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2 Math

2.1 Basic Arithmetic

```
1 typedef long long ll;
2 typedef unsigned long long ull;
4 // calculate lg2(a)
5 inline int lg2(ll a)
6 {
7
       return 63 - builtin clzll(a);
8 }
9
10 // calculate the number of 1-bits
11 inline int bitcount(ll a)
12 {
13
       return __builtin_popcountl1(a);
14 }
15
16 // calculate ceil(a/b)
|17|/|a|, |b| \le (2^63)-1 (does not dover -2^63)
18 ll ceildiv(ll a, ll b) {
      if (b < 0) return ceildiv(-a, -b);</pre>
19
20
       if (a < 0) return (-a) / b;
21
       return ((ull)a + (ull)b - 1ull) / b;
22 }
23
24 // calculate floor(a/b)
25 // |a|, |b| <= (2^63)-1 (does not cover -2^63)
26 ll floordiv(ll a, ll b) {
27
      if (b < 0) return floordiv(-a, -b);</pre>
28
      if (a >= 0) return a / b;
29
       return -(11)(((ull)(-a) + b - 1) / b);
30 }
32 // calculate a*b % m
33 // x86-64 only
34 ll large_mod_mul(ll a, ll b, ll m)
35 {
36
       return 11(( int128)a*( int128)b%m);
37 }
38
39 // calculate a*b % m
40 // |m| < 2^62, x86 available
41 // O(Logb)
42 ll large_mod_mul(ll a, ll b, ll m)
43 {
44
       a \% = m; b \% = m; 11 r = 0, v = a;
45
       while (b) {
           if (b\&1) r = (r + v) \% m;
```

```
47
           b >>= 1:
           v = (v << 1) \% m;
48
49
50
       return r;
51 }
52
53 // calculate n^k % m
54 ll modpow(ll n, ll k, ll m) {
       ll ret = 1;
56
       n \% = m;
57
       while (k) {
58
           if (k & 1) ret = large mod mul(ret, n, m);
59
           n = large_mod_mul(n, n, m);
60
           k /= 2;
61
       }
62
       return ret;
63 }
64
65 // calculate gcd(a, b)
66 ll gcd(ll a, ll b) {
       return b == 0 ? a : gcd(b, a % b);
68 }
69
70 // find a pair (c, d) s.t. ac + bd = gcd(a, b)
71 pair<ll, ll> extended gcd(ll a, ll b) {
      if (b == 0) return { 1, 0 };
72
73
       auto t = extended_gcd(b, a % b);
       return { t.second, t.first - t.second * (a / b) };
75 }
76
77 // find x in [0,m) s.t. ax === gcd(a, m) \pmod{m}
78 ll modinverse(ll a, ll m) {
       return (extended gcd(a, m).first % m + m) % m;
79
80 }
82 // calculate modular inverse for 1 ~ n
83 void calc_range_modinv(int n, int mod, int ret[]) {
84
       ret[1] = 1;
85
       for (int i = 2; i <= n; ++i)
           ret[i] = (11)(mod - mod/i) * ret[mod%i] % mod;
86
87 }
```

2.2 Sieve Methods: Prime, Divisor, Euler phi

```
1 // find prime numbers in 1 ~ n
2 // ret[x] = false -> x is prime
3 // O(n*loglogn)
4 void sieve(int n, bool ret[]) {
5    for (int i = 2; i * i <= n; ++i)
6         if (!ret[i])
7         for (int j = i * i; j <= n; j += i)
8               ret[i] = true;
9 }
10</pre>
```

```
11 // calculate number of divisors for 1 \sim n
12 // when you need to calculate sum, change += 1 to += i
13 // O(n*Loan)
14 void num of divisors(int n, int ret[]) {
      for (int i = 1; i <= n; ++i)
           for (int j = i; j <= n; j += i)
17
               ret[i] += 1;
18 }
20 // calculate euler totient function for 1 ~ n
21 // phi(n) = number of x s.t. 0 < x < n && gcd(n, x) = 1
22 // O(n*LogLogn)
23 void euler phi(int n, int ret[]) {
      for (int i = 1; i <= n; ++i) ret[i] = i;
25
      for (int i = 2; i <= n; ++i)
26
           if (ret[i] == i)
27
               for (int j = i; j <= n; j += i)
28
                   ret[j] -= ret[j] / i;
29 }
```

2.3 Primality Test

```
1 bool test witness(ull a, ull n, ull s) {
      if (a >= n) a %= n;
      if (a <= 1) return true;
      ull d = n \gg s:
      ull x = modpow(a, d, n);
      if (x == 1 || x == n-1) return true;
       while (s-- > 1) {
           x = large_mod_mul(x, x, n);
           if (x == 1) return false;
9
10
           if (x == n-1) return true;
11
12
       return false;
13 }
14
15 // test whether n is prime
16 // based on miller-rabin test
17 // O(logn*logn)
18 bool is_prime(ull n) {
19
      if (n == 2) return true;
      if (n < 2 || n % 2 == 0) return false;</pre>
20
21
22
      ull d = n \gg 1, s = 1;
23
      for(; (d&1) == 0; s++) d >>= 1;
24
25 #define T(a) test witness(a##ull, n, s)
      if (n < 4759123141ull) return T(2) && T(7) && T(61);</pre>
26
27
       return T(2) && T(325) && T(9375) && T(28178)
           && T(450775) && T(9780504) && T(1795265022);
28
29 #undef T
30 }
```

2.4 Chinese Remainder Theorem

```
1 // find x s.t. x === a[0] (mod n[0])
2 //
                     === a[1] \pmod{n[1]}
3 //
4 // assumption: qcd(n[i], n[i]) = 1
5 ll chinese remainder(ll* a, ll* n, int size) {
      if (size == 1) return *a;
      ll tmp = modinverse(n[0], n[1]);
      ll tmp2 = (tmp * (a[1] - a[0]) % n[1] + n[1]) % n[1];
9
      ll ora = a[1];
10
      11 tgcd = gcd(n[0], n[1]);
      a[1] = a[0] + n[0] / tgcd * tmp2;
11
12
      n[1] *= n[0] / tgcd;
13
      11 ret = chinese remainder(a + 1, n + 1, size - 1);
14
      n[1] /= n[0] / tgcd;
15
      a[1] = ora;
16
      return ret:
17 }
```

2.5 Modular Equation

 $x \equiv a \pmod{m}, x \equiv b \pmod{n}$ 을 만족시키는 x를 구하는 방법.

m과 n을 소인수분해한 후 소수의 제곱꼴의 합동식들로 각각 쪼갠다. 이 때 특정 소수에 대하여 모순이 생기면 불가능한 경우고, 모든 소수에 대해서 모순이 생기지 않으면 전체식을 CRT로 합치면 된다. 이제 $x\equiv x_1\pmod{p^{k_1}}$ 과 $x\equiv x_2\pmod{p^{k_2}}$ 가 모순이 생길조건은 $k_1\leq k_2$ 라고 했을 때, $x_1\not\equiv x_2\pmod{p^{k_1}}$ 인 경우이다. 모순이 생기지 않았을 때답을 구하려면 CRT로 합칠 때 $x\equiv x_2\pmod{p^{k_2}}$ 만을 남기고 합쳐주면 된다.

2.6 Rational Number Class

```
1 struct rational {
       long long p, q;
       void red() {
           if (q < 0) {
               p = -p;
               q = -q;
8
9
           11 t = gcd((p >= 0 ? p : -p), q);
10
           p /= t;
11
           q /= t;
12
13
14
       rational(): p(0), q(1) {}
15
       rational(long long p_): p(p_), q(1) {}
16
       rational(long long p_, long long q_): p(p_), q(q_) { red(); }
17
18
       bool operator==(const rational& rhs) const {
           return p == rhs.p && q == rhs.q;
19
```

```
20
21
       bool operator!=(const rational& rhs) const {
22
           return p != rhs.p || q != rhs.q;
23
24
       bool operator<(const rational& rhs) const {</pre>
25
           return p * rhs.q < rhs.p * q;</pre>
26
27
       rational operator+(const rational& rhs) const {
28
           ll g = gcd(q, rhs.q);
29
           return rational(p * (rhs.q / g) + rhs.p * (q / g), (q / g) * rhs.q);
30
31
       rational operator-(const rational& rhs) const {
32
           11 g = gcd(q, rhs.q);
33
           return rational(p * (rhs.q / g) - rhs.p * (q / g), (q / g) * rhs.q);
34
35
       rational operator*(const rational& rhs) const {
36
           return rational(p * rhs.p, q * rhs.q);
37
38
       rational operator/(const rational& rhs) const {
39
           return rational(p * rhs.q, q * rhs.p);
40
41 };
```

2.7 Burnside's Lemma

경우의 수를 세는데, 특정 transform operation(회전, 반사, ..) 해서 같은 경우들은 하나로 ³² 친다. 전체 경우의 수는?

- 각 operation마다 이 operation을 했을 때 변하지 않는 경우의 수를 센다 (단, "아무것도 하지 않는다"라는 operation도 있어야 함!)
- 전체 경우의 수를 더한 후, operation의 수로 나눈다. (답이 맞다면 항상 나누어 떨어져야 한다)

2.8 Kirchoff's Theorem

그래프의 스패닝 트리의 개수를 구하는 정리.

무향 그래프의 Laplacian matrix L를 만든다. 이것은 (정점의 차수 대각 행렬) - (인접행렬) 이다. L에서 행과 열을 하나씩 제거한 것을 L'라 하자. 어느 행/열이든 관계 없다. 그래프의 스패닝 트리의 개수는 det(L')이다.

