n describing which side the vertex belongs to.
 90
                    auto nflow = min(dist[cur].second, e.residual capacity);
                                                                                         9 struct MinCutMatrix
 91
                    if (dist[next].first <= ncost) continue;</pre>
                                                                                        10 {
 92
                    dist[next] = make pair(ncost, nflow);
                                                                                        11
                                                                                               typedef int cap t;
 93
                    from[next] = e.revid;
                                                                                        12
 94
                    pq.emplace(dist[next].first, next);
                                                                                        13
                                                                                               vector<vector<cap_t>> graph;
 95
                }
                                                                                        14
 96
                                                                                        15
                                                                                               void init(int _n) {
 97
            /** augment the shortest path **/
                                                                                        16
                                                                                                    n = n;
                                                                                        17
 98
            auto p = e;
                                                                                                    graph = vector<vector<cap t>>(n, vector<cap t>(n, 0));
 99
            auto pathcost = dist[p].first + pi[p] - pi[s];
                                                                                        18
100
            auto flow = dist[p].second;
                                                                                        19
                                                                                               void addEdge(int a, int b, cap t w) {
101
            if (iszerocap(flow)|| (flow_limit <= 0 && pathcost >= 0)) return pair
                                                                                        20
                                                                                                    if (a == b) return;
              cost t, cap t>(0, 0);
                                                                                        21
                                                                                                    graph[a][b] += w;
102
            if (flow limit > 0) flow = min(flow, flow limit);
                                                                                        22
                                                                                                    graph[b][a] += w;
                                                                                        23
103
            /* update potential */
                                                                                               }
104
            for (int i = 0; i < n; i++) {
                                                                                        24
105
                if (iszerocap(dist[i].second)) continue;
                                                                                        25
                                                                                               pair<cap t, pair<int, int>> stMinCut(vector<int> &active) {
106
                pi[i] += dist[i].first;
                                                                                        26
                                                                                                    vector<cap_t> key(n);
                                                                                        27
107
                                                                                                    vector<int> v(n);
                                                                                        28
                                                                                                    int s = -1, t = -1;
108
            while (from[p] != -1) {
109
                                                                                        29
                                                                                                    for (int i = 0; i < active.size(); i++) {</pre>
                auto nedge = from[p];
110
                auto np = graph[p][nedge].target;
                                                                                        30
                                                                                                        cap t maxv = -1;
111
                auto fedge = graph[p][nedge].revid;
                                                                                        31
                                                                                                        int cur = -1;
112
                graph[p][nedge].residual_capacity += flow;
                                                                                        32
                                                                                                        for (auto j : active) {
                                                                                        33
                                                                                                            if (v[i] == 0 && maxv < key[i]) {</pre>
113
                graph[np][fedge].residual capacity -= flow;
                                                                                        34
114
                p = np;
                                                                                                                maxv = key[j];
115
                                                                                        35
                                                                                                                cur = j;
116
            return make_pair(pathcost * flow, flow);
                                                                                        36
                                                                                                            }
117
       }
                                                                                        37
118
                                                                                        38
                                                                                                        t = s; s = cur;
119
        pair<cost_t,cap_t> solve(int s, int e, cap_t flow_minimum = numeric_limits
                                                                                                        v[cur] = 1;
          cap t>::max()) {
                                                                                                        for (auto j : active) key[j] += graph[cur][j];
120
            cost t total cost = 0;
                                                                                        41
121
            cap_t total_flow = 0;
                                                                                        42
                                                                                                    return make_pair(key[s], make_pair(s, t));
122
            for(;;) {
                                                                                        43
123
                auto res = AugmentShortest(s, e, flow_minimum - total_flow);
                                                                                        44
124
                if (res.second <= 0) break;</pre>
                                                                                        45
                                                                                               vector<int> cut;
                total cost += res.first;
125
                                                                                        46
126
                total flow += res.second;
                                                                                        47
                                                                                               cap t solve() {
127
                                                                                        48
                                                                                                    cap_t res = numeric_limits<cap_t>::max();
128
            return make pair(total cost, total flow);
                                                                                        49
                                                                                                    vector<vector<int>> grps;
129
                                                                                        50
                                                                                                    vector<int> active;
                                                                                        51
130 };
                                                                                                    cut.resize(n);
                                                                                        52
                                                                                                    for (int i = 0; i < n; i++) grps.emplace_back(1, i);</pre>
                                                                                        53
                                                                                                    for (int i = 0; i < n; i++) active.push back(i);</pre>
   5.11 General Min-cut (Stoer-Wagner)
                                                                                        54
                                                                                                    while (active.size() >= 2) {
                                                                                        55
                                                                                                        auto stcut = stMinCut(active);
                                                                                        56
                                                                                                        if (stcut.first < res) {</pre>
 1 // implementation of Stoer-Wagner algorithm
                                                                                        57
                                                                                                            res = stcut.first;
 2 // O(V^3)
                                                                                        58
                                                                                                            fill(cut.begin(), cut.end(), 0);
 3 //usage
                                                                                        59
                                                                                                            for (auto v : grps[stcut.second.first]) cut[v] = 1;
 4 // MinCut mc;
                                                                                                        }
 5 // mc.init(n);
```

