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2 Math

2.1 Basic Arithmetic

```
1 typedef long long 11;
2 typedef unsigned long long ull;
4 // calculate lg2(a)
5 inline int lg2(ll a)
6 {
7
       return 63 - builtin clzll(a);
8 }
9
10 // calculate the number of 1-bits
11 inline int bitcount(ll a)
12 {
13
       return builtin popcountl1(a);
14 }
15
16 // calculate ceil(a/b)
|17|/|a|, |b| \le (2^63)-1 (does not dover -2^63)
18 ll ceildiv(ll a, ll b) {
      if (b < 0) return ceildiv(-a, -b);</pre>
      if (a < 0) return (-a) / b;
       return ((ull)a + (ull)b - 1ull) / b;
21
22 }
24 // calculate floor(a/b)
25 // |a|, |b| <= (2^63)-1 (does not cover -2^63)
27
      if (b < 0) return floordiv(-a, -b);</pre>
      if (a >= 0) return a / b;
29
       return -(11)(((ull)(-a) + b - 1) / b);
30 }
31
32 // calculate a*b % m
33 // x86-64 only
34 ll large mod mul(ll a, ll b, ll m)
35 {
36
       return ll((__int128)a*(__int128)b%m);
37 }
39 // calculate a*b % m
40 // |m| < 2^62, x86 available
41 // O(Logb)
42 ll large mod mul(ll a, ll b, ll m)
43 {
44
      a \% = m; b \% = m; 11 r = 0, v = a;
45
       while (b) {
46
          if (b\&1) r = (r + v) \% m;
47
          b >>= 1:
48
          v = (v << 1) \% m;
49
50
       return r;
```

```
51 }
52
53 // calculate n^k % m
54 ll modpow(ll n, ll k, ll m) {
      ll ret = 1;
56
       n \% = m;
57
       while (k) {
58
           if (k & 1) ret = large_mod_mul(ret, n, m);
59
           n = large mod mul(n, n, m);
60
           k /= 2;
61
       }
62
       return ret;
63 }
65 // calculate gcd(a, b)
66 ll gcd(ll a, ll b) {
       return b == 0 ? a : gcd(b, a % b);
67
68 }
70 // find a pair (c, d) s.t. ac + bd = qcd(a, b)
71 pair<ll, 1l> extended_gcd(ll a, ll b) {
      if (b == 0) return { 1, 0 };
73
       auto t = extended gcd(b, a % b);
74
       return { t.second, t.first - t.second * (a / b) };
75 }
76
77 // find x in [0,m) s.t. ax === gcd(a, m) \pmod{m}
78 ll modinverse(ll a, ll m) {
       return (extended_gcd(a, m).first % m + m) % m;
80 }
81
82 // calculate modular inverse for 1 ~ n
83 void calc range modinv(int n, int mod, int ret[]) {
       ret[1] = 1;
85
       for (int i = 2; i <= n; ++i)
           ret[i] = (11)(mod - mod/i) * ret[mod%i] % mod;
87 }
  2.2 Sieve Methods: Prime, Divisor, Euler phi
 1 // find prime numbers in 1 ~ n
 2 // ret[x] = false -> x is prime
 3 // O(n*loglogn)
 4 void sieve(int n, bool ret[]) {
       for (int i = 2; i * i <= n; ++i)
 6
           if (!ret[i])
 7
               for (int j = i * i; j <= n; j += i)
 8
                   ret[i] = true;
9 }
11 // calculate number of divisors for 1 \sim n
12 // when you need to calculate sum, change += 1 to += i
13 // O(n*logn)
14 void num of divisors(int n, int ret[]) {
       for (int i = 1; i <= n; ++i)
```

```
for (int j = i; j <= n; j += i)
17
               ret[i] += 1;
18 }
20 // calculate euler totient function for 1 ~ n
21 // phi(n) = number of x s.t. 0 < x < n && <math>gcd(n, x) = 1
22 // O(n*loglogn)
23 void euler phi(int n, int ret[]) {
       for (int i = 1; i <= n; ++i) ret[i] = i;</pre>
       for (int i = 2; i <= n; ++i)</pre>
           if (ret[i] == i)
26
27
               for (int j = i; j <= n; j += i)
28
                    ret[j] -= ret[j] / i;
29 }
```

2.3 Primality Test

```
1 bool test witness(ull a, ull n, ull s) {
       if (a >= n) a %= n;
       if (a <= 1) return true;</pre>
      ull d = n \gg s;
       ull x = modpow(a, d, n);
      if (x == 1 || x == n-1) return true;
7
       while (s-- > 1) {
8
          x = large_mod_mul(x, x, n);
9
           x = x * x % n:
10
          if (x == 1) return false;
          if (x == n-1) return true;
11
12
13
       return false;
14 }
15
16 // test whether n is prime
17 // based on miller-rabin test
18 // O(logn*logn)
19 bool is prime(ull n) {
      if (n == 2) return true;
21
      if (n < 2 | | n % 2 == 0) return false;
22
23
       ull d = n \gg 1, s = 1;
24
       for(; (d&1) == 0; s++) d >>= 1;
25
26 #define T(a) test_witness(a##ull, n, s)
       if (n < 4759123141ull) return T(2) && T(7) && T(61);</pre>
28
       return T(2) && T(325) && T(9375) && T(28178)
29
           && T(450775) && T(9780504) && T(1795265022);
30 #undef T
31 }
```

2.4 Chinese Remainder Theorem

```
1 // find x s.t. x === a[0] (mod n[0])
2 // === a[1] (mod n[1])
3 // ...
```

```
4 // assumption: gcd(n[i], n[j]) = 1
5 11 chinese_remainder(ll* a, ll* n, int size) {
      if (size == 1) return *a;
      ll tmp = modinverse(n[0], n[1]);
      ll tmp2 = (tmp * (a[1] - a[0]) % n[1] + n[1]) % n[1];
9
      ll ora = a[1];
10
      11 tgcd = gcd(n[0], n[1]);
11
      a[1] = a[0] + n[0] / tgcd * tmp2;
12
      n[1] *= n[0] / tgcd;
13
      ll ret = chinese_remainder(a + 1, n + 1, size - 1);
14
      n[1] /= n[0] / tgcd;
15
      a[1] = ora;
16
      return ret;
17 }
```