2.9 Fast Fourier Transform

```
void fft(int sign, int n, double *real, double *imag) {
double theta = sign * 2 * pi / n;
for (int m = n; m >= 2; m >>= 1, theta *= 2) {
    double wr = 1, wi = 0, c = cos(theta), s = sin(theta);
    for (int i = 0, mh = m >> 1; i < mh; ++i) {</pre>
```

```
for (int j = i; j < n; j += m) {
                   int k = j + mh;
                   double xr = real[j] - real[k], xi = imag[j] - imag[k];
                   real[j] += real[k], imag[j] += imag[k];
                   real[k] = wr * xr - wi * xi, imag[k] = wr * xi + wi * xr;
10
11
               double wr = wr * c - wi * s, wi = wr * s + wi * c;
12
13
               wr = \_wr, wi = \_wi;
14
15
16
       for (int i = 1, j = 0; i < n; ++i) {
17
           for (int k = n >> 1; k > (j ^= k); k >>= 1);
18
           if (j < i) swap(real[i], real[j]), swap(imag[i], imag[j]);</pre>
19
20 }
21 // Compute Poly(a)*Poly(b), write to r; Indexed from 0
22 // O(n*Logn)
23 int mult(int *a, int n, int *b, int m, int *r) {
       const int maxn = 100;
       static double ra[maxn], rb[maxn], ia[maxn], ib[maxn];
       int fn = 1;
26
27
       while (fn < n + m) fn <<= 1; // n + m: interested Length
       for (int i = 0; i < n; ++i) ra[i] = a[i], ia[i] = 0;
       for (int i = n; i < fn; ++i) ra[i] = ia[i] = 0;</pre>
29
30
       for (int i = 0; i < m; ++i) rb[i] = b[i], ib[i] = 0;
       for (int i = m; i < fn; ++i) rb[i] = ib[i] = 0;
       fft(1, fn, ra, ia);
       fft(1, fn, rb, ib);
       for (int i = 0; i < fn; ++i) {</pre>
           double real = ra[i] * rb[i] - ia[i] * ib[i];
           double imag = ra[i] * ib[i] + rb[i] * ia[i];
37
           ra[i] = real, ia[i] = imag;
       fft(-1, fn, ra, ia);
       for (int i = 0; i < fn; ++i) r[i] = (int)floor(ra[i] / fn + 0.5);</pre>
41
       return fn:
42 }
```

2.10 Matrix Operations

```
1 const int MATSZ = 100;
2
3 inline bool is_zero(double a) { return fabs(a) < 1e-9; }
4
5 // out = A^(-1), returns det(A)
6 // A becomes invalid after call this
7 // O(n^3)
8 double inverse_and_det(int n, double A[][MATSZ], double out[][MATSZ]) {
9     double det = 1;
10     for (int i = 0; i < n; i++) {
11         for (int j = 0; j < n; j++) out[i][j] = 0;
12         out[i][i] = 1;
13     }
14     for (int i = 0; i < n; i++) {</pre>
```

```
15
           if (is_zero(A[i][i])) {
16
                double maxv = 0;
17
               int maxid = -1;
18
                for (int j = i + 1; j < n; j++) {
                    auto cur = fabs(A[j][i]);
19
20
                    if (maxv < cur) {</pre>
21
                        maxv = cur;
22
                        maxid = j;
23
                    }
24
25
                if (maxid == -1 || is_zero(A[maxid][i])) return 0;
26
                for (int k = 0; k < n; k++) {
27
                    A[i][k] += A[maxid][k];
28
                    out[i][k] += out[maxid][k];
29
30
31
           det *= A[i][i];
32
           double coeff = 1.0 / A[i][i];
33
           for (int j = 0; j < n; j++) A[i][j] *= coeff;</pre>
34
           for (int j = 0; j < n; j++) out[i][j] *= coeff;</pre>
35
           for (int j = 0; j < n; j++) if (j != i) {
36
                double mp = A[j][i];
37
               for (int k = 0; k < n; k++) A[j][k] -= A[i][k] * mp;
38
                for (int k = 0; k < n; k++) out[j][k] -= out[i][k] * mp;
39
           }
40
       }
41
       return det;
42 }
```

2.11 Gaussian Elimination

```
1 const double EPS = 1e-10;
2 typedef vector<vector<double>> VVD;
4 // Gauss-Jordan elimination with full pivoting.
5 // solving systems of linear equations (AX=B)
6 // INPUT:
               a[][] = an n*n matrix
                b[][] = an n*m matrix
7 //
8 // OUTPUT: X
                      = an n*m matrix (stored in b[][])
9 //
                A^{-1} = an n*n matrix (stored in a[][])
10 // O(n^3)
11 bool gauss_jordan(VVD& a, VVD& b) {
12
       const int n = a.size();
13
       const int m = b[0].size();
       vector<int> irow(n), icol(n), ipiv(n);
14
15
16
      for (int i = 0; i < n; i++) {
17
           int pj = -1, pk = -1;
18
           for (int j = 0; j < n; j++) if (!ipiv[j])</pre>
19
               for (int k = 0; k < n; k++) if (!ipiv[k])
20
                   if (pj == -1 \mid | fabs(a[j][k]) > fabs(a[pj][pk])) { pj = j; pk = 26}
                     k: }
21
           if (fabs(a[pj][pk]) < EPS) return false; // matrix is singular</pre>
22
           ipiv[pk]++;
```

```
23
           swap(a[pj], a[pk]);
24
           swap(b[pj], b[pk]);
25
           irow[i] = pj;
26
           icol[i] = pk;
27
28
           double c = 1.0 / a[pk][pk];
29
           a[pk][pk] = 1.0;
30
           for (int p = 0; p < n; p++) a[pk][p] *= c;
31
           for (int p = 0; p < m; p++) b[pk][p] *= c;
32
           for (int p = 0; p < n; p++) if (p != pk) {
33
               c = a[p][pk];
34
               a[p][pk] = 0;
35
               for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c;
36
               for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c;
37
           }
38
39
       for (int p = n - 1; p >= 0; p --) if (irow[p] != icol[p]) {
40
           for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][icol[p]]);
41
       }
42
       return true:
43 }
```

2.12 Simplex Algorithm

```
1 // Two-phase simplex algorithm for solving linear programs of the form
          maximize
2 //
                       c^T x
3 //
          subject to
                       Ax <= b
                        x >= 0
4 //
5 // INPUT: A -- an m x n matrix
 6 //
            b -- an m-dimensional vector
7 //
             c -- an n-dimensional vector
8 //
            x -- a vector where the optimal solution will be stored
9 // OUTPUT: value of the optimal solution (infinity if unbounded
              above, nan if infeasible)
11 // To use this code, create an LPSolver object with A, b, and c as
12 // arguments. Then, call Solve(x).
13 typedef vector<double> VD;
14 typedef vector < VD > VVD;
15 typedef vector<int> VI;
16 const double EPS = 1e-9;
17
18 struct LPSolver {
19
       int m, n;
20
       VI B, N;
21
       VVD D;
22
23
       LPSolver(const VVD& A, const VD& b, const VD& c):
24
           m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2, VD(n + 2)) 
25
           for (int i = 0; i < m; i++) for (int j = 0; j < n; j++) D[i][j] = A[i][j]
            ];
           for (int i = 0; i < m; i++) { B[i] = n + i; D[i][n] = -1; D[i][n + 1] =
27
           for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
28
           N[n] = -1; D[m + 1][n] = 1;
```

```
29
      }
30
31
       void pivot(int r, int s) {
32
           double inv = 1.0 / D[r][s];
33
           for (int i = 0; i < m + 2; i++) if (i != r)
34
               for (int j = 0; j < n + 2; j++) if (j != s)
35
                   D[i][j] -= D[r][j] * D[i][s] * inv;
36
           for (int j = 0; j < n + 2; j++) if (j != s) D[r][j] *= inv;
           for (int i = 0; i < m + 2; i++) if (i != r) D[i][s] *= -inv;
37
38
           D[r][s] = inv;
39
           swap(B[r], N[s]);
40
      }
41
42
       bool simplex(int phase) {
43
           int x = phase == 1 ? m + 1 : m;
44
           while (true) {
45
               int s = -1;
               for (int j = 0; j <= n; j++) {
46
47
                   if (phase == 2 && N[j] == -1) continue;
                   if (s == -1 \mid \mid D[x][j] < D[x][s] \mid \mid D[x][j] == D[x][s] && N[j] <
                      N[s]) s = j;
49
50
               if (D[x][s] > -EPS) return true;
51
               int r = -1;
               for (int i = 0; i < m; i++) {</pre>
52
53
                   if (D[i][s] < EPS) continue;</pre>
                   if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][s] ||</pre>
55
                        (D[i][n + 1] / D[i][s]) == (D[r][n + 1] / D[r][s]) && B[i] <
                          B[r]) r = i;
56
57
               if (r == -1) return false;
58
               pivot(r, s);
59
60
      }
61
62
       double solve(VD& x) {
63
64
           for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r][n + 1]) r = i;
65
           if (D[r][n + 1] < -EPS) {
66
               pivot(r, n);
67
               if (!simplex(1) || D[m + 1][n + 1] < -EPS)
68
                   return -numeric limits<double>::infinity();
69
               for (int i = 0; i < m; i++) if (B[i] == -1) {
70
                   int s = -1;
71
                   for (int j = 0; j <= n; j++)
72
                        if (s == -1 || D[i][j] < D[i][s] || D[i][j] == D[i][s] && N[
                         j] < N[s]) s = j;
73
                   pivot(i, s);
74
               }
75
76
           if (!simplex(2))
77
               return numeric limits<double>::infinity();
78
           x = VD(n);
79
           for (int i = 0; i < m; i++) if (B[i] < n) \times [B[i]] = D[i][n + 1];
80
           return D[m][n + 1];
```

3 Data Structure

81

82 };

3.1 Order statistic tree

```
1 #include <ext/pb_ds/assoc_container.hpp>
 2 #include <ext/pb_ds/tree_policy.hpp>
 3 #include <ext/pb_ds/detail/standard_policies.hpp>
 4 #include <functional>
 5 #include <iostream>
 6 using namespace __gnu_pbds;
 7 using namespace std;
 9 // tree<key_type, value_type(set if null), comparator, ...>
10 using ordered_set = tree<int, null_type, less<int>, rb_tree_tag,
       tree_order_statistics_node_update>;
12
13 int main()
14 {
15
       ordered set X;
16
       for (int i = 1; i < 10; i += 2) X.insert(i); // 1 3 5 7 9
17
       cout << boolalpha;</pre>
       cout << *X.find_by_order(2) << endl; // 5</pre>
       cout << *X.find_by_order(4) << endl; // 9</pre>
20
       cout << (X.end() == X.find by order(5)) << endl; // true</pre>
21
22
       cout << X.order_of_key(-1) << endl; // 0</pre>
23
       cout << X.order_of_key(1) << endl; // 0</pre>
24
       cout << X.order_of_key(4) << endl; // 2</pre>
25
       X.erase(3);
26
       cout << X.order of key(4) << endl; // 1</pre>
27
       for (int t : X) printf("%d", t); // 1 5 7 9
28 }
```

3.2 Fenwick Tree

```
1 const int TSIZE = 100000;
2 int tree[TSIZE + 1];
3
4 // Returns the sum from index 1 to p, inclusive
5 int query(int p) {
6    int ret = 0;
7    for (; p > 0; p -= p & -p) ret += tree[p];
8    return ret;
9 }
10
11 // Adds val to element with index pos
12 void add(int p, int val) {
13    for (; p <= TSIZE; p += p & -p) tree[p] += val;
14 }</pre>
```