```
61
62
               int s = stcut.second.first, t = stcut.second.second;
63
               if (grps[s].size() < grps[t].size()) swap(s, t);</pre>
65
               active.erase(find(active.begin(), active.end(), t));
               grps[s].insert(grps[s].end(), grps[t].begin(), grps[t].end());
               for (int i = 0; i < n; i++) { graph[i][s] += graph[i][t]; graph[i][t 40 inline double inner(const Point& a, const Point& b) {
67
                1 = 0;
68
               for (int i = 0; i < n; i++) { graph[s][i] += graph[t][i]; graph[t][i 42 }</pre>
                ] = 0; }
69
               graph[s][s] = 0;
70
71
           return res;
72
73 };
```

Geometry

6.1 Basic Operations

```
1 const double eps = 1e-9;
3 inline int diff(double lhs, double rhs) {
       if (lhs - eps < rhs && rhs < lhs + eps) return 0;
       return (lhs < rhs) ? -1 : 1;
6 }
8 inline bool is_between(double check, double a, double b) {
       if (a < b)
10
           return (a - eps < check && check < b + eps);</pre>
11
12
           return (b - eps < check && check < a + eps);</pre>
13 }
14
15 struct Point {
16
       double x, v;
17
       bool operator==(const Point& rhs) const {
18
           return diff(x, rhs.x) == 0 && diff(y, rhs.y) == 0;
19
20
       Point operator+(const Point& rhs) const {
21
           return Point{ x + rhs.x, y + rhs.y };
22
23
       Point operator-(const Point& rhs) const {
24
           return Point{ x - rhs.x, y - rhs.y };
25
26
       Point operator*(double t) const {
27
           return Point{ x * t, y * t };
28
       }
29 };
30
31 struct Circle {
32
       Point center;
       double r;
```

```
Point pos, dir;
38 };
       return a.x * b.x + a.y * b.y;
44 inline double outer(const Point& a, const Point& b) {
       return a.x * b.y - a.y * b.x;
46 }
48 inline int ccw_line(const Line& line, const Point& point) {
       return diff(outer(line.dir, point - line.pos), 0);
50 }
51
52 inline int ccw(const Point& a, const Point& b, const Point& c) {
       return diff(outer(b - a, c - a), 0);
54 }
55
56 inline double dist(const Point& a, const Point& b) {
       return sqrt(inner(a - b, a - b));
58 }
59
60 inline double dist2(const Point &a, const Point &b) {
       return inner(a - b, a - b);
62 }
63
64 inline double dist(const Line& line, const Point& point, bool segment = false) {
       double c1 = inner(point - line.pos, line.dir);
       if (segment && diff(c1, 0) <= 0) return dist(line.pos, point);</pre>
66
       double c2 = inner(line.dir, line.dir);
67
       if (segment && diff(c2, c1) <= 0) return dist(line.pos + line.dir, point);</pre>
68
69
       return dist(line.pos + line.dir * (c1 / c2), point);
70 }
71
72 bool get_cross(const Line& a, const Line& b, Point& ret) {
       double mdet = outer(b.dir, a.dir);
73
74
       if (diff(mdet, 0) == 0) return false;
75
       double t2 = outer(a.dir, b.pos - a.pos) / mdet;
76
       ret = b.pos + b.dir * t2;
77
       return true;
78 }
80 bool get_segment_cross(const Line& a, const Line& b, Point& ret) {
81
       double mdet = outer(b.dir, a.dir);
82
       if (diff(mdet, 0) == 0) return false;
83
       double t1 = -outer(b.pos - a.pos, b.dir) / mdet;
       double t2 = outer(a.dir, b.pos - a.pos) / mdet;
85
       if (!is between(t1, 0, 1) || !is between(t2, 0, 1)) return false;
86
       ret = b.pos + b.dir * t2;
87
       return true;
```

34 };

36 struct Line {

35

37