2.5 Rational Number Class

```
1 struct rational {
       long long p, q;
       void red() {
5
           if (q < 0) {
6
               p = -p;
7
               q = -q;
8
9
           11 t = gcd((p >= 0 ? p : -p), q);
10
           p /= t;
11
           q /= t;
12
13
14
       rational(): p(0), q(1) {}
15
       rational(long long p_): p(p_), q(1) {}
16
       rational(long long p_, long long q_): p(p_), q(q_) { red(); }
17
18
       bool operator==(const rational& rhs) const {
19
           return p == rhs.p && q == rhs.q;
20
21
       bool operator!=(const rational& rhs) const {
           return p != rhs.p || a != rhs.a:
22
23
       bool operator<(const rational& rhs) const {</pre>
24
25
           return p * rhs.q < rhs.p * q;</pre>
26
       rational operator+(const rational& rhs) const {
27
28
           ll g = gcd(q, rhs.q);
29
           return rational(p * (rhs.q / g) + rhs.p * (q / g), (q / g) * rhs.q);
30
31
       rational operator-(const rational& rhs) const {
32
           11 g = gcd(q, rhs.q);
33
           return rational(p * (rhs.q / g) - rhs.p * (q / g), (q / g) * rhs.q);
34
35
       rational operator*(const rational& rhs) const {
36
           return rational(p * rhs.p, q * rhs.q);
37
       rational operator/(const rational& rhs) const {
```

```
39          return rational(p * rhs.q, q * rhs.p);
40     }
41 };
```

2.6 Burnside's Lemma

경우의 수를 세는데, 특정 transform operation(회전, 반사, ..) 해서 같은 경우들은 하나로 33 친다. 전체 경우의 수는?

- 각 operation마다 이 operation을 했을 때 변하지 않는 경우의 수를 센다 (단, "아무것도 하지 않는다"라는 operation도 있어야 함!)
- 전체 경우의 수를 더한 후, operation의 수로 나눈다. (답이 맞다면 항상 나누어 떨어져야 한다)

2.7 Kirchoff's Theorem

그래프의 스패닝 트리의 개수를 구하는 정리.

무향 그래프의 Laplacian matrix L를 만든다. 이것은 (정점의 차수 대각 행렬) - (인접행렬) 이다. L에서 행과 열을 하나씩 제거한 것을 L'라 하자. 어느 행/열이든 관계 없다. 그래프의 스패닝 트리의 개수는 det(L')이다.

2.8 Fast Fourier Transform

```
1 void fft(int sign, int n, double *real, double *imag) {
       double theta = sign * 2 * pi / n;
       for (int m = n; m >= 2; m >>= 1, theta *= 2) {
           double wr = 1, wi = 0, c = cos(theta), s = sin(theta);
           for (int i = 0, mh = m >> 1; i < mh; ++i) {
               for (int j = i; j < n; j += m) {</pre>
                   int k = j + mh;
                   double xr = real[j] - real[k], xi = imag[j] - imag[k];
9
                   real[j] += real[k], imag[j] += imag[k];
                   real[k] = wr * xr - wi * xi, imag[k] = wr * xi + wi * xr;
10
11
12
               double _wr = wr * c - wi * s, _wi = wr * s + wi * c;
13
               wr = \_wr, wi = \_wi;
          }
14
15
16
      for (int i = 1, j = 0; i < n; ++i) {
           for (int k = n \gg 1; k \gg (j = k); k \gg 1);
17
18
           if (j < i) swap(real[i], real[j]), swap(imag[i], imag[j]);</pre>
19
20 }
21 // Compute Poly(a)*Poly(b), write to r; Indexed from 0
22 // O(n*Loan)
23 int mult(int *a, int n, int *b, int m, int *r) {
       const int maxn = 100;
25
       static double ra[maxn], rb[maxn], ia[maxn], ib[maxn];
```

```
int fn = 1:
27
       while (fn < n + m) fn <<= 1; // n + m: interested length
28
       for (int i = 0; i < n; ++i) ra[i] = a[i], ia[i] = 0;
29
       for (int i = n; i < fn; ++i) ra[i] = ia[i] = 0;
       for (int i = 0; i < m; ++i) rb[i] = b[i], ib[i] = 0;
31
       for (int i = m; i < fn; ++i) rb[i] = ib[i] = 0;
       fft(1, fn, ra, ia);
       fft(1, fn, rb, ib);
       for (int i = 0; i < fn; ++i) {
35
           double real = ra[i] * rb[i] - ia[i] * ib[i];
           double imag = ra[i] * ib[i] + rb[i] * ia[i];
           ra[i] = real, ia[i] = imag;
38
       fft(-1, fn, ra, ia);
       for (int i = 0; i < fn; ++i) r[i] = (int)floor(ra[i] / fn + 0.5);</pre>
       return fn;
42 }
```