3.3 Segment Tree with Lazy Propagation

```
1 // example implementation of sum tree
2 const int TSIZE = 131072; // always 2^k form && n <= TSIZE
3 int segtree[TSIZE * 2], prop[TSIZE * 2];
4 void seg init(int nod, int 1, int r) {
                                                                                            9
       if (1 == r) segtree[nod] = dat[1];
                                                                                           10
6
       else {
                                                                                           11
           int m = (1 + r) >> 1;
                                                                                           12
8
           seg init(nod << 1, 1, m);</pre>
                                                                                           13
9
           seg init(nod \langle\langle 1 | 1, m + 1, r \rangle\rangle;
                                                                                           14
10
           segtree[nod] = segtree[nod << 1] + segtree[nod << 1 | 1];</pre>
                                                                                           15
11
                                                                                           16
12 }
                                                                                           17
13 void seg_relax(int nod, int 1, int r) {
                                                                                           18
       if (prop[nod] == 0) return;
                                                                                           19
15
       if (1 < r) {
                                                                                           20
16
           int m = (1 + r) >> 1;
                                                                                           21
17
           segtree[nod \langle\langle 1] += (m - l + 1) * prop[nod];
                                                                                           22
           prop[nod << 1] += prop[nod];</pre>
18
                                                                                           23
19
           segtree[nod \langle\langle 1 \mid 1] += (r - m) * prop[nod];
                                                                                           24
20
           prop[nod << 1 | 1] += prop[nod];</pre>
                                                                                           25
21
                                                                                           26
22
       prop[nod] = 0;
                                                                                           27
23 }
                                                                                           28
24 int seg_query(int nod, int 1, int r, int s, int e) {
                                                                                           29
       if (r < s || e < 1) return 0;
                                                                                           30
       if (s <= 1 && r <= e) return segtree[nod];</pre>
26
                                                                                           31
27
       seg relax(nod, 1, r);
                                                                                           32
28
       int m = (1 + r) >> 1;
       return seg_query(nod << 1, 1, m, s, e) + seg_query(nod << 1 | 1, m + 1, r, s
                                                                                                  }
         , e);
                                                                                           35
30 }
                                                                                           36
31 void seg update(int nod, int l, int r, int s, int e, int val) {
       if (r < s || e < 1) return;
                                                                                           38
33
       if (s <= 1 && r <= e) {
                                                                                           39
34
           segtree[nod] += (r - l + 1) * val;
                                                                                           40
35
           prop[nod] += val;
                                                                                           41
36
           return;
                                                                                           42
37
                                                                                           43
38
       seg_relax(nod, 1, r);
                                                                                           44
39
       int m = (1 + r) >> 1;
                                                                                           45
40
       seg update(nod << 1, 1, m, s, e, val);</pre>
                                                                                           46
       seg_update(nod << 1 | 1, m + 1, r, s, e, val);
                                                                                           47
42
       segtree[nod] = segtree[nod << 1] + segtree[nod << 1 | 1];</pre>
                                                                                           48
43 }
                                                                                           49
44 // usage:
                                                                                           50
45 // seg_update(1, 0, n - 1, qs, qe, val);
                                                                                           51
46 // seg_query(1, 0, n - 1, qs, qe);
                                                                                           52
                                                                                           53
   3.4 Persistent Segment Tree
                                                                                           54
                                                                                           55
                                                                                           56
```

```
1 // persistent segment tree impl: sum tree
2 namespace pstree {
```

```
typedef int val t;
const int DEPTH = 18;
const int TSIZE = 1 << 18;</pre>
const int MAX QUERY = 262144;
struct node {
    val t v;
    node *1, *r;
} npoll[TSIZE * 2 + MAX QUERY * (DEPTH + 1)];
int pptr, last q;
node *head[MAX QUERY + 1];
int q[MAX QUERY + 1];
int lqidx;
void init() {
    // zero-initialize, can be changed freely
    memset(&npoll[TSIZE - 1], 0, sizeof(node) * TSIZE);
    for (int i = TSIZE - 2; i >= 0; i--) {
        npoll[i].v = 0;
        npoll[i].l = &npoll[i*2+1];
        npoll[i].r = &npoll[i*2+2];
    head[0] = &npoll[0];
    last q = 0;
    pptr = 2 * TSIZE - 1;
    q[0] = 0;
    lqidx = 0;
// update val to pos at time t
// 0 <= t <= MAX QUERY, 0 <= pos < TSIZE
void update(int pos, int val, int t, int prev) {
    head[++last_q] = &npoll[pptr++];
    node *old = head[q[prev]], *now = head[last q];
    while (lqidx < t) q[lqidx++] = q[prev];</pre>
    q[t] = last_q;
    int flag = 1 << DEPTH;</pre>
    for (;;) {
        now->v = old->v + val;
        flag >>= 1;
        if (flag==0) {
            now->l = now->r = nullptr; break;
        if (flag & pos) {
            now->1 = old->1;
            now->r = &npoll[pptr++];
            now = now ->r, old = old ->r;
       } else {
            now->r = old->r;
            now->1 = &npoll[pptr++];
```

57

```
58
                      now = now ->1, old = old->1;
                                                                                                    32
                                                                                                            node* p = x-p;
59
                                                                                                    33
                                                                                                            node* b = nullptr;
                                                                                                    34
60
             }
                                                                                                            if (x == p->1) {
        }
                                                                                                    35
61
                                                                                                                 p->1 = b = x->r;
62
                                                                                                    36
                                                                                                                 x->r = p;
63
        val t query(int s, int e, int l, int r, node *n) {
                                                                                                    37
                                                                                                            }
             if (s == 1 \&\& e == r) return n \rightarrow v;
64
                                                                                                    38
                                                                                                            else {
65
             int m = (1 + r) / 2;
                                                                                                    39
                                                                                                                 p->r = b = x->1;
66
             if (m >= e) return query(s, e, 1, m, n->1);
                                                                                                    40
                                                                                                                 x \rightarrow 1 = p;
67
             else if (m < s) return query(s, e, m + 1, r, n->r);
                                                                                                    41
68
             else return query(s, m, l, m, n->1) + query(m + 1, e, m + 1, r, n->r);
                                                                                                            x \rightarrow p = p \rightarrow p;
69
        }
                                                                                                            p \rightarrow p = x;
70
                                                                                                    44
                                                                                                            if (b) b \rightarrow p = p;
71
        // query summation of [s, e] at time t
                                                                                                    45
                                                                                                            x->p? (p == x->p->1 ? x->p->1 : x->p->r) = x : (root = x);
72
        val_t query(int s, int e, int t) {
                                                                                                    46
                                                                                                            update(p);
73
             s = max(0, s); e = min(TSIZE - 1, e);
                                                                                                    47
                                                                                                            update(x);
74
             if (s > e) return 0;
                                                                                                    48 }
75
                                                                                                    49
             return query(s, e, 0, TSIZE - 1, head[q[t]]);
76
        }
                                                                                                    50 // make x into root
77 }
                                                                                                    51 void splav(node* x) {
                                                                                                            while (x->p) {
                                                                                                    52
                                                                                                    53
                                                                                                                 node* p = x->p;
          Splay Tree
                                                                                                    54
                                                                                                                 node* g = p->p;
                                                                                                    55
                                                                                                                 if (g) rotate((x == p \rightarrow 1) == (p == g \rightarrow 1) ? p : x);
                                                                                                    56
                                                                                                                 rotate(x);
1 // example : https://www.acmicpc.net/problem/13159
                                                                                                    57
                                                                                                            }
2 struct node {
                                                                                                    58 }
        node* 1, * r, * p;
                                                                                                    59
        int cnt, min, max, val;
                                                                                                    60 void relax_lazy(node* x) {
 5
        long long sum;
                                                                                                    61
                                                                                                            if (!x->inv) return;
 6
        bool inv;
                                                                                                    62
                                                                                                            swap(x->1, x->r);
        node(int val) :
                                                                                                    63
                                                                                                            x->inv = false;
 8
             cnt(1), sum( val), min( val), max( val), val( val), inv(false),
                                                                                                    64
                                                                                                            if (x\rightarrow 1) x\rightarrow 1\rightarrow inv = !x\rightarrow 1\rightarrow inv;
9
             l(nullptr), r(nullptr), p(nullptr) {
                                                                                                    65
                                                                                                            if (x->r) x->r->inv = !x->r->inv;
10
        }
                                                                                                    66 }
11 };
                                                                                                    67
12 node* root;
                                                                                                    68 // find kth node in splay tree
13
                                                                                                    69 void find kth(int k) {
14 void update(node* x) {
                                                                                                    70
                                                                                                            node* x = root;
15
        x \rightarrow cnt = 1;
                                                                                                    71
                                                                                                            relax_lazy(x);
16
        x \rightarrow sum = x \rightarrow min = x \rightarrow max = x \rightarrow val;
                                                                                                            while (true) {
                                                                                                    72
17
        if (x->1) {
                                                                                                    73
                                                                                                                 while (x->1 && x->1->cnt > k) {
18
            x\rightarrow cnt += x\rightarrow l\rightarrow cnt;
                                                                                                    74
                                                                                                                     x = x \rightarrow 1;
19
             x \rightarrow sum += x \rightarrow 1 \rightarrow sum;
                                                                                                    75
                                                                                                                      relax lazy(x);
20
             x-\min = \min(x-\min, x->l->\min);
                                                                                                    76
21
             x -> max = max(x -> max, x -> 1 -> max);
                                                                                                    77
                                                                                                                 if (x->1) k -= x->1->cnt;
22
                                                                                                    78
                                                                                                                 if (!k--) break;
23
        if (x->r) {
                                                                                                    79
                                                                                                                 x = x - r;
24
            x \rightarrow cnt += x \rightarrow r \rightarrow cnt;
                                                                                                    80
                                                                                                                 relax_lazy(x);
25
             x \rightarrow sum += x \rightarrow r \rightarrow sum;
                                                                                                    81
26
            x->min = min(x->min, x->r->min);
                                                                                                    82
                                                                                                            splay(x);
27
             x->max = max(x->max, x->r->max);
                                                                                                    83 }
28
        }
29 }
                                                                                                    85 // collect [l, r] nodes into one subtree and return its root
                                                                                                    86 node* interval(int 1, int r) {
31 void rotate(node* x) {
```

```
87
        find kth(1 - 1);
        node* x = root;
 88
 89
        root = x->r;
        root->p = nullptr;
 90
 91
        find_kth(r - l + 1);
 92
        x->r = root;
 93
        root -> p = x;
 94
        root = x;
 95
        return root->r->l;
 96 }
 97
 98 void traverse(node* x) {
 99
        relax_lazy(x);
100
        if (x->1) {
101
             traverse(x->1);
102
103
        // do something
        if (x->r) {
104
105
             traverse(x->r);
106
107 }
108
109 void uptree(node* x) {
110
        if (x->p) {
111
             uptree(x->p);
112
113
        relax_lazy(x);
114 }
```

3.6 Link/Cut Tree

4 DP

4.1 Convex Hull Optimization

4.1.1 requirement

```
O(n^2) 	o O(n \log n) 조건 1) DP 점화식 꼴 D[i] = \min_{j < i} (D[j] + b[j] * a[i]) 조건 2) b[j] \le b[j+1] 특수조건) a[i] \le a[i+1] 도 만족하는 경우, 마지막 쿼리의 위치를 저장해두면 이분검색이 필요없어지기 때문에 amortized O(n) 에 해결할 수 있음
```