```
143
                                                                                                 Line{ Point{ tmp / cdiff.x, 0 }, Point{ -cdiff.y, cdiff.x } });
 90 Point inner_center(const Point &a, const Point &b, const Point &c) {
                                                                                     144 }
                                                                                     145
 91
        double wa = dist(b, c), wb = dist(c, a), wc = dist(a, b);
        double w = wa + wb + wc;
                                                                                     146 Circle circle from 3pts(const Point& a, const Point& b, const Point& c) {
 92
        return Point{ (wa * a.x + wb * b.x + wc * c.x) / w, (wa * a.y + wb * b.y +
                                                                                             Point ba = b - a, cb = c - b;
                                                                                     147
         wc * c.y) / w };
                                                                                     148
                                                                                             Line p\{(a + b) * 0.5, Point\{ba.y, -ba.x\}\};
                                                                                             Line q\{ (b + c) * 0.5, Point\{ cb.y, -cb.x \} \};
 94 }
                                                                                     149
 95
                                                                                     150
                                                                                             Circle circle;
 96 Point outer center(const Point &a, const Point &b, const Point &c) {
                                                                                     151
                                                                                             if (!get cross(p, q, circle.center))
       Point d1 = b - a, d2 = c - a;
                                                                                     152
                                                                                                 circle.r = -1;
 98
        double area = outer(d1, d2);
                                                                                     153
                                                                                             else
        double dx = d1.x * d1.x * d2.y - d2.x * d2.x * d1.y
                                                                                     154
99
                                                                                                 circle.r = dist(circle.center, a);
            + d1.y * d2.y * (d1.y - d2.y);
100
                                                                                     155
                                                                                             return circle;
101
        double dy = d1.y * d1.y * d2.x - d2.y * d2.y * d1.x
                                                                                     156 }
102
            + d1.x * d2.x * (d1.x - d2.y);
                                                                                     157
        return Point{ a.x + dx / area / 2.0, a.y - dy / area / 2.0 };
103
                                                                                     158 Circle circle from 2pts rad(const Point& a, const Point& b, double r) {
104 }
                                                                                     159
                                                                                             double det = r * r / dist2(a, b) - 0.25;
105
                                                                                     160
                                                                                             Circle circle;
106 vector<Point> circle line(const Circle& circle, const Line& line) {
                                                                                     161
                                                                                             if (det < 0)
        vector<Point> result:
107
                                                                                     162
                                                                                                 circle.r = -1:
108
        double a = 2 * inner(line.dir, line.dir);
                                                                                     163
                                                                                             else {
        double b = 2 * (line.dir.x * (line.pos.x - circle.center.x)
109
                                                                                     164
                                                                                                 double h = sqrt(det);
            + line.dir.y * (line.pos.y - circle.center.y));
110
                                                                                     165
                                                                                                 // center is to the left of a->b
        double c = inner(line.pos - circle.center, line.pos - circle.center)
                                                                                     166
                                                                                                 circle.center = (a + b) * 0.5 + Point{a.v - b.v, b.x - a.x} * h;
111
112
            - circle.r * circle.r:
                                                                                     167
                                                                                                 circle.r = r:
113
        double det = b * b - 2 * a * c;
                                                                                     168
114
       int pred = diff(det, 0);
                                                                                     169
                                                                                             return circle;
       if (pred == 0)
                                                                                     170 }
115
            result.push back(line.pos + line.dir * (-b / a));
116
117
       else if (pred > 0) {
                                                                                               Compare angles
118
            det = sqrt(det);
119
            result.push_back(line.pos + line.dir * ((-b + det) / a));
            result.push back(line.pos + line.dir * ((-b - det) / a));
120
                                                                                         6.3 Convex Hull
121
        return result;
122
123 }
                                                                                       1 // find convex hull
124
                                                                                       2 // O(n*Logn)
125 vector<Point> circle circle(const Circle& a, const Circle& b) {
                                                                                       3 vector<Point> convex hull(vector<Point>& dat) {
126
        vector<Point> result;
                                                                                             if (dat.size() <= 3) return dat;</pre>
        int pred = diff(dist(a.center, b.center), a.r + b.r);
127
                                                                                             vector<Point> upper, lower;
128
       if (pred > 0) return result;
                                                                                             sort(dat.begin(), dat.end(), [](const Point& a, const Point& b) {
129
       if (pred == 0) {
                                                                                                 return (a.x == b.x)? a.y < b.y: a.x < b.x;
            result.push back((a.center * b.r + b.center * a.r) * (1 / (a.r + b.r));
130
                                                                                             });
131
            return result;
                                                                                             for (const auto& p : dat) {
132
                                                                                      10
                                                                                                 while (upper.size() >= 2 && ccw(*++upper.rbegin(), *upper.rbegin(), p)
        double aa = a.center.x * a.center.x + a.center.y * a.center.y - a.r * a.r;
133
                                                                                                   >= 0) upper.pop_back();
134
        double bb = b.center.x * b.center.x + b.center.y * b.center.y - b.r * b.r;
                                                                                                 while (lower.size() >= 2 && ccw(*++lower.rbegin(), *lower.rbegin(), p)
        double tmp = (bb - aa) / 2.0;
135
                                                                                                   <= 0) lower.pop back();
       Point cdiff = b.center - a.center;
136
                                                                                      12
                                                                                                 upper.emplace back(p);
137
       if (diff(cdiff.x, 0) == 0) {
                                                                                      13
                                                                                                 lower.emplace back(p);
138
            if (diff(cdiff.y, 0) == 0)
                                                                                      14
139
                return result; // if (diff(a.r, b.r) == 0): same circle
                                                                                      15
                                                                                             upper.insert(upper.end(), ++lower.rbegin(), --lower.rend());
            return circle_line(a, Line{ Point{ 0, tmp / cdiff.y }, Point{ 1, 0 } }); 16
140
                                                                                             return upper;
141
                                                                                      17 }
142
        return circle_line(a,
```

6.4 Polygon Cut