2.9 Matrix Operations

```
1 const int MATSZ = 100;
 3 inline bool is zero(double a) { return fabs(a) < 1e-9; }</pre>
 5 // out = A^{(-1)}, returns det(A)
 6 // A becomes invalid after call this
7 // O(n^3)
8 double inverse and det(int n, double A[][MATSZ], double out[][MATSZ]) {
       double det = 1;
       for (int i = 0; i < n; i++) {
11
           for (int j = 0; j < n; j++) out[i][j] = 0;
12
           out[i][i] = 1;
13
14
       for (int i = 0; i < n; i++) {</pre>
15
           if (is zero(A[i][i])) {
               double maxv = 0;
16
17
               int maxid = -1;
               for (int j = i + 1; j < n; j++) {
18
                    auto cur = fabs(A[j][i]);
19
20
                   if (maxv < cur) {</pre>
                        maxv = cur;
21
22
                        maxid = j;
23
                   }
24
               if (maxid == -1 || is_zero(A[maxid][i])) return 0;
25
               for (int k = 0; k < n; k++) {
27
                   A[i][k] += A[maxid][k];
28
                    out[i][k] += out[maxid][k];
29
               }
30
31
           det *= A[i][i];
32
           double coeff = 1.0 / A[i][i];
33
           for (int j = 0; j < n; j++) A[i][j] *= coeff;</pre>
34
           for (int j = 0; j < n; j++) out[i][j] *= coeff;</pre>
35
           for (int j = 0; j < n; j++) if (j != i) {
```

2.10 Gaussian Elimination

```
1 const double EPS = 1e-10;
2 typedef vector<vector<double>> VVD;
4 // Gauss-Jordan elimination with full pivotina.
5 // solving systems of linear equations (AX=B)
6 // INPUT:
               a[][] = an n*n matrix
                b[][] = an n*m matrix
7 //
8 // OUTPUT:
                      = an n*m matrix (stored in b[1][1])
9 //
                A^{-1} = an n*n matrix (stored in a[][])
10 // O(n^3)
11 bool gauss_jordan(VVD& a, VVD& b) {
       const int n = a.size();
13
       const int m = b[0].size();
14
       vector<int> irow(n), icol(n), ipiv(n);
15
16
       for (int i = 0; i < n; i++) {
17
          int pj = -1, pk = -1;
18
           for (int j = 0; j < n; j++) if (!ipiv[j])
19
               for (int k = 0; k < n; k++) if (!ipiv[k])
20
                   if (pj == -1 || fabs(a[j][k]) > fabs(a[pj][pk])) { pj = j; pk}
21
          if (fabs(a[pj][pk]) < EPS) return false; // matrix is singular</pre>
22
          ipiv[pk]++;
23
           swap(a[pj], a[pk]);
24
          swap(b[pi], b[pk]);
25
          irow[i] = pj;
26
          icol[i] = pk;
27
28
           double c = 1.0 / a[pk][pk];
29
           a[pk][pk] = 1.0;
30
           for (int p = 0; p < n; p++) a[pk][p] *= c;
          for (int p = 0; p < m; p++) b[pk][p] *= c;</pre>
31
32
          for (int p = 0; p < n; p++) if (p != pk) {
33
               c = a[p][pk];
34
               a[p][pk] = 0;
35
               for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c;
36
               for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c;
37
          }
38
39
       for (int p = n - 1; p >= 0; p --) if (irow[p] != icol[p]) {
40
           for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][icol[p]]);
41
42
       return true:
43 }
```

2.11 Simplex Algorithm

```
1 // Two-phase simplex algorithm for solving linear programs of the form
          maximize
 2 //
                       c^T x
 3 //
          subject to Ax <= b
 4 //
                        x >= 0
 5 // INPUT: A -- an m x n matrix
 6 //
             b -- an m-dimensional vector
7 //
             c -- an n-dimensional vector
 8 //
             x -- a vector where the optimal solution will be stored
 9 // OUTPUT: value of the optimal solution (infinity if unbounded
10 //
              above, nan if infeasible)
11 // To use this code, create an LPSolver object with A, b, and c as
12 // arguments. Then, call Solve(x).
13 typedef vector<double> VD;
14 typedef vector < VD > VVD;
15 typedef vector<int> VI;
16 const double EPS = 1e-9;
17
18 struct LPSolver {
19
       int m, n;
20
       VI B, N;
21
       VVD D;
22
23
       LPSolver(const VVD& A, const VD& b, const VD& c):
24
           m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2, VD(n + 2)) 
25
           for (int i = 0; i < m; i++) for (int j = 0; j < n; j++) D[i][j] = A[i]
             ][j];
26
           for (int i = 0; i < m; i++) { B[i] = n + i; D[i][n] = -1; D[i][n + 1]
             = b[i]; }
27
           for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
28
           N[n] = -1; D[m + 1][n] = 1;
29
       }
30
31
       void pivot(int r, int s) {
32
           double inv = 1.0 / D[r][s];
33
           for (int i = 0; i < m + 2; i++) if (i != r)
34
               for (int j = 0; j < n + 2; j++) if (j != s)
35
                   D[i][j] -= D[r][j] * D[i][s] * inv;
36
           for (int j = 0; j < n + 2; j++) if (j != s) D[r][j] *= inv;
37
           for (int i = 0; i < m + 2; i++) if (i != r) D[i][s] *= -inv;
38
           D[r][s] = inv;
39
           swap(B[r], N[s]);
40
41
42
       bool simplex(int phase) {
43
           int x = phase == 1 ? m + 1 : m;
44
           while (true) {
               int s = -1;
45
46
               for (int j = 0; j <= n; j++) {
                   if (phase == 2 && N[j] == -1) continue;
48
                   if (s == -1 || D[x][i] < D[x][s] || D[x][i] == D[x][s] && N[i]
                      \langle N[s] \rangle s = j;
49
               }
50
               if (D[x][s] > -EPS) return true;
```

```
int r = -1;
52
               for (int i = 0; i < m; i++) {
53
                    if (D[i][s] < EPS) continue;</pre>
54
                    if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][s]
                     Ш
                        (D[i][n + 1] / D[i][s]) == (D[r][n + 1] / D[r][s]) && B[i]
                           \langle B[r] \rangle r = i;
56
57
               if (r == -1) return false;
58
               pivot(r, s);
59
           }
       }
60
61
62
       double solve(VD& x) {
63
           int r = 0:
           for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r][n + 1]) r = i;
64
65
           if (D[r][n + 1] < -EPS) {</pre>
66
               pivot(r, n);
               if (!simplex(1) || D[m + 1][n + 1] < -EPS)
67
68
                    return -numeric limits<double>::infinity();
               for (int i = 0; i < m; i++) if (B[i] == -1) {
69
70
                    int s = -1:
                    for (int j = 0; j <= n; j++)
                        if (s == -1 || D[i][j] < D[i][s] || D[i][j] == D[i][s] &&</pre>
72
                         N[j] < N[s]) s = j;
73
                    pivot(i, s);
74
               }
75
           }
           if (!simplex(2))
76
77
               return numeric limits<double>::infinity();
78
79
           for (int i = 0; i < m; i++) if (B[i] < n) \times [B[i]] = D[i][n + 1];
80
           return D[m][n + 1];
81
82 };
```