4.1.2 Source Code

```
1 //0(n^3) \rightarrow 0(n^2)
3 #define sz 100001
4 long long s[sz];
5 long long dp[2][sz];
 6 //deque {index, x pos }
7 int dqi[sz];
8 long long dqm[sz];
9 //pointer to deque
10 int ql,qr;
11 //dp[i][j] = max(dp[i][k] + s[j]*s[k] - s[k]^2)
12 //let y = dp[i][j], x = s[j] -> y = max(s[k]*x + dp[i][k] - s[k]^2);
13
14 //push new value to deque
15 //i = index, x = current x pos
16 void setq(int i, int x)
17 {
18
       //a1,b1 = prv line, a2,b2 = new line
19
       int a1, a2 = s[i];
       long long b1, b2 = dp[0][i] - s[i] * s[i], r;
20
21
       //renew deque
       while (qr>=ql)
22
23
       {
24
           //last line enqueued
25
           a1 = s[dqi[qr]];
26
           b1 = dp[0][dqi[qr]] - s[dqi[qr]] * s[dqi[qr]];
27
           //tie breaking to newer one
28
           if (a1 == a2)
29
30
               dqi[qr] = i;
31
               return;
32
33
           // x intersection between last line and new line
34
           r = (b1 - b2) / (a2 - a1);
35
           if ((b1 - b2) % (a2 - a1)) r++;
36
           //last line is not needed
37
           if (r <= dqm[qr])
38
39
               qr--;
40
41
           else break;
42
       if (r < 0) r = 0;
43
       //push back new line
44
45
       if (dqm[qr] < s[n - 1] && r <= s[n - 1])
46
47
           dqi[++qr] = i;
           dqm[qr] = r;
       //discard old lines
       while (qr-ql && dqm[ql+1] <= x)
51
52
       {
53
           q1++;
54
55 }
```

```
57 int main()
58 {
59
       for (int j = 0; j < k; j++)
60
61
           ql = 0;
62
           qr = 1;
63
           dqi[0] = dqm[0] = 0;
64
           for (int i = 1; i < n; i++)
65
               //get line used by current x pos
67
               setq(i, s[i]);
68
               //line index to use
69
               int g = dqi[ql];
70
               //set dp value
71
               dp[1][i] = dp[0][g] + s[g] * (s[i] - s[g]);
72
73
           for (int i = 0; i < n; i++)
74
75
               dp[0][i] = dp[1][i];
76
               dp[1][i] = 0;
77
78
79 }
```

4.2 Divide & Conquer Optimization

```
O(kn^2) 	o O(kn\log n) 조건 1) DP 점화식 꼴 D[t][i] = \min_{j < i} (D[t-1][j] + C[j][i]) 조건 2) A[t][i] \vdash D[t][i]의 답이 되는 최소의 j라 할 때, 아래의 부등식을 만족해야 함 A[t][i] \le A[t][i+1] 조건 2-1) 비용C가 다음의 사각부등식을 만족하는 경우도 조건 2)를 만족하게 됨 C[a][c] + C[b][d] \le C[a][d] + C[b][c] \ \ (a \le b \le c \le d)
```

4.3 Knuth Optimization

```
O(n^3) \to O(n^2)
조건 1) DP 점화식 꼴 D[i][j] = \min_{i < k < j} (D[i][k] + D[k][j]) + C[i][j] 조건 2) 사각 부등식 C[a][c] + C[b][d] \le C[a][d] + C[b][c] \ \ (a \le b \le c \le d) 조건 3) 단조성
```

```
C[b][c] \le C[a][d] \ (a \le b \le c \le d)
```

결론) 조건 2, 3을 만족한다면 A[i][j]를 D[i][j]의 답이 되는 최소의 k라 할 때, 아래의 부등 식을 만족하게 됨

 $A[i][j-1] \le A[i][j] \le A[i+1][j]$

3중 루프를 돌릴 때 위 조건을 이용하면 최종적으로 시간복잡도가 $O(n^2)$ 이 됨

5 Graph

5.1 SCC (Tarjan)

```
1 const int MAXN = 100;
 2 vector<int> graph[MAXN];
 3 int up[MAXN], visit[MAXN], vtime;
 4 vector<int> stk;
 5 int scc_idx[MAXN], scc_cnt;
  void dfs(int nod) {
       up[nod] = visit[nod] = ++vtime;
9
       stk.push back(nod);
10
       for (int next : graph[nod]) {
           if (visit[next] == 0) {
11
12
               dfs(next);
13
               up[nod] = min(up[nod], up[next]);
14
15
           else if (scc idx[next] == 0)
               up[nod] = min(up[nod], visit[next]);
16
17
18
       if (up[nod] == visit[nod]) {
19
           ++scc_cnt;
20
           int t;
21
22
               t = stk.back();
23
               stk.pop_back();
24
               scc_idx[t] = scc_cnt;
25
           } while (!stk.empty() && t != nod);
26
27 }
29 // find SCCs in given directed graph
30 // O(V+E)
31 void get_scc() {
       vtime = 0;
33
       memset(visit, 0, sizeof(visit));
       scc_cnt = 0;
       memset(scc_idx, 0, sizeof(scc_idx));
35
       for (int i = 0; i < n; ++i)
           if (visit[i] == 0) dfs(i);
37
38 }
```

5.2 SCC (Kosaraju)

```
1 const int MAXN = 100;
2 vector<int> graph[MAXN], grev[MAXN];
3 int visit[MAXN], vcnt;
4 int scc idx[MAXN], scc cnt;
5 vector<int> emit;
7 void dfs(int nod, vector<int> graph[]) {
       visit[nod] = vcnt;
9
       for (int next : graph[nod]) {
           if (visit[next] == vcnt) continue;
10
11
           dfs(next, graph);
12
      }
13
       emit.push_back(nod);
14 }
16 // find SCCs in given graph
17 // O(V+E)
18 void get_scc() {
19
       scc_cnt = 0;
20
       vcnt = 1;
21
       emit.clear();
22
       memset(visit, 0, sizeof(visit));
23
24
       for (int i = 0; i < n; i++) {
25
           if (visit[i] == vcnt) continue;
26
           dfs(i, graph);
27
      }
28
29
       ++vcnt;
30
       for (auto st : vector<int>(emit.rbegin(), emit.rend())) {
31
           if (visit[st] == vcnt) continue;
32
           emit.clear();
33
           dfs(st, grev);
34
           ++scc cnt;
35
           for (auto node : emit)
36
               scc idx[node] = scc cnt;
37
38 }
```

5.3 2-SAT

 $(b_x \lor b_y) \land (\neg b_x \lor b_z) \land (b_z \lor \neg b_x) \land \cdots$ 같은 form을 2-CNF라고 함. 주어진 2-CNF 식을 37 참으로 하는 $\{b_1,b_2,\cdots\}$ 가 존재하는지, 존재한다면 그 값은 무엇인지 구하는 문제를 2-SAT 38 이라 함.

boolean variable b_i 마다 b_i 를 나타내는 정점, $\neg b_i$ 를 나타내는 정점 2개를 만듦. 각 clause $b_i \lor b_j$ 마다 $\neg b_i \to b_j$, $\neg b_j \to b_i$ 이렇게 edge를 이어줌. 그렇게 만든 그래프에서 SCC를 다 43 // 0(V+E) 구함. 어떤 SCC 안에 b_i 와 $\neg b_i$ 가 같이 포함되어있다면 해가 존재하지 않음. 아니라면 해가 44 void get_bcc() { 존재함.

해가 존재할 때 구체적인 해를 구하는 방법. 위에서 SCC를 구하면서 SCC DAG를 만들어 47

준다. 거기서 위상정렬을 한 후, 앞에서부터 SCC를 하나씩 봐준다. 현재 보고있는 SCC에 b_i 가 속해있는데 얘가 $\neg b_i$ 보다 먼저 등장했다면 b_i = false, 반대의 경우라면 b_i = true, 이미 값이 assign되었다면 pass.

5.4 BCC, Cut vertex, Bridge

```
1 const int MAXN = 100;
 2 vector<pair<int, int>> graph[MAXN]; // { next vertex id, edge id }
 3 int up[MAXN], visit[MAXN], vtime;
 4 vector<pair<int, int>> stk;
 6 int is_cut[MAXN];
                               // v is cut vertex if is_cut[v] > 0
                               // list of edge ids
7 vector<int> bridge;
 8 vector<int> bcc idx[MAXN]; // list of bccids for vertex i
9 int bcc_cnt;
10
11 void dfs(int nod, int par_edge) {
       up[nod] = visit[nod] = ++vtime;
12
13
       int child = 0;
       for (const auto& e : graph[nod]) {
15
           int next = e.first, edge id = e.second;
16
           if (edge id == par edge) continue;
17
           if (visit[next] == 0) {
18
               stk.push_back({ nod, next });
19
               ++child;
20
               dfs(next, edge_id);
21
               if (up[next] == visit[next]) bridge.push_back(edge_id);
22
               if (up[next] >= visit[nod]) {
23
                   ++bcc_cnt;
24
                   do {
25
                       auto last = stk.back();
26
                       stk.pop back();
27
                       bcc_idx[last.second].push_back(bcc_cnt);
28
                       if (last == pair<int, int>{ nod, next }) break;
29
                   } while (!stk.empty());
30
                   bcc_idx[nod].push_back(bcc_cnt);
31
                   is cut[nod]++;
32
33
               up[nod] = min(up[nod], up[next]);
34
35
           else
36
               up[nod] = min(up[nod], visit[next]);
       if (par_edge == -1 && is_cut[nod] == 1)
39
           is_cut[nod] = 0;
40 }
42 // find BCCs & cut vertexs & bridges in undirected graph
       vtime = 0:
       memset(visit, 0, sizeof(visit));
       memset(is_cut, 0, sizeof(is_cut));
```

5.5 Shortest Path Faster Algorithm

```
1 // shortest path faster algorithm
2 // average for random graph : O(E) , worst : O(VE)
4 \text{ const int MAXN} = 20001;
5 const int INF = 100000000;
6 int n, m;
7 vector<pair<int, int>> graph[MAXN];
8 bool inqueue[MAXN];
9 int dist[MAXN];
10
11 void spfa(int st) {
12
       for (int i = 0; i < n; ++i) {
13
           dist[i] = INF;
14
15
       dist[st] = 0;
16
17
       queue<int> q;
18
       q.push(st);
19
       inqueue[st] = true;
20
       while (!q.empty()) {
21
           int u = q.front();
22
           q.pop();
23
           inqueue[u] = false;
24
           for (auto& e : graph[u]) {
25
               if (dist[u] + e.second < dist[e.first]) {</pre>
26
                    dist[e.first] = dist[u] + e.second;
27
                    if (!inqueue[e.first]) {
28
                        q.push(e.first);
29
                        inqueue[e.first] = true;
30
31
               }
32
           }
33
34 }
```

5.6 Lowest Common Ancestor

```
1 const int MAXN = 100;
2 const int MAXLN = 9;
3 vector<int> tree[MAXN];
4 int depth[MAXN];
5 int par[MAXLN][MAXN];
```

```
7 void dfs(int nod, int parent) {
       for (int next : tree[nod]) {
 9
           if (next == parent) continue;
           depth[next] = depth[nod] + 1;
10
11
           par[0][next] = nod;
12
           dfs(next, nod);
13
14 }
15
16 void prepare_lca() {
17
       const int root = 0;
18
       dfs(root, -1);
19
       par[0][root] = root;
20
       for (int i = 1; i < MAXLN; ++i)
21
           for (int j = 0; j < n; ++j)
22
               par[i][j] = par[i - 1][par[i - 1][j]];
23 }
25 // find lowest common ancestor in tree between u & v
26 // assumption : must call 'prepare_lca' once before call this
27 // O(LogV)
28 int lca(int u, int v) {
29
       if (depth[u] < depth[v]) swap(u, v);</pre>
       if (depth[u] > depth[v]) {
30
31
           for (int i = MAXLN - 1; i >= 0; --i)
32
               if (depth[u] - (1 << i) >= depth[v])
33
                   u = par[i][u];
34
35
       if (u == v) return u;
36
       for (int i = MAXLN - 1; i >= 0; --i) {
37
           if (par[i][u] != par[i][v]) {
38
               u = par[i][u];
39
               v = par[i][v];
40
           }
41
42
       return par[0][u];
43 }
```