```
1 // left side of a->b
2 vector<Point> cut_polygon(const vector<Point>& polygon, Line line) {
       if (!polygon.size()) return polygon;
       typedef vector<Point>::const_iterator piter;
       piter la, lan, fi, fip, i, j;
      la = lan = fi = fip = polygon.end();
      i = polygon.end() - 1;
      bool lastin = diff(ccw_line(line, polygon[polygon.size() - 1]), 0) > 0;
9
       for (j = polygon.begin(); j != polygon.end(); j++) {
10
           bool thisin = diff(ccw_line(line, *j), 0) > 0;
11
           if (lastin && !thisin) {
12
               la = i;
13
               lan = j;
14
15
           if (!lastin && thisin) {
               fi = j;
16
17
               fip = i;
18
19
           i = j;
20
           lastin = thisin;
21
22
      if (fi == polygon.end()) {
23
           if (!lastin) return vector<Point>();
24
           return polygon;
25
      }
26
      vector<Point> result;
27
       for (i = fi ; i != lan ; i++) {
28
           if (i == polygon.end()) {
29
               i = polygon.begin();
30
               if (i == lan) break;
31
32
           result.push_back(*i);
33
34
      Point lc, fc;
35
       get_cross(Line{ *la, *lan - *la }, line, lc);
36
       get cross(Line{ *fip, *fi - *fip }, line, fc);
37
      result.push_back(lc);
38
       if (diff(dist2(lc, fc), 0) != 0) result.push_back(fc);
39
       return result;
40 }
```

6.5 Pick's theorem

격자점으로 구성된 simple polygon이 주어짐. i는 polygon 내부의 격자점 수, b는 polygon 11 선분 위 격자점 수, A는 polygon의 넓이라고 할 때, 다음과 같은 식이 성립한다.

$$A = i + \frac{b}{2} - 1$$

7 String

7.1 KMP