3 Data Structure

3.1 Order statistic tree

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>
#include <functional>
#include <iostream>
using namespace __gnu_pbds;
using namespace std;

// tree<key_type, value_type(set if null), comparator, ...>
using ordered_set = tree<int, null_type, less<int>, rb_tree_tag,
tree_order_statistics_node_update>;

int main()
```

```
14 {
15
                                     ordered set X;
16
                                     for (int i = 1; i < 10; i += 2) X.insert(i); // 1 3 5 7 9
17
                                     cout << boolalpha;</pre>
                                     cout << *X.find_by_order(2) << endl; // 5</pre>
                                     cout << *X.find by order(4) << endl; // 9</pre>
                                      cout << (X.end() == X.find by order(5)) << endl; // true</pre>
 21
 22
                                     cout << X.order_of_key(-1) << endl; // 0</pre>
 23
                                     cout << X.order_of_key(1) << endl; // 0</pre>
 24
                                     cout << X.order_of_key(4) << endl; // 2</pre>
 25
                                     X.erase(3);
 26
                                     cout << X.order_of_key(4) << endl; // 1</pre>
27
                                     for (int t : X) printf("%d<sub>\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\under</sub>
 28 }
```

3.2 Fenwick Tree

```
1 const int TSIZE = 100000;
2 int tree[TSIZE + 1];
3
4 // Returns the sum from index 1 to p, inclusive
5 int query(int p) {
6    int ret = 0;
7    for (; p > 0; p -= p & -p) ret += tree[p];
8     return ret;
9 }
10
1 // Adds val to element with index pos
12 void add(int p, int val) {
13    for (; p <= TSIZE; p += p & -p) tree[p] += val;
14 }</pre>
```

3.3 Segment Tree with Lazy Propagation

```
1 // example implementation of sum tree
 2 const int TSIZE = 131072; // always 2<sup>k</sup> form && n <= TSIZE
 3 int segtree[TSIZE * 2], prop[TSIZE * 2];
 4 void seg_init(int nod, int 1, int r) {
       if (1 == r) segtree[nod] = dat[1];
 6
       else {
 7
            int m = (1 + r) >> 1;
 8
            seg init(nod << 1, 1, m);</pre>
 9
            seg_init(nod << 1 | 1, m + 1, r);
10
            segtree[nod] = segtree[nod << 1] + segtree[nod << 1 | 1];</pre>
11
12 }
13 void seg relax(int nod, int 1, int r) {
14
       if (prop[nod] == 0) return;
15
       if (1 < r) {
16
            int m = (1 + r) >> 1;
17
            segtree[nod \langle\langle 1] += (m - 1 + 1) * prop[nod];
18
            prop[nod << 1] += prop[nod];</pre>
19
            segtree[nod << 1 | 1] += (r - m) * prop[nod];
```