5.7 Heavy-Light Decomposition

```
1 // heavy-light decomposition
2 //
3 // hld h;
4 // insert edges to tree[0~n-1];
5 // h.init(n);
6 // h.decompose(root);
7 // h.hldquery(u, v); // edges from u to v
8 struct hld {
       static const int MAXLN = 18;
10
       static const int MAXN = 1 << (MAXLN - 1);</pre>
11
       vector<int> tree[MAXN];
       int subsize[MAXN], depth[MAXN], pa[MAXLN][MAXN];
12
13
```

```
14
       int chead[MAXN], cidx[MAXN];
                                                                                        69
                                                                                                   int diff = depth[u] - depth[v];
15
       int lchain;
                                                                                        70
                                                                                                   for (int i = logu; i >= 0; --i) {
16
                                                                                        71
       int flatpos[MAXN + 1], fptr;
                                                                                                        if ((diff >> i) & 1) u = pa[i][u];
17
                                                                                        72
18
                                                                                        73
                                                                                                   if (u == v) return u;
       void dfs(int u, int par) {
19
           pa[0][u] = par;
                                                                                        74
                                                                                        75
20
           subsize[u] = 1;
                                                                                                   for (int i = logu; i >= 0; --i) {
21
           for (int v : tree[u]) {
                                                                                        76
                                                                                                        if (pa[i][u] != pa[i][v]) {
                                                                                        77
22
               if (v == pa[0][u]) continue;
                                                                                                            u = pa[i][u];
23
               depth[v] = depth[u] + 1;
                                                                                        78
                                                                                                           v = pa[i][v];
24
                                                                                        79
               dfs(v, u);
                                                                                                       }
25
               subsize[u] += subsize[v];
                                                                                        80
26
           }
                                                                                        81
                                                                                                   return pa[0][u];
27
       }
                                                                                        82
                                                                                               }
28
                                                                                        83
29
                                                                                        84
       void init(int size)
                                                                                               // TODO: implement query functions
30
                                                                                        85
                                                                                               inline int query(int s, int e) {
31
           lchain = fptr = 0;
                                                                                        86
                                                                                                   return 0;
32
           dfs(0, -1);
                                                                                        87
                                                                                               }
33
           memset(chead, -1, sizeof(chead));
                                                                                        88
34
                                                                                        89
                                                                                               int subquery(int u, int v, int t) {
35
           for (int i = 1; i < MAXLN; i++) {</pre>
                                                                                        90
                                                                                                   int uchain, vchain = cidx[v];
                                                                                        91
36
               for (int j = 0; j < size; j++) {
                                                                                                   int ret = 0;
37
                                                                                        92
                   if (pa[i - 1][j] != -1) {
                                                                                                   for (;;) {
38
                       pa[i][j] = pa[i - 1][pa[i - 1][j]];
                                                                                        93
                                                                                                        uchain = cidx[u];
39
                                                                                        94
                                                                                                        if (uchain == vchain) {
                                                                                        95
40
               }
                                                                                                            ret += query(flatpos[v], flatpos[u]);
41
                                                                                        96
           }
                                                                                                            break;
                                                                                        97
42
       }
                                                                                                       }
43
                                                                                        98
44
                                                                                        99
                                                                                                        ret += query(flatpos[chead[uchain]], flatpos[u]);
       void decompose(int u) {
45
           if (chead[lchain] == -1) chead[lchain] = u;
                                                                                       100
                                                                                                        u = pa[0][chead[uchain]];
46
           cidx[u] = lchain;
                                                                                       101
47
           flatpos[u] = ++fptr;
                                                                                       102
                                                                                                   return ret;
48
                                                                                               }
                                                                                       103
49
           int maxchd = -1;
                                                                                       104
50
                                                                                       105
                                                                                               inline int hldquery(int u, int v) {
           for (int v : tree[u]) {
51
               if (v == pa[0][u]) continue;
                                                                                       106
                                                                                                   int p = lca(u, v);
52
               if (maxchd == -1 || subsize[maxchd] < subsize[v]) maxchd = v;</pre>
                                                                                       107
                                                                                                   return subquery(u, p) + subquery(v, p) - query(flatpos[p], flatpos[p]);
53
                                                                                       108
54
           if (maxchd != -1) decompose(maxchd);
                                                                                       109 };
55
           for (int v : tree[u]) {
56
                                                                                                 Bipartite Matching (Hopcroft-Karp)
57
               if (v == pa[0][u] || v == maxchd) continue;
58
               ++lchain; decompose(v);
59
                                                                                         1 // in: n, m, graph
60
       }
                                                                                         2 // out: match, matched
61
                                                                                         3 // vertex cover: (reached[0][left_node] == 0) || (reached[1][right_node] == 1)
62
       int lca(int u, int v) {
                                                                                         4 // O(E*sqrt(V))
63
           if (depth[u] < depth[v]) swap(u, v);</pre>
                                                                                         5 struct BipartiteMatching {
64
                                                                                               int n, m;
65
           int logu;
                                                                                         7
                                                                                               vector<vector<int>> graph;
66
           for (logu = 1; 1 << logu <= depth[u]; logu++);</pre>
                                                                                               vector<int> matched, match, edgeview, level;
67
           logu--;
                                                                                         9
                                                                                               vector<int> reached[2];
68
                                                                                               BipartiteMatching(int n, int m) : n(n), m(m), graph(n), matched(m, -1),
```

```
match(n, -1) {}
11
12
       bool assignLevel() {
13
           bool reachable = false;
14
           level.assign(n, -1);
15
           reached[0].assign(n, 0);
           reached[1].assign(m, 0);
16
17
           queue<int> q;
18
           for (int i = 0; i < n; i++) {
19
               if (match[i] == -1) {
20
                   level[i] = 0;
21
                   reached[0][i] = 1;
22
                   q.push(i);
23
               }
24
25
           while (!q.empty()) {
26
               auto cur = q.front(); q.pop();
27
               for (auto adj : graph[cur]) {
28
                   reached[1][adj] = 1;
29
                   auto next = matched[adj];
30
                   if (next == -1) {
31
                        reachable = true;
32
33
                   else if (level[next] == -1) {
34
                        level[next] = level[cur] + 1;
35
                        reached[0][next] = 1;
36
                        q.push(next);
37
                   }
38
               }
39
           }
40
           return reachable;
41
      }
42
43
       int findpath(int nod) {
44
           for (int &i = edgeview[nod]; i < graph[nod].size(); i++) {</pre>
45
               int adj = graph[nod][i];
46
               int next = matched[adj];
47
               if (next >= 0 && level[next] != level[nod] + 1) continue;
48
               if (next == -1 || findpath(next)) {
                   match[nod] = adj;
49
                   matched[adj] = nod;
50
51
                   return 1;
52
               }
53
54
           return 0;
55
       }
56
57
       int solve() {
58
           int ans = 0;
59
           while (assignLevel()) {
60
                edgeview.assign(n, 0);
61
               for (int i = 0; i < n; i++)
62
                   if (match[i] == -1)
63
                        ans += findpath(i);
64
           }
```

```
65 return ans;
66 }
67 };
```