```
1 typedef vector<int> seg t;
 3 void calculate pi(vector<int>& pi, const seg t& str) {
       pi[0] = -1;
       for (int i = 1, j = -1; i < str.size(); i++) {
           while (j >= 0 && str[i] != str[j + 1]) j = pi[j];
           if (str[i] == str[j + 1])
 8
               pi[i] = ++j;
9
           else
10
               pi[i] = -1;
11
       }
12 }
13
14 // returns all positions matched
15 // O(|text|+|pattern|)
16 vector<int> kmp(const seq_t& text, const seq_t& pattern) {
       vector<int> pi(pattern.size()), ans;
       if (pattern.size() == 0) return ans;
18
19
       calculate_pi(pi, pattern);
20
       for (int i = 0, j = -1; i < text.size(); i++) {
21
           while (j >= 0 && text[i] != pattern[j + 1]) j = pi[j];
22
           if (text[i] == pattern[j + 1]) {
23
               j++;
24
               if (j + 1 == pattern.size()) {
25
                   ans.push_back(i - j);
26
                   j = pi[j];
27
               }
28
           }
29
       }
30
       return ans;
31 }
```

7.2 Aho-Corasick

```
1 #include <algorithm>
2 #include <vector>
 3 #include <queue>
 4 using namespace std;
 6 struct AhoCorasick
7 {
       const int alphabet;
9
       struct node {
           node() {}
           explicit node(int alphabet) : next(alphabet) {}
           vector<int> next, report;
12
13
           int back = 0, output_link = 0;
       };
14
       int maxid = 0;
```

```
16
       vector<node> dfa;
                                                                                      3 // calculates suffix array.
17
       explicit AhoCorasick(int alphabet) : alphabet(alphabet), dfa(1, node(
                                                                                      4 // O(n*logn)
                                                                                      5 vector<int> suffix array(const vector<T>& in) {
18
       template<typename InIt, typename Fn> void add(int id, InIt first, InIt last,
                                                                                            int n = (int)in.size(), c = 0;
                                                                                            vector<int> temp(n), pos2bckt(n), bckt(n), bpos(n), out(n);
          Fn func) {
19
           int cur = 0;
                                                                                            for (int i = 0; i < n; i++) out[i] = i;
           for ( ; first != last; ++first) {
                                                                                      9
20
                                                                                            sort(out.begin(), out.end(), [&](int a, int b) { return in[a] < in[b]; });</pre>
21
               auto s = func(*first);
                                                                                     10
                                                                                            for (int i = 0; i < n; i++) {
22
               if (auto next = dfa[cur].next[s]) cur = next;
                                                                                     11
                                                                                                 bckt[i] = c;
23
                                                                                     12
                                                                                                 if (i + 1 == n || in[out[i]] != in[out[i + 1]]) c++;
24
                                                                                     13
                   cur = dfa[cur].next[s] = (int)dfa.size();
25
                   dfa.emplace_back(alphabet);
                                                                                     14
                                                                                            for (int h = 1; h < n && c < n; h <<= 1) {
26
               }
                                                                                     15
                                                                                                 for (int i = 0; i < n; i++) pos2bckt[out[i]] = bckt[i];</pre>
                                                                                                 for (int i = n - 1; i >= 0; i--) bpos[bckt[i]] = i;
27
                                                                                     16
28
           dfa[cur].report.push_back(id);
                                                                                     17
                                                                                                 for (int i = 0; i < n; i++)</pre>
29
                                                                                     18
           maxid = max(maxid, id);
                                                                                                     if (out[i] >= n - h) temp[bpos[bckt[i]]++] = out[i];
30
                                                                                     19
                                                                                                 for (int i = 0; i < n; i++)
31
                                                                                     20
                                                                                                     if (out[i] >= h) temp[bpos[pos2bckt[out[i] - h]]++] = out[i] - h;
       void build() {
32
                                                                                     21
           queue<int> q;
33
                                                                                     22
           vector<char> visit(dfa.size());
                                                                                                 for (int i = 0; i + 1 < n; i++) {
34
                                                                                     23
                                                                                                     int a = (bckt[i] != bckt[i + 1]) || (temp[i] >= n - h)
           visit[0] = 1;
35
           q.push(0);
                                                                                     24
                                                                                                             while(!q.empty()) {
                                                                                     25
36
                                                                                                     bckt[i] = c;
37
                                                                                     26
               auto cur = q.front(); q.pop();
                                                                                                     c += a;
                                                                                     27
38
               dfa[cur].output link = dfa[cur].back;
39
               if (dfa[dfa[cur].back].report.empty())
                                                                                     28
                                                                                                 bckt[n - 1] = c++;
                   dfa[cur].output_link = dfa[dfa[cur].back].output_link;
                                                                                     29
                                                                                                 temp.swap(out);
                                                                                     30
41
               for (int s = 0; s < alphabet; s++) {</pre>
42
                   auto &next = dfa[cur].next[s];
                                                                                     31
                                                                                            return out;
43
                   if (next == 0) next = dfa[dfa[cur].back].next[s];
                                                                                     32 }
                   if (visit[next]) continue;
                   if (cur) dfa[next].back = dfa[dfa[cur].back].next[s];
45
                                                                                     34 // calculates lcp array. it needs suffix array & original sequence.
46
                   visit[next] = 1;
                                                                                     35 // O(n)
47
                   q.push(next);
                                                                                     36 vector<int> lcp(const vector<T>& in, const vector<int>& sa) {
48
               }
                                                                                            int n = (int)in.size();
49
           }
                                                                                     38
                                                                                            if (n == 0) return vector<int>();
50
                                                                                     39
                                                                                            vector<int> rank(n), height(n - 1);
51
       template<typename InIt, typename Fn> vector<int> countMatch(InIt first, InIt 40
                                                                                            for (int i = 0; i < n; i++) rank[sa[i]] = i;
         last, Fn func) {
                                                                                     41
                                                                                            for (int i = 0, h = 0; i < n; i++) {
           int cur = 0;
                                                                                     42
52
                                                                                                 if (rank[i] == 0) continue;
53
           vector<int> ret(maxid+1);
                                                                                     43
                                                                                                 int j = sa[rank[i] - 1];
54
                                                                                     44
           for (; first != last; ++first) {
                                                                                                 while (i + h < n \& k j + h < n \& k in[i + h] == in[j + h]) h++;
                                                                                                 height[rank[i] - 1] = h;
55
               cur = dfa[cur].next[func(*first)];
                                                                                     45
                                                                                     46
56
               for (int p = cur; p; p = dfa[p].output link)
                                                                                                 if (h > 0) h--;
57
                   for (auto id : dfa[p].report) ret[id]++;
                                                                                     47
58
                                                                                     48
                                                                                            return height;
59
           return ret;
60
61 };
```

7.3 Suffix Array with LCP

1 typedef char T;

7.5 Manacher's Algorithm

Suffix Tree

1 // find longest palindromic span for each element in str