```
npoll[i].l = &npoll[i*2+1];
20
           prop[nod << 1 | 1] += prop[nod];</pre>
                                                                                       25
21
       }
                                                                                       26
                                                                                                       npoll[i].r = &npoll[i*2+2];
22
       prop[nod] = 0;
                                                                                       27
23 }
                                                                                       28
24 int seg_query(int nod, int 1, int r, int s, int e) {
                                                                                       29
                                                                                                  head[0] = &npoll[0];
25
       if (r < s || e < 1) return 0;
                                                                                       30
                                                                                                  last_q = 0;
26
       if (s <= 1 && r <= e) return segtree[nod];</pre>
                                                                                       31
                                                                                                  pptr = 2 * TSIZE - 1;
27
       seg relax(nod, 1, r);
                                                                                       32
                                                                                                  a[0] = 0;
28
       int m = (1 + r) >> 1;
                                                                                       33
                                                                                                  lqidx = 0;
29
       return seg_query(nod << 1, 1, m, s, e) + seg_query(nod << 1 | 1, m + 1, r,
                                                                                       34
                                                                                       35
30 }
                                                                                       36
                                                                                              // update val to pos at time t
31 void seg_update(int nod, int 1, int r, int s, int e, int val) {
                                                                                       37
                                                                                              // 0 <= t <= MAX OUERY, 0 <= pos < TSIZE
       if (r < s \mid | e < 1) return;
                                                                                       38
                                                                                              void update(int pos, int val, int t, int prev) {
33
                                                                                       39
                                                                                                  head[++last q] = &npoll[pptr++];
       if (s <= 1 && r <= e) {
34
           segtree[nod] += (r - l + 1) * val;
                                                                                       40
                                                                                                  node *old = head[q[prev]], *now = head[last_q];
35
                                                                                       41
                                                                                                  while (lqidx < t) q[lqidx++] = q[prev];</pre>
           prop[nod] += val;
36
           return;
                                                                                       42
                                                                                                  q[t] = last_q;
37
                                                                                       43
                                                                                       44
38
       seg relax(nod, 1, r);
                                                                                                  int flag = 1 << DEPTH;</pre>
39
       int m = (1 + r) >> 1;
                                                                                       45
                                                                                                  for (;;) {
40
       seg_update(nod << 1, 1, m, s, e, val);</pre>
                                                                                       46
                                                                                                      now -> v = old -> v + val;
                                                                                       47
       seg update(nod << 1 | 1, m + 1, r, s, e, val);
                                                                                                       flag >>= 1;
42
       segtree[nod] = segtree[nod << 1] + segtree[nod << 1 | 1];</pre>
                                                                                       48
                                                                                                       if (flag==0) {
43 }
                                                                                       49
                                                                                                           now->1 = now->r = nullptr; break;
44 // usage:
                                                                                       50
                                                                                                      }
45 // seg_update(1, 0, n - 1, qs, qe, val);
                                                                                       51
                                                                                                      if (flag & pos) {
                                                                                       52
                                                                                                           now->1 = old->1;
46 // seg_query(1, 0, n - 1, qs, qe);
                                                                                       53
                                                                                                           now->r = &npoll[pptr++];
                                                                                       54
                                                                                                           now = now -> r, old = old -> r;
   3.4 Persistent Segment Tree
                                                                                       55
                                                                                                      } else {
                                                                                       56
                                                                                                           now->r = old->r;
1 // persistent segment tree impl: sum tree
                                                                                       57
                                                                                                           now \rightarrow 1 = &npoll[pptr++];
2 namespace pstree {
                                                                                       58
                                                                                                           now = now ->1, old = old->1;
       typedef int val_t;
                                                                                       59
                                                                                                      }
       const int DEPTH = 18;
                                                                                       60
                                                                                                  }
5
       const int TSIZE = 1 << 18;</pre>
                                                                                              }
                                                                                       61
6
       const int MAX_QUERY = 262144;
                                                                                       62
7
                                                                                       63
                                                                                              val t query(int s, int e, int l, int r, node *n) {
8
       struct node {
                                                                                       64
                                                                                                  if (s == 1 \&\& e == r) return n \rightarrow v;
9
                                                                                       65
           val_t v;
                                                                                                  int m = (1 + r) / 2;
10
           node *1, *r;
                                                                                                  if (m \ge e) return query(s, e, 1, m, n \ge 1);
                                                                                       66
11
       } npoll[TSIZE * 2 + MAX_QUERY * (DEPTH + 1)];
                                                                                       67
                                                                                                  else if (m < s) return query(s, e, m + 1, r, n->r);
12
                                                                                       68
                                                                                                  else return query(s, m, l, m, n->l) + query(m + 1, e, m + 1, r, n->r);
13
       int pptr, last q;
                                                                                       69
                                                                                              }
14
                                                                                       70
15
       node *head[MAX QUERY + 1];
                                                                                       71
                                                                                              // query summation of [s, e] at time t
16
       int q[MAX QUERY + 1];
                                                                                       72
                                                                                              val_t query(int s, int e, int t) {
17
       int lqidx;
                                                                                       73
                                                                                                  s = max(0, s); e = min(TSIZE - 1, e);
18
                                                                                       74
                                                                                                  if (s > e) return 0;
19
       void init() {
                                                                                       75
                                                                                                  return query(s, e, 0, TSIZE - 1, head[q[t]]);
20
           // zero-initialize, can be changed freely
                                                                                       76
                                                                                              }
21
           memset(&npoll[TSIZE - 1], 0, sizeof(node) * TSIZE);
                                                                                       77 }
22
23
           for (int i = TSIZE - 2; i >= 0; i--) {
24
               npoll[i].v = 0;
```

3.5 Splay Tree

```
1 // example : https://www.acmicpc.net/problem/13159
 2 struct node {
         node* 1, * r, * p;
 4
         int cnt, min, max, val;
 5
         long long sum;
 6
         bool inv;
 7
         node(int val) :
 8
               cnt(1), sum( val), min( val), max( val), val( val), inv(false),
 9
               l(nullptr), r(nullptr), p(nullptr) {
10
11 };
12 node* root;
13
14 void update(node* x) {
         x \rightarrow cnt = 1;
16
         x \rightarrow sum = x \rightarrow min = x \rightarrow max = x \rightarrow val;
17
         if (x\rightarrow 1) {
18
               x \rightarrow cnt += x \rightarrow 1 \rightarrow cnt;
19
               x \rightarrow sum += x \rightarrow 1 \rightarrow sum;
20
               x \rightarrow min = min(x \rightarrow min, x \rightarrow 1 \rightarrow min);
21
               x->max = max(x->max, x->1->max);
22
23
         if(x->r) {
24
              x \rightarrow cnt += x \rightarrow r \rightarrow cnt;
25
               x \rightarrow sum += x \rightarrow r \rightarrow sum;
26
               x - \min = \min(x - \min, x - r - \min);
27
               x -> max = max(x -> max, x -> r -> max);
28
         }
29 }
30
31 void rotate(node* x) {
         node* p = x-p;
33
         node* b = nullptr;
34
         if (x == p -> 1) {
35
               p->1 = b = x->r;
36
               x \rightarrow r = p;
37
38
         else {
39
               p->r = b = x->1;
40
               x \rightarrow 1 = p;
41
42
         x \rightarrow p = p \rightarrow p;
43
         p \rightarrow p = x;
         if (b) b \rightarrow p = p;
45
         x \rightarrow p? (p == x \rightarrow p \rightarrow 1 ? x \rightarrow p \rightarrow 1 : x \rightarrow p \rightarrow r) = x : (root = x);
46
         update(p);
47
          update(x);
48 }
50 // make x into root
51 void splay(node* x) {
52
         while (x->p) {
53
               node* p = x-p;
```