5.9 Maximum Flow (Dinic)

```
1 // usage:
 2 // MaxFlowDinic::init(n);
3 // MaxFlowDinic::add edge(0, 1, 100, 100); // for bidirectional edge
4 // MaxFlowDinic::add_edge(1, 2, 100); // directional edge
5 // result = MaxFlowDinic::solve(0, 2); // source -> sink
6 // graph[i][edgeIndex].res -> residual
7 //
8 // in order to find out the minimum cut, use `l'.
9 // if L[i] == 0, i is unrechable.
10 //
11 // O(V*V*E)
12 // with unit capacities, O(\min(V^{(2/3)}, E^{(1/2)}) * E)
13 struct MaxFlowDinic {
14
       typedef int flow t;
15
       struct Edge {
16
           int next;
17
           int inv; /* inverse edge index */
18
           flow_t res; /* residual */
19
       };
20
21
       vector<vector<Edge>> graph;
22
       vector<int> q, 1, start;
23
24
       void init(int n) {
25
           n = n;
26
           graph.resize(n);
27
           for (int i = 0; i < n; i++) graph[i].clear();</pre>
28
29
       void add_edge(int s, int e, flow_t cap, flow_t caprev = 0) {
30
           Edge forward{ e, graph[e].size(), cap };
31
           Edge reverse{ s, graph[s].size(), caprev };
32
           graph[s].push back(forward);
33
           graph[e].push back(reverse);
34
35
       bool assign level(int source, int sink) {
36
           int t = 0;
37
           memset(&1[0], 0, sizeof(1[0]) * 1.size());
38
           1[source] = 1;
39
           q[t++] = source;
40
           for (int h = 0; h < t && !1[sink]; h++) {</pre>
41
               int cur = q[h];
42
               for (const auto& e : graph[cur]) {
43
                   if (1[e.next] || e.res == 0) continue;
44
                   l[e.next] = l[cur] + 1;
45
                   q[t++] = e.next;
46
               }
47
           return l[sink] != 0;
```

```
49
                                                                                      23
                                                                                             int n:
50
      flow_t block_flow(int cur, int sink, flow_t current) {
                                                                                      24
                                                                                             vector<vector<edge>> graph;
51
           if (cur == sink) return current;
                                                                                      25
                                                                                             vector<cost t> pi;
52
           for (int& i = start[cur]; i < graph[cur].size(); i++) {</pre>
                                                                                      26
                                                                                             bool needNormalize, ranbefore;
53
               auto& e = graph[cur][i];
                                                                                      27
                                                                                             int lastStart;
               if (e.res == 0 | | l[e.next] != l[cur] + 1) continue;
54
                                                                                      28
               if (flow t res = block flow(e.next, sink, min(e.res, current))) {
55
                                                                                      29
                                                                                             MinCostFlow(int n) : graph(n), n(n), pi(n, 0), needNormalize(false),
56
                   e.res -= res;
                                                                                                ranbefore(false) {}
                   graph[e.next][e.inv].res += res;
                                                                                             void addEdge(int s, int e, cost t cost, cap t cap)
57
                                                                                      30
58
                                                                                      31
                   return res;
59
               }
                                                                                      32
                                                                                                  if (s == e) return;
                                                                                                  edge forward={e, cost, cap, cap, graph[e].size()};
                                                                                      33
60
61
           return 0;
                                                                                      34
                                                                                                  edge backward={s, -cost, 0, 0, graph[s].size()};
                                                                                                  if (cost < 0 || ranbefore) needNormalize = true;</pre>
62
                                                                                      35
63
      flow_t solve(int source, int sink) {
                                                                                      36
                                                                                                  graph[s].emplace_back(forward);
                                                                                      37
                                                                                                  graph[e].emplace back(backward);
64
           q.resize(n);
65
           1.resize(n);
                                                                                      38
                                                                                      39
                                                                                             bool normalize(int s) {
66
           start.resize(n);
67
           flow_t ans = 0;
                                                                                      40
                                                                                                  auto infinite cost = numeric limits<cost t>::max();
           while (assign level(source, sink)) {
                                                                                                  vector<cost t> dist(n, infinite cost);
68
                                                                                      41
69
               memset(&start[0], 0, sizeof(start[0]) * n);
                                                                                      42
                                                                                                  dist[s] = 0;
               while (flow t flow = block flow(source, sink, numeric limits<flow t</pre>
70
                                                                                                  queue<int> q;
                                                                                                  vector<int> v(n), relax_count(n);
                >::max()))
                   ans += flow;
                                                                                      45
                                                                                                  v[s] = 1; q.push(s);
71
72
                                                                                                  while(!q.empty()) {
                                                                                      46
73
           return ans;
                                                                                      47
                                                                                                      int cur = q.front();
74
                                                                                      48
                                                                                                      v[cur] = 0; q.pop();
75 };
                                                                                      49
                                                                                                      if (++relax count[cur] >= n) return false;
                                                                                      50
                                                                                                      for (const auto &e : graph[cur]) {
                                                                                      51
                                                                                                          if (iszerocap(e.residual_capacity)) continue;
   5.10 Min-cost Maximum Flow
                                                                                      52
                                                                                                          auto next = e.target;
                                                                                                          auto ncost = dist[cur] + e.cost;
                                                                                      53
                                                                                      54
                                                                                                          if (dist[next] > ncost) {
1 // precondition: there is no negative cycle.
                                                                                      55
                                                                                                              dist[next] = ncost;
2 // usage:
                                                                                      56
                                                                                                              if (v[next]) continue;
3 // MinCostFlow mcf(n);
                                                                                      57
                                                                                                              v[next] = 1; q.push(next);
4 // for(each edges) mcf.addEdge(from, to, cost, capacity);
                                                                                      58
5 // mcf.solve(source, sink); // min cost max flow
                                                                                      59
                                                                                                      }
6 // mcf.solve(source, sink, 0); // min cost flow
                                                                                      60
7 // mcf.solve(source, sink, goal flow); // min cost flow with total flow >=
                                                                                      61
                                                                                                  for (int i = 0; i < n; i++) pi[i] = dist[i];</pre>
    goal flow if possible
                                                                                      62
                                                                                                  return true:
8 struct MinCostFlow
                                                                                      63
                                                                                             }
9 {
                                                                                      64
10
       typedef int cap_t;
                                                                                      65
                                                                                             pair<cost t, cap t> AugmentShortest(int s, int e, cap t flow limit) {
11
       typedef int cost_t;
                                                                                                  auto infinite_cost = numeric_limits<cost_t>::max();
                                                                                      66
12
                                                                                      67
                                                                                                  auto infinite_flow = numeric_limits<cap_t>::max();
13
       bool iszerocap(cap_t cap) { return cap == 0; }
                                                                                      68
                                                                                                  typedef pair<cost_t, int> pq_t;
14
                                                                                      69
                                                                                                  priority_queue<pq_t, vector<pq_t>, greater<pq_t>> pq;
15
       struct edge {
                                                                                                  vector<pair<cost_t, cap_t>> dist(n, make_pair(infinite_cost, 0));
                                                                                      70
16
           int target;
                                                                                      71
                                                                                                  vector<int> from(n, -1), v(n);
17
           cost t cost;
                                                                                      72
           cap_t residual_capacity;
18
                                                                                      73
                                                                                                  if (needNormalize || (ranbefore && lastStart != s))
19
           cap_t orig_capacity;
                                                                                      74
                                                                                                      normalize(s);
20
           size t revid;
                                                                                      75
                                                                                                  ranbefore = true:
21
      };
                                                                                      76
                                                                                                  lastStart = s;
22
```

```
77
                                                                                       130 };
 78
            dist[s] = pair<cost_t, cap_t>(0, infinite_flow);
 79
            pq.emplace(dist[s].first, s);
 80
            while(!pq.empty()) {
 81
                auto cur = pq.top().second; pq.pop();
 82
                if (v[cur]) continue;
 83
                v[cur] = 1;
 84
                if (cur == e) continue;
                for (const auto &e : graph[cur]) {
 85
                    auto next = e.target;
 86
 87
                    if (v[next]) continue;
                    if (iszerocap(e.residual capacity)) continue;
 88
 89
                    auto ncost = dist[cur].first + e.cost - pi[next] + pi[cur];
 90
                    auto nflow = min(dist[cur].second, e.residual capacity);
 91
                    if (dist[next].first <= ncost) continue;</pre>
                    dist[next] = make_pair(ncost, nflow);
 92
 93
                    from[next] = e.revid;
                    pq.emplace(dist[next].first, next);
 95
                }
 96
 97
            /** augment the shortest path **/
 98
            auto p = e;
 99
            auto pathcost = dist[p].first + pi[p] - pi[s];
100
            auto flow = dist[p].second;
            if (iszerocap(flow)|| (flow limit <= 0 && pathcost >= 0)) return pair
101
              cost_t, cap_t>(0, 0);
102
            if (flow limit > 0) flow = min(flow, flow limit);
            /* update potential */
103
104
            for (int i = 0; i < n; i++) {
105
                if (iszerocap(dist[i].second)) continue;
106
                pi[i] += dist[i].first;
107
108
            while (from[p] != -1) {
109
                auto nedge = from[p];
                auto np = graph[p][nedge].target;
110
111
                auto fedge = graph[p][nedge].revid;
112
                graph[p][nedge].residual_capacity += flow;
113
                graph[np][fedge].residual_capacity -= flow;
114
                p = np;
115
116
            return make pair(pathcost * flow, flow);
117
       }
118
119
        pair<cost t,cap t> solve(int s, int e, cap t flow minimum = numeric limits
          cap_t>::max()) {
120
            cost t total cost = 0;
121
            cap_t total_flow = 0;
122
            for(;;) {
123
                auto res = AugmentShortest(s, e, flow minimum - total flow);
124
                if (res.second <= 0) break;</pre>
125
                total cost += res.first;
126
                total flow += res.second;
127
128
            return make pair(total cost, total flow);
129
        }
```

5.11 General Min-cut (Stoer-Wagner)

```
1 // implementation of Stoer-Wagner algorithm
2 // O(V^3)
3 //usage
 4 // MinCut mc:
5 // mc.init(n);
6 // for (each edge) mc.addEdge(a,b,weight);
7 // mincut = mc.solve();
 8 // mc.cut = \{0,1\}^n describing which side the vertex belongs to.
9 struct MinCutMatrix
10 {
11
       typedef int cap t;
12
       int n;
13
       vector<vector<cap t>> graph;
14
15
       void init(int _n) {
16
           n = n;
17
           graph = vector<vector<cap_t>>(n, vector<cap_t>(n, 0));
18
19
       void addEdge(int a, int b, cap t w) {
20
           if (a == b) return;
21
           graph[a][b] += w;
22
           graph[b][a] += w;
23
24
25
       pair<cap_t, pair<int, int>> stMinCut(vector<int> &active) {
26
           vector<cap t> key(n);
27
           vector<int> v(n);
28
           int s = -1, t = -1;
29
           for (int i = 0; i < active.size(); i++) {</pre>
30
               cap_t maxv = -1;
31
               int cur = -1;
32
               for (auto j : active) {
33
                   if (v[j] == 0 && maxv < key[j]) {</pre>
34
                        maxv = key[j];
35
                        cur = i:
36
                   }
37
38
               t = s; s = cur;
39
               v[cur] = 1;
40
               for (auto j : active) key[j] += graph[cur][j];
41
42
           return make_pair(key[s], make_pair(s, t));
43
       }
44
45
       vector<int> cut;
46
47
       cap_t solve() {
48
           cap_t res = numeric_limits<cap_t>::max();
49
           vector<vector<int>> grps;
50
           vector<int> active;
```

```
51
           cut.resize(n);
                                                                                        27
52
                                                                                        28
           for (int i = 0; i < n; i++) grps.emplace_back(1, i);</pre>
                                                                                               int augment(int r) {
                                                                                        29
53
           for (int i = 0; i < n; i++) active.push_back(i);</pre>
                                                                                                    int h, t = 0;
54
           while (active.size() >= 2) {
                                                                                        30
                                                                                                    v[r] = ++vcnt;
55
               auto stcut = stMinCut(active);
                                                                                        31
                                                                                                    q[t++] = r;
56
               if (stcut.first < res) {</pre>
                                                                                        32
                                                                                                    for (h = 0; h < t; h ++) {
57
                   res = stcut.first;
                                                                                        33
                                                                                                        int qh = q[h];
58
                                                                                        34
                   fill(cut.begin(), cut.end(), 0);
                                                                                                        for (int j = 0; j < n; j + +) {
59
                   for (auto v : grps[stcut.second.first]) cut[v] = 1;
                                                                                        35
                                                                                                            if (dat[qh][j] != 0) continue;
60
               }
                                                                                        36
                                                                                                            int next = matched[j];
61
                                                                                        37
                                                                                                            if (next == -1) {
62
               int s = stcut.second.first, t = stcut.second.second;
                                                                                        38
                                                                                                                for (;;) {
63
               if (grps[s].size() < grps[t].size()) swap(s, t);</pre>
                                                                                        39
                                                                                                                     int org = match[qh];
64
                                                                                        40
                                                                                                                    match[qh] = j; matched[j] = qh;
65
               active.erase(find(active.begin(), active.end(), t));
                                                                                        41
                                                                                                                    if (qh == r) return 1;
66
               grps[s].insert(grps[s].end(), grps[t].begin(), grps[t].end());
                                                                                        42
                                                                                                                    qh = f[qh]; j = org;
67
               for (int i = 0; i < n; i++) { graph[i][s] += graph[i][t]; graph[i][t 43
                 1 = 0; 
               for (int i = 0; i < n; i++) { graph[s][i] += graph[t][i]; graph[t][i  45
                                                                                                            else if (v[next] != vcnt) {
68
                 1 = 0; }
                                                                                                                v[next] = vcnt, f[next] = qh, q[t++] = next;
                                                                                        47
69
               graph[s][s] = 0;
70
                                                                                        48
                                                                                                        }
71
                                                                                        49
           return res;
72
                                                                                        50
                                                                                                    for (int i = 0; i<n; i++)
73 };
                                                                                        51
                                                                                                        if (v[i] == vcnt) {
                                                                                        52
                                                                                                            reach[i] = rcnt;
                                                                                        53
                                                                                                            if (i != r) reach2[match[i]] = rcnt;
  5.12 Hungarian Algorithm
                                                                                        54
                                                                                        55
                                                                                                    return 0;
                                                                                        56
                                                                                               }
1 namespace hung {
                                                                                        57
                                                                                        58
                                                                                               cost_t solve() {
       * alternative cost t example
                                                                                        59
                                                                                                    cost t ans = 0;
                                                                                        60
                                                                                                    for (int i = 0; i<n; i++) match[i] = matched[i] = -1;</pre>
5
       typedef pair<int,int> cost_t;
                                                                                        61
                                                                                                    for (int i = 0; i<n; i++) {
       cost_t MAX_COST = make_pair(2,0);
                                                                                        62
                                                                                                        cost_t minv = *min_element(dat[i], dat[i] + n);
       pair<int,int> &operator += (pair<int,int> &a, const pair<int,int> &b) {
                                                                                        63
                                                                                                        for (int j = 0; j < n; j++) dat[i][j] -= minv;</pre>
8
           a.first += b.first; a.second += b.second;
                                                                                        64
                                                                                                        ans += minv;
9
           return a;
                                                                                        65
                                                                                                        minv = dat[0][i];
10
                                                                                        66
                                                                                                        for (int j = 1; j<n; j++) minv = min(minv, dat[j][i]);</pre>
11
       pair<int,int> &operator -= (pair<int,int> &a, const pair<int,int> &b) {
                                                                                                        for (int j = 0; j<n; j++) dat[j][i] -= minv;</pre>
12
           a.first -= b.first; a.second -= b.second;
                                                                                        68
                                                                                                        ans += minv;
13
           return a;
                                                                                        69
14
                                                                                        70
                                                                                                    for (;;) {
       */
15
                                                                                        71
                                                                                                        ++rcnt;
16
       typedef int cost t;
                                                                                        72
                                                                                                        bool needMore = false;
17
       cost_t MAX_COST = numeric_limits<cost_t>::max() / 2;
                                                                                        73
                                                                                                        for (int i = 0; i<n; i++) {
18
       // input: n, dat(which is NOT const)
                                                                                        74
                                                                                                            if (match[i] >= 0) continue;
19
       // output: call solve(), match, matched
                                                                                        75
                                                                                                            if (!augment(i)) needMore = true;
20
       // minimum matching 계산이다.
                                                                                        76
21
       const int MAX N = 500;
                                                                                        77
                                                                                                        if (!needMore) break;
22
                                                                                        78
                                                                                                        cost t minv = MAX COST;
23
       int n, match[MAX_N], matched[MAX_N];
                                                                                        79
                                                                                                        for (int i = 0; i<n; i++) {
24
       cost_t dat[MAX_N][MAX_N];
                                                                                        80
                                                                                                            if (reach[i] != rcnt) continue;
25
       int q[MAX_N], v[MAX_N], vcnt;
                                                                                        81
                                                                                                            for (int j = 0; j < n; j + +) {
26
       int f[MAX_N], reach[MAX_N], reach2[MAX_N], rcnt;
```

```
82
                        if (reach2[j] == rcnt) continue;
83
                        minv = min(minv, dat[i][j]);
84
                    }
85
86
                for (int i = 0; i<n; i++) {
87
                    if (match[i]<0) ans += minv;</pre>
                    for (int j = 0; j < n; j + +) {
89
                        if (reach[i] != rcnt) dat[i][j] += minv;
                        if (reach2[j] != rcnt) dat[i][j] -= minv;
90
91
92
                }
93
94
           return ans;
95
96 }
```