```
2 // O(|str|)
3 void manacher(const string& str, int plen[]) {
       int r = -1, p = -1;
       for (int i = 0; i < str.length(); ++i) {</pre>
           if (i <= r)</pre>
 6
                plen[i] = min((2 * p - i >= 0) ? plen[2 * p - i] : 0, r - i);
           else
 9
                plen[i] = 0;
10
           while (i - plen[i] - 1 >= 0 && i + plen[i] + 1 < str.length()</pre>
11
                    && str[i - plen[i] - 1] == str[i + plen[i] + 1]) {
12
                plen[i] += 1;
13
14
           if (i + plen[i] > r) {
15
               r = i + plen[i];
16
               p = i;
17
           }
18
19 }
```

8 Miscellaneous

8.1 Fast I/O

```
1 namespace fio {
       const int BSIZE = 524288;
       char buffer[BSIZE];
       int p = BSIZE;
       inline char readChar() {
 6
           if(p == BSIZE) {
               fread(buffer, 1, BSIZE, stdin);
8
               p = 0;
9
10
           return buffer[p++];
11
12
       int readInt() {
13
           char c = readChar();
14
           while ((c < '0' | | c > '9') \&\& c != '-') {
15
               c = readChar();
16
17
           int ret = 0; bool neg = c == '-';
18
           if (neg) c = readChar();
19
           while (c >= '0' \&\& c <= '9') {
20
               ret = ret * 10 + c - '0';
21
               c = readChar();
22
23
           return neg ? -ret : ret;
24
25 }
```

8.2 Magic Numbers

```
소수: 10007, 10009, 10111, 31567, 70001, 1000003, 1000033, 4000037, 1000000007, 10000000009
```

8.3 Java Examples

```
1 import java.util.Scanner;
 3 public class example
4 {
       public static void main(String[] args)
           Scanner in = new Scanner(System.in);
           int T = in.nextInt();
9
           while (T --> 0)
10
11
               String str = in.next();
12
               if (str.matches("[A-F]?A+F+C+[A-F]?"))
13
                   System.out.println("Infected!");
14
15
                   System.out.println("Good");
16
17
18 }
```