```
54
             node* g = p - p;
55
             if (g) rotate((x == p \rightarrow 1) == (p == g \rightarrow 1) ? p : x);
56
             rotate(x);
57
        }
 58 }
59
 60 void relax lazy(node* x) {
        if (!x->inv) return;
 62
        swap(x->1, x->r);
 63
        x->inv = false;
 64
        if (x\rightarrow 1) x\rightarrow 1\rightarrow inv = !x\rightarrow 1\rightarrow inv;
 65
        if (x->r) x->r->inv = !x->r->inv;
 66 }
 67
 68 // find kth node in splay tree
 69 void find_kth(int k) {
        node* x = root;
70
71
        relax_lazy(x);
72
        while (true) {
73
             while (x->1 && x->1->cnt > k) {
74
                 x = x \rightarrow 1;
75
                  relax_lazy(x);
76
77
             if (x->1) k -= x->1->cnt;
             if (!k--) break;
78
79
             x = x - r;
 80
             relax_lazy(x);
81
        }
 82
        splay(x);
 83 }
 85 // collect [l, r] nodes into one subtree and return its root
 86 node* interval(int 1, int r) {
        find_kth(l - 1);
 88
        node* x = root;
 89
        root = x - r;
 90
        root->p = nullptr;
91
        find_kth(r - l + 1);
92
        x \rightarrow r = root;
 93
        root -> p = x;
 94
        root = x;
 95
        return root->r->l;
 96 }
98 void traverse(node* x) {
99
        relax lazy(x);
100
        if (x->1) {
101
             traverse(x->1);
102
        }
103
        // do something
104
        if (x->r) {
105
             traverse(x->r);
106
107 }
108
```

3.6 Link/Cut Tree

4 DP

4.1 Convex Hull Optimization

필요없어지기 때문에 amortized O(n) 에 해결할 수 있음

4.1.1 requirement

 $O(n^2) \to O(n \log n)$

```
조건 1) DP 점화식 꼴 D[i] = \min_{j < i} (D[j] + b[j] * a[i]) 조건 2) b[j] \le b[j+1] 특수조건) a[i] \le a[i+1] 도 만족하는 경우, 마지막 쿼리의 위치를 저장해두면 이분검색이
```

4.1.2 Source Code

```
1 //0(n^3) \rightarrow 0(n^2)
3 #define sz 100001
4 long long s[sz];
5 long long dp[2][sz];
6 //deque {index, x pos }
7 int dqi[sz];
8 long long dqm[sz];
9 //pointer to deque
10 int ql,qr;
11 //dp[i][j] = max(dp[i][k] + s[j]*s[k] - s[k]^2)
12 //Let y = dp[i][j], x = s[j] \rightarrow y = max(s[k]*x + dp[i][k] - s[k]^2);
14 //push new value to deque
15 //i = index, x = current x pos
16 void setq(int i, int x)
17 {
18
       //a1,b1 = prv line, a2,b2 = new line
19
       int a1, a2 = s[i];
20
       long long b1, b2 = dp[0][i] - s[i] * s[i], r;
21
      //renew deque
22
       while (qr>=ql)
```

```
23
24
           //last line enqueued
25
           a1 = s[dqi[qr]];
26
           b1 = dp[0][dqi[qr]] - s[dqi[qr]] * s[dqi[qr]];
27
           //tie breaking to newer one
28
           if (a1 == a2)
29
30
               dqi[qr] = i;
31
               return;
32
33
           // x intersection between last line and new line
34
           r = (b1 - b2) / (a2 - a1);
35
           if ((b1 - b2) % (a2 - a1)) r++;
36
           //last line is not needed
37
           if (r <= dqm[qr])
38
39
               qr--;
40
41
           else break:
42
43
       if (r < 0) r = 0;
       //push back new line
45
       if (dqm[qr] < s[n - 1] && r <= s[n - 1])
46
47
           dqi[++qr] = i;
48
           dqm[qr] = r;
       //discard old lines
       while (qr-ql && dqm[ql+1] <= x)
51
52
       {
53
           q1++;
54
55 }
56
57 int main()
58 {
59
       for (int j = 0; j < k; j++)
60
61
           ql = 0;
62
           qr = 1;
63
           dqi[0] = dqm[0] = 0;
64
           for (int i = 1; i < n; i++)</pre>
65
               //get line used by current x pos
67
               setq(i, s[i]);
68
               //line index to use
69
               int g = dqi[ql];
70
               //set dp value
71
               dp[1][i] = dp[0][g] + s[g] * (s[i] - s[g]);
72
73
           for (int i = 0; i < n; i++)</pre>
74
75
               dp[0][i] = dp[1][i];
76
                dp[1][i] = 0;
77
```

```
78 }
79 }
```

4.2 Divide & Conquer Optimization

```
O(kn^2) 	o O(kn\log n) 조건 1) DP 점화식 꼴 D[t][i] = \min_{j < i} (D[t-1][j] + C[j][i]) 조건 2) A[t][i] \vdash D[t][i]의 답이 되는 최소의 j라 할 때, 아래의 부등식을 만족해야 함 A[t][i] \le A[t][i+1] 조건 2-1) 비용C가 다음의 사각부등식을 만족하는 경우도 조건 2)를 만족하게 됨 C[a][c] + C[b][d] \le C[a][d] + C[b][c] \ \ (a \le b \le c \le d)
```

4.3 Knuth Optimization

```
O(n^3) 	o O(n^2) 조건 1) DP 점화식 꼴 D[i][j] = \min_{i < k < j} (D[i][k] + D[k][j]) + C[i][j] 조건 2) 사각 부등식 C[a][c] + C[b][d] \le C[a][d] + C[b][c] \ (a \le b \le c \le d) 조건 3) 단조성 C[b][c] \le C[a][d] \ (a \le b \le c \le d) 결론) 조건 2, 3을 만족한다면 A[i][j]를 D[i][j]의 답이 되는 최소의 k라 할 때, 아래의 부등식을 만족하게 됨 A[i][j-1] \le A[i][j] \le A[i+1][j] 3중 루프를 돌릴 때 위 조건을 이용하면 최종적으로 시간복잡도가 O(n^2) 이 됨
```

5 Graph

5.1 SCC (Tarjan)