${f 6}$ Geometry

6.1 Basic Operations

```
1 const double eps = 1e-9;
3 inline int diff(double lhs, double rhs) {
       if (lhs - eps < rhs && rhs < lhs + eps) return 0;
       return (lhs < rhs) ? -1 : 1;
6 }
8 inline bool is_between(double check, double a, double b) {
       if (a < b)
10
           return (a - eps < check && check < b + eps);</pre>
11
12
           return (b - eps < check && check < a + eps);</pre>
13 }
14
15 struct Point {
16
       double x, y;
17
       bool operator==(const Point& rhs) const {
18
           return diff(x, rhs.x) == 0 && diff(y, rhs.y) == 0;
19
20
       Point operator+(const Point& rhs) const {
21
           return Point{ x + rhs.x, y + rhs.y };
22
23
       Point operator-(const Point& rhs) const {
24
           return Point{ x - rhs.x, y - rhs.y };
25
26
       Point operator*(double t) const {
27
           return Point{ x * t, y * t };
28
       }
29 };
30
31 struct Circle {
32
       Point center;
       double r;
```

```
34 };
35
36 struct Line {
37
       Point pos, dir;
38 };
39
40 inline double inner(const Point& a, const Point& b) {
       return a.x * b.x + a.y * b.y;
42 }
44 inline double outer(const Point& a, const Point& b) {
       return a.x * b.y - a.y * b.x;
46 }
48 inline int ccw_line(const Line& line, const Point& point) {
       return diff(outer(line.dir, point - line.pos), 0);
50 }
51
52 inline int ccw(const Point& a, const Point& b, const Point& c) {
       return diff(outer(b - a, c - a), 0);
54 }
55
56 inline double dist(const Point& a, const Point& b) {
       return sqrt(inner(a - b, a - b));
58 }
59
60 inline double dist2(const Point &a, const Point &b) {
       return inner(a - b, a - b);
62 }
63
64 inline double dist(const Line& line, const Point& point, bool segment = false) {
       double c1 = inner(point - line.pos, line.dir);
       if (segment && diff(c1, 0) <= 0) return dist(line.pos, point);</pre>
66
67
       double c2 = inner(line.dir, line.dir);
68
       if (segment && diff(c2, c1) <= 0) return dist(line.pos + line.dir, point);</pre>
69
       return dist(line.pos + line.dir * (c1 / c2), point);
70 }
71
72 bool get_cross(const Line& a, const Line& b, Point& ret) {
       double mdet = outer(b.dir, a.dir);
73
74
       if (diff(mdet, 0) == 0) return false;
75
       double t2 = outer(a.dir, b.pos - a.pos) / mdet;
76
       ret = b.pos + b.dir * t2;
77
       return true;
78 }
80 bool get_segment_cross(const Line& a, const Line& b, Point& ret) {
81
       double mdet = outer(b.dir, a.dir);
82
       if (diff(mdet, 0) == 0) return false;
       double t1 = -outer(b.pos - a.pos, b.dir) / mdet;
       double t2 = outer(a.dir, b.pos - a.pos) / mdet;
85
       if (!is between(t1, 0, 1) || !is between(t2, 0, 1)) return false;
86
       ret = b.pos + b.dir * t2;
87
       return true;
```

```
143
                                                                                                 Line{ Point{ tmp / cdiff.x, 0 }, Point{ -cdiff.y, cdiff.x } });
 90 Point inner_center(const Point &a, const Point &b, const Point &c) {
                                                                                     144 }
                                                                                     145
 91
        double wa = dist(b, c), wb = dist(c, a), wc = dist(a, b);
        double w = wa + wb + wc;
                                                                                     146 Circle circle from 3pts(const Point& a, const Point& b, const Point& c) {
 92
        return Point{ (wa * a.x + wb * b.x + wc * c.x) / w, (wa * a.y + wb * b.y +
                                                                                             Point ba = b - a, cb = c - b;
                                                                                     147
         wc * c.y) / w };
                                                                                     148
                                                                                             Line p\{(a + b) * 0.5, Point\{ba.y, -ba.x\}\};
                                                                                             Line q{(b + c) * 0.5, Point{cb.y, -cb.x}};
 94 }
                                                                                     149
 95
                                                                                     150
                                                                                             Circle circle;
 96 Point outer center(const Point &a, const Point &b, const Point &c) {
                                                                                     151
                                                                                             if (!get cross(p, q, circle.center))
       Point d1 = b - a, d2 = c - a;
                                                                                     152
                                                                                                 circle.r = -1;
 98
        double area = outer(d1, d2);
                                                                                     153
                                                                                             else
        double dx = d1.x * d1.x * d2.y - d2.x * d2.x * d1.y
                                                                                     154
99
                                                                                                 circle.r = dist(circle.center, a);
            + d1.y * d2.y * (d1.y - d2.y);
100
                                                                                     155
                                                                                             return circle;
101
        double dy = d1.y * d1.y * d2.x - d2.y * d2.y * d1.x
                                                                                     156 }
102
            + d1.x * d2.x * (d1.x - d2.y);
                                                                                     157
        return Point{ a.x + dx / area / 2.0, a.y - dy / area / 2.0 };
103
                                                                                     158 Circle circle from 2pts rad(const Point& a, const Point& b, double r) {
104 }
                                                                                     159
                                                                                             double det = r * r / dist2(a, b) - 0.25;
105
                                                                                     160
                                                                                             Circle circle;
106 vector<Point> circle line(const Circle& circle, const Line& line) {
                                                                                     161
                                                                                             if (det < 0)
        vector<Point> result:
107
                                                                                     162
                                                                                                 circle.r = -1:
108
        double a = 2 * inner(line.dir, line.dir);
                                                                                     163
                                                                                             else {
        double b = 2 * (line.dir.x * (line.pos.x - circle.center.x)
109
                                                                                     164
                                                                                                 double h = sqrt(det);
            + line.dir.y * (line.pos.y - circle.center.y));
110
                                                                                     165
                                                                                                 // center is to the left of a->b
        double c = inner(line.pos - circle.center, line.pos - circle.center)
                                                                                     166
                                                                                                 circle.center = (a + b) * 0.5 + Point{a.v - b.v, b.x - a.x} * h;
111
112
            - circle.r * circle.r:
                                                                                     167
                                                                                                 circle.r = r:
113
        double det = b * b - 2 * a * c;
                                                                                     168
114
       int pred = diff(det, 0);
                                                                                     169
                                                                                             return circle;
       if (pred == 0)
                                                                                     170 }
115
            result.push back(line.pos + line.dir * (-b / a));
116
117
       else if (pred > 0) {
                                                                                               Compare angles
118
            det = sqrt(det);
            result.push_back(line.pos + line.dir * ((-b + det) / a));
119
            result.push back(line.pos + line.dir * ((-b - det) / a));
120
                                                                                         6.3 Convex Hull
121
        return result;
122
123 }
                                                                                       1 // find convex hull
124
                                                                                       2 // O(n*Logn)
125 vector<Point> circle circle(const Circle& a, const Circle& b) {
                                                                                       3 vector<Point> convex hull(vector<Point>& dat) {
126
        vector<Point> result;
                                                                                             if (dat.size() <= 3) return dat;</pre>
        int pred = diff(dist(a.center, b.center), a.r + b.r);
127
                                                                                             vector<Point> upper, lower;
128
       if (pred > 0) return result;
                                                                                             sort(dat.begin(), dat.end(), [](const Point& a, const Point& b) {
129
       if (pred == 0) {
                                                                                                 return (a.x == b.x)? a.y < b.y: a.x < b.x;
            result.push back((a.center * b.r + b.center * a.r) * (1 / (a.r + b.r));
130
                                                                                             });
131
            return result;
                                                                                             for (const auto& p : dat) {
132
                                                                                      10
                                                                                                 while (upper.size() >= 2 && ccw(*++upper.rbegin(), *upper.rbegin(), p)
        double aa = a.center.x * a.center.x + a.center.y * a.center.y - a.r * a.r;
133
                                                                                                   >= 0) upper.pop_back();
134
        double bb = b.center.x * b.center.x + b.center.y * b.center.y - b.r * b.r;
                                                                                                 while (lower.size() >= 2 && ccw(*++lower.rbegin(), *lower.rbegin(), p)
        double tmp = (bb - aa) / 2.0;
135
                                                                                                   <= 0) lower.pop back();
       Point cdiff = b.center - a.center;
136
                                                                                      12
                                                                                                 upper.emplace back(p);
137
       if (diff(cdiff.x, 0) == 0) {
                                                                                      13
                                                                                                 lower.emplace back(p);
138
            if (diff(cdiff.y, 0) == 0)
                                                                                      14
139
                return result; // if (diff(a.r, b.r) == 0): same circle
                                                                                      15
                                                                                             upper.insert(upper.end(), ++lower.rbegin(), --lower.rend());
            return circle_line(a, Line{ Point{ 0, tmp / cdiff.y }, Point{ 1, 0 } }); 16
140
                                                                                             return upper;
141
                                                                                      17 }
142
        return circle_line(a,
```

6.4 Polygon Cut

```
1 // left side of a->b
2 vector<Point> cut_polygon(const vector<Point>& polygon, Line line) {
       if (!polygon.size()) return polygon;
       typedef vector<Point>::const_iterator piter;
       piter la, lan, fi, fip, i, j;
      la = lan = fi = fip = polygon.end();
      i = polygon.end() - 1;
      bool lastin = diff(ccw_line(line, polygon[polygon.size() - 1]), 0) > 0;
9
       for (j = polygon.begin(); j != polygon.end(); j++) {
10
           bool thisin = diff(ccw_line(line, *j), 0) > 0;
11
           if (lastin && !thisin) {
12
               la = i;
13
               lan = j;
14
15
           if (!lastin && thisin) {
               fi = j;
16
17
               fip = i;
18
19
           i = j;
20
           lastin = thisin;
21
22
      if (fi == polygon.end()) {
23
           if (!lastin) return vector<Point>();
24
           return polygon;
25
      }
26
      vector<Point> result;
27
       for (i = fi ; i != lan ; i++) {
28
           if (i == polygon.end()) {
29
               i = polygon.begin();
30
               if (i == lan) break;
31
32
           result.push_back(*i);
33
34
      Point lc, fc;
35
       get_cross(Line{ *la, *lan - *la }, line, lc);
36
       get cross(Line{ *fip, *fi - *fip }, line, fc);
37
      result.push_back(lc);
38
       if (diff(dist2(lc, fc), 0) != 0) result.push_back(fc);
39
       return result;
40 }
```

6.5 Pick's theorem

격자점으로 구성된 simple polygon이 주어짐. i는 polygon 내부의 격자점 수, b는 polygon 11 선분 위 격자점 수, A는 polygon의 넓이라고 할 때, 다음과 같은 식이 성립한다.

$$A = i + \frac{b}{2} - 1$$

7 String

7.1 KMP

```
1 typedef vector<int> seg t;
 3 void calculate pi(vector<int>& pi, const seg t& str) {
       pi[0] = -1;
       for (int i = 1, j = -1; i < str.size(); i++) {
           while (j >= 0 && str[i] != str[j + 1]) j = pi[j];
           if (str[i] == str[j + 1])
 8
               pi[i] = ++j;
9
           else
10
               pi[i] = -1;
11
       }
12 }
13
14 // returns all positions matched
15 // O(|text|+|pattern|)
16 vector<int> kmp(const seq_t& text, const seq_t& pattern) {
       vector<int> pi(pattern.size()), ans;
       if (pattern.size() == 0) return ans;
18
19
       calculate_pi(pi, pattern);
20
       for (int i = 0, j = -1; i < text.size(); i++) {
21
           while (j >= 0 && text[i] != pattern[j + 1]) j = pi[j];
22
           if (text[i] == pattern[j + 1]) {
23
               j++;
24
               if (j + 1 == pattern.size()) {
25
                   ans.push_back(i - j);
26
                   j = pi[j];
27
               }
28
           }
29
       }
30
       return ans;
31 }
```

7.2 Aho-Corasick

```
1 #include <algorithm>
2 #include <vector>
 3 #include <queue>
 4 using namespace std;
 6 struct AhoCorasick
7 {
       const int alphabet;
9
       struct node {
           node() {}
           explicit node(int alphabet) : next(alphabet) {}
           vector<int> next, report;
12
13
           int back = 0, output_link = 0;
       };
14
       int maxid = 0;
```

```
16
       vector<node> dfa;
                                                                                      3 // calculates suffix array.
17
       explicit AhoCorasick(int alphabet) : alphabet(alphabet), dfa(1, node(
                                                                                      4 // O(n*logn)
                                                                                      5 vector<int> suffix array(const vector<T>& in) {
                                                                                            int n = (int)in.size(), c = 0;
18
       template<typename InIt, typename Fn> void add(int id, InIt first, InIt last,
                                                                                            vector<int> temp(n), pos2bckt(n), bckt(n), bpos(n), out(n);
          Fn func) {
19
           int cur = 0;
                                                                                            for (int i = 0; i < n; i++) out[i] = i;
           for ( ; first != last; ++first) {
                                                                                      9
20
                                                                                            sort(out.begin(), out.end(), [&](int a, int b) { return in[a] < in[b]; });</pre>
21
               auto s = func(*first);
                                                                                     10
                                                                                            for (int i = 0; i < n; i++) {
22
               if (auto next = dfa[cur].next[s]) cur = next;
                                                                                     11
                                                                                                 bckt[i] = c;
23
                                                                                     12
                                                                                                 if (i + 1 == n || in[out[i]] != in[out[i + 1]]) c++;
24
                                                                                     13
                   cur = dfa[cur].next[s] = (int)dfa.size();
25
                   dfa.emplace_back(alphabet);
                                                                                     14
                                                                                            for (int h = 1; h < n && c < n; h <<= 1) {
26
               }
                                                                                     15
                                                                                                 for (int i = 0; i < n; i++) pos2bckt[out[i]] = bckt[i];</pre>
27
                                                                                     16
                                                                                                 for (int i = n - 1; i >= 0; i--) bpos[bckt[i]] = i;
28
           dfa[cur].report.push_back(id);
                                                                                     17
                                                                                                 for (int i = 0; i < n; i++)</pre>
29
                                                                                     18
           maxid = max(maxid, id);
                                                                                                     if (out[i] >= n - h) temp[bpos[bckt[i]]++] = out[i];
30
                                                                                     19
                                                                                                 for (int i = 0; i < n; i++)
31
                                                                                     20
                                                                                                     if (out[i] >= h) temp[bpos[pos2bckt[out[i] - h]]++] = out[i] - h;
       void build() {
32
                                                                                     21
           queue<int> q;
33
                                                                                     22
           vector<char> visit(dfa.size());
                                                                                                 for (int i = 0; i + 1 < n; i++) {
34
                                                                                     23
                                                                                                     int a = (bckt[i] != bckt[i + 1]) || (temp[i] >= n - h)
           visit[0] = 1;
35
           q.push(0);
                                                                                     24
                                                                                                             while(!q.empty()) {
                                                                                     25
36
                                                                                                     bckt[i] = c;
37
                                                                                     26
               auto cur = q.front(); q.pop();
                                                                                                     c += a;
                                                                                     27
38
               dfa[cur].output link = dfa[cur].back;
39
               if (dfa[dfa[cur].back].report.empty())
                                                                                     28
                                                                                                 bckt[n - 1] = c++;
                   dfa[cur].output_link = dfa[dfa[cur].back].output_link;
                                                                                     29
                                                                                                 temp.swap(out);
                                                                                     30
41
               for (int s = 0; s < alphabet; s++) {</pre>
42
                   auto &next = dfa[cur].next[s];
                                                                                     31
                                                                                            return out;
43
                   if (next == 0) next = dfa[dfa[cur].back].next[s];
                                                                                     32 }
                   if (visit[next]) continue;
                   if (cur) dfa[next].back = dfa[dfa[cur].back].next[s];
45
                                                                                     34 // calculates lcp array. it needs suffix array & original sequence.
46
                   visit[next] = 1;
                                                                                     35 // O(n)
47
                   q.push(next);
                                                                                     36 vector<int> lcp(const vector<T>& in, const vector<int>& sa) {
48
               }
                                                                                            int n = (int)in.size();
49
           }
                                                                                     38
                                                                                            if (n == 0) return vector<int>();
50
                                                                                     39
                                                                                            vector<int> rank(n), height(n - 1);
51
       template<typename InIt, typename Fn> vector<int> countMatch(InIt first, InIt 40
                                                                                            for (int i = 0; i < n; i++) rank[sa[i]] = i;
         last, Fn func) {
                                                                                     41
                                                                                            for (int i = 0, h = 0; i < n; i++) {
           int cur = 0;
                                                                                     42
52
                                                                                                 if (rank[i] == 0) continue;
53
           vector<int> ret(maxid+1);
                                                                                     43
                                                                                                 int j = sa[rank[i] - 1];
54
                                                                                     44
           for (; first != last; ++first) {
                                                                                                 while (i + h < n \& k j + h < n \& k in[i + h] == in[j + h]) h++;
                                                                                                 height[rank[i] - 1] = h;
55
               cur = dfa[cur].next[func(*first)];
                                                                                     45
56
                                                                                     46
               for (int p = cur; p; p = dfa[p].output link)
                                                                                                if (h > 0) h--;
57
                   for (auto id : dfa[p].report) ret[id]++;
                                                                                     47
58
                                                                                     48
                                                                                            return height;
59
           return ret;
60
61 };
```

7.3 Suffix Array with LCP

1 typedef char T;

7.5 Manacher's Algorithm

Suffix Tree

1 // find longest palindromic span for each element in str

```
2 // O(|str|)
3 void manacher(const string& str, int plen[]) {
       int r = -1, p = -1;
       for (int i = 0; i < str.length(); ++i) {</pre>
           if (i <= r)</pre>
 6
                plen[i] = min((2 * p - i >= 0) ? plen[2 * p - i] : 0, r - i);
           else
 9
                plen[i] = 0;
10
           while (i - plen[i] - 1 >= 0 && i + plen[i] + 1 < str.length()</pre>
11
                    && str[i - plen[i] - 1] == str[i + plen[i] + 1]) {
12
                plen[i] += 1;
13
14
           if (i + plen[i] > r) {
15
               r = i + plen[i];
16
               p = i;
17
           }
18
19 }
```

8 Miscellaneous

8.1 Fast I/O

```
1 namespace fio {
       const int BSIZE = 524288;
       char buffer[BSIZE];
       int p = BSIZE;
       inline char readChar() {
 6
           if(p == BSIZE) {
               fread(buffer, 1, BSIZE, stdin);
8
               p = 0;
9
10
           return buffer[p++];
11
12
       int readInt() {
13
           char c = readChar();
14
           while ((c < '0' | | c > '9') \&\& c != '-') {
15
               c = readChar();
16
17
           int ret = 0; bool neg = c == '-';
18
           if (neg) c = readChar();
19
           while (c >= '0' \&\& c <= '9') {
20
               ret = ret * 10 + c - '0';
21
               c = readChar();
22
23
           return neg ? -ret : ret;
24
25 }
```

8.2 Magic Numbers

```
소수: 10007, 10009, 10111, 31567, 70001, 1000003, 1000033, 4000037, 1000000007, 1000000009
```

8.3 Java Examples

```
1 import java.util.Scanner;
 3 public class example
4 {
       public static void main(String[] args)
           Scanner in = new Scanner(System.in);
           int T = in.nextInt();
9
           while (T --> 0)
10
11
               String str = in.next();
12
               if (str.matches("[A-F]?A+F+C+[A-F]?"))
13
                   System.out.println("Infected!");
14
15
                   System.out.println("Good");
16
17
18 }
```