```
1 const int MAXN = 100;
2 vector<int> graph[MAXN];
3 int up[MAXN], visit[MAXN], vtime;
4 vector<int> stk;
5 int scc_idx[MAXN], scc_cnt;
```

```
6
7 void dfs(int nod) {
       up[nod] = visit[nod] = ++vtime;
       stk.push back(nod);
10
       for (int next : graph[nod]) {
11
           if (visit[next] == 0) {
12
               dfs(next);
13
               up[nod] = min(up[nod], up[next]);
14
15
           else if (scc_idx[next] == 0)
               up[nod] = min(up[nod], visit[next]);
16
17
18
       if (up[nod] == visit[nod]) {
19
           ++scc_cnt;
20
           int t;
21
22
               t = stk.back();
23
               stk.pop_back();
24
               scc_idx[t] = scc_cnt;
25
           } while (!stk.empty() && t != nod);
26
27 }
29 // find SCCs in given directed graph
30 // O(V+E)
31 void get_scc() {
       vtime = 0;
       memset(visit, 0, sizeof(visit));
       scc_cnt = 0;
35
       memset(scc_idx, 0, sizeof(scc_idx));
       for (int i = 0; i < n; ++i)
37
           if (visit[i] == 0) dfs(i);
38 }
```

5.2 SCC (Kosaraju)

```
1 const int MAXN = 100;
 2 vector<int> graph[MAXN], grev[MAXN];
 3 int visit[MAXN], vcnt;
 4 int scc_idx[MAXN], scc_cnt;
 5 vector<int> emit;
 7 void dfs(int nod, vector<int> graph[]) {
       visit[nod] = vcnt;
       for (int next : graph[nod]) {
10
           if (visit[next] == vcnt) continue;
11
           dfs(next, graph);
12
13
       emit.push back(nod);
14 }
16 // find SCCs in given graph
17 // O(V+E)
18 void get scc() {
       scc cnt = 0;
```

```
20
       vcnt = 1;
21
       emit.clear();
22
       memset(visit, 0, sizeof(visit));
23
24
       for (int i = 0; i < n; i++) {
25
           if (visit[i] == vcnt) continue;
26
           dfs(i, graph);
27
      }
28
29
       ++vcnt;
30
       for (auto st : vector<int>(emit.rbegin(), emit.rend())) {
31
           if (visit[st] == vcnt) continue;
32
           emit.clear();
33
           dfs(st, grev);
34
           ++scc cnt:
35
           for (auto node : emit)
36
               scc idx[node] = scc cnt;
37
      }
38 }
```

5.3 2-SAT

 $(b_x \lor b_y) \land (\neg b_x \lor b_z) \land (b_z \lor \neg b_x) \land \cdots$ 같은 form을 2-CNF라고 함. 주어진 2-CNF 식을 ³⁷ 참으로 하는 $\{b_1, b_2, \cdots\}$ 가 존재하는지, 존재한다면 그 값은 무엇인지 구하는 문제를 2-SAT ³⁸ 이라 함.

boolean variable b_i 마다 b_i 를 나타내는 정점, $\neg b_i$ 를 나타내는 정점 2개를 만듦. 각 clause $b_i \lor b_j$ 마다 $\neg b_i \to b_j$, $\neg b_j \to b_i$ 이렇게 edge를 이어줌. 그렇게 만든 그래프에서 SCC를 43 // o(V+E) 다 구함. 어떤 SCC 안에 b_i 와 $\neg b_i$ 가 같이 포함되어있다면 해가 존재하지 않음. 아니라면 44 void get_bcc() { 해가 존재함.

해가 존재할 때 구체적인 해를 구하는 방법. 위에서 SCC를 구하면서 SCC DAG를 만들어 47 준다. 거기서 위상정렬을 한 후, 앞에서부터 SCC를 하나씩 봐준다. 현재 보고있는 SCC 48 49 에 49 가 속해있는데 얘가 49 보다 먼저 등장했다면 49 등이미 값이 assign되었다면 pass.

5.4 BCC, Cut vertex, Bridge

```
1 const int MAXN = 100;
2 vector<pair<int, int>> graph[MAXN]; // { next vertex id, edge id }
3 int up[MAXN], visit[MAXN], vtime;
4 vector<pair<int, int>> stk;
5
6 int is_cut[MAXN]; // v is cut vertex if is_cut[v] > 0
7 vector<int>> bridge; // List of edge ids
8 vector<int>> bcc_idx[MAXN]; // List of bccids for vertex i
9 int bcc_cnt;
10
11 void dfs(int nod, int par_edge) {
12  up[nod] = visit[nod] = ++vtime;
13  int child = 0;
```

```
14
       for (const auto& e : graph[nod]) {
15
           int next = e.first, edge_id = e.second;
16
           if (edge_id == par_edge) continue;
           if (visit[next] == 0) {
17
18
               stk.push_back({ nod, next });
19
               ++child;
20
               dfs(next, edge id);
21
               if (up[next] == visit[next]) bridge.push_back(edge_id);
22
               if (up[next] >= visit[nod]) {
23
                   ++bcc_cnt;
24
                   do {
25
                       auto last = stk.back();
26
                       stk.pop_back();
27
                       bcc_idx[last.second].push_back(bcc_cnt);
28
                       if (last == pair<int, int>{ nod, next }) break;
29
                   } while (!stk.empty());
30
                   bcc idx[nod].push back(bcc cnt);
31
                   is_cut[nod]++;
32
33
               up[nod] = min(up[nod], up[next]);
34
35
           else
               up[nod] = min(up[nod], visit[next]);
       if (par edge == -1 && is cut[nod] == 1)
           is cut[nod] = 0;
42 // find BCCs & cut vertexs & bridges in undirected graph
46
       memset(visit, 0, sizeof(visit));
       memset(is_cut, 0, sizeof(is_cut));
       bridge.clear();
       for (int i = 0; i < n; ++i) bcc idx[i].clear();</pre>
       bcc cnt = 0;
51
       for (int i = 0; i < n; ++i) {
52
           if (visit[i] == 0)
53
               dfs(i, -1);
54
       }
55 }
```

5.5 Shortest Path Faster Algorithm

```
1 // shortest path faster algorithm
2 // average for random graph : O(E) , worst : O(VE)
3
4 const int MAXN = 20001;
5 const int INF = 100000000;
6 int n, m;
7 vector<pair<int, int>> graph[MAXN];
8 bool inqueue[MAXN];
9 int dist[MAXN];
10
```

```
11 void spfa(int st) {
12
       for (int i = 0; i < n; ++i) {
13
           dist[i] = INF;
14
15
       dist[st] = 0;
16
17
       queue<int> q;
18
       q.push(st);
19
       inqueue[st] = true;
20
       while (!q.empty()) {
21
           int u = q.front();
22
           q.pop();
23
           inqueue[u] = false;
           for (auto& e : graph[u]) {
24
25
               if (dist[u] + e.second < dist[e.first]) {</pre>
26
                   dist[e.first] = dist[u] + e.second;
27
                   if (!inqueue[e.first]) {
28
                       q.push(e.first);
29
                        inqueue[e.first] = true;
30
31
               }
32
33
34 }
```

5.6 Lowest Common Ancestor

```
1 const int MAXN = 100:
2 const int MAXLN = 9;
3 vector<int> tree[MAXN];
4 int depth[MAXN];
5 int par[MAXLN][MAXN];
7 void dfs(int nod, int parent) {
      for (int next : tree[nod]) {
9
          if (next == parent) continue;
          depth[next] = depth[nod] + 1;
10
11
          par[0][next] = nod;
12
           dfs(next, nod);
13
      }
14 }
15
16 void prepare_lca() {
17
       const int root = 0;
      dfs(root, -1);
18
19
      par[0][root] = root;
20
       for (int i = 1; i < MAXLN; ++i)
21
           for (int j = 0; j < n; ++j)
22
               par[i][j] = par[i - 1][par[i - 1][j]];
23 }
25 // find lowest common ancestor in tree between u & v
26 // assumption : must call 'prepare_lca' once before call this
27 // O(LogV)
28 int lca(int u, int v) {
```

```
if (depth[u] < depth[v]) swap(u, v);</pre>
29
30
       if (depth[u] > depth[v]) {
31
           for (int i = MAXLN - 1; i >= 0; --i)
32
               if (depth[u] - (1 << i) >= depth[v])
33
                   u = par[i][u];
34
35
       if (u == v) return u;
36
       for (int i = MAXLN - 1; i >= 0; --i) {
37
           if (par[i][u] != par[i][v]) {
38
               u = par[i][u];
39
               v = par[i][v];
40
41
       }
42
       return par[0][u];
43 }
```

5.7 Heavy-Light Decomposition

```
1 // heavy-light decomposition
2 //
3 // hld h;
4 // insert edges to tree[0~n-1];
5 // h.init(n);
6 // h.decompose(root);
7 // h.hldquery(u, v); // edges from u to v
8 struct hld {
9
       static const int MAXLN = 18;
       static const int MAXN = 1 << (MAXLN - 1);</pre>
10
11
       vector<int> tree[MAXN];
12
       int subsize[MAXN], depth[MAXN], pa[MAXLN][MAXN];
13
14
       int chead[MAXN], cidx[MAXN];
15
       int lchain:
       int flatpos[MAXN + 1], fptr;
16
17
18
       void dfs(int u, int par) {
19
           pa[0][u] = par;
20
           subsize[u] = 1;
21
           for (int v : tree[u]) {
22
               if (v == pa[0][u]) continue;
23
               depth[v] = depth[u] + 1;
24
               dfs(v, u);
25
               subsize[u] += subsize[v];
26
27
       }
28
29
       void init(int size)
30
31
           lchain = fptr = 0;
32
           dfs(0, -1);
33
           memset(chead, -1, sizeof(chead));
34
35
           for (int i = 1; i < MAXLN; i++) {</pre>
36
               for (int j = 0; j < size; j++) {</pre>
37
                   if (pa[i - 1][j] != -1) {
```

```
38
                       pa[i][j] = pa[i - 1][pa[i - 1][j]];
39
40
               }
41
          }
42
      }
43
44
       void decompose(int u) {
45
           if (chead[lchain] == -1) chead[lchain] = u;
46
           cidx[u] = lchain:
47
           flatpos[u] = ++fptr;
48
49
           int maxchd = -1;
50
           for (int v : tree[u]) {
51
               if (v == pa[0][u]) continue;
52
               if (maxchd == -1 || subsize[maxchd] < subsize[v]) maxchd = v;</pre>
53
54
           if (maxchd != -1) decompose(maxchd);
55
56
           for (int v : tree[u]) {
57
               if (v == pa[0][u] || v == maxchd) continue;
58
               ++lchain; decompose(v);
59
           }
60
       }
61
62
       int lca(int u, int v) {
63
           if (depth[u] < depth[v]) swap(u, v);</pre>
64
65
           int logu;
           for (logu = 1; 1 << logu <= depth[u]; logu++);</pre>
66
67
           logu--;
68
69
           int diff = depth[u] - depth[v];
70
           for (int i = logu; i >= 0; --i) {
71
               if ((diff >> i) & 1) u = pa[i][u];
72
73
           if (u == v) return u;
74
75
           for (int i = logu; i >= 0; --i) {
76
               if (pa[i][u] != pa[i][v]) {
77
                   u = pa[i][u];
78
                   v = pa[i][v];
79
               }
80
          }
81
           return pa[0][u];
82
83
84
      // TODO: implement query functions
85
       inline int query(int s, int e) {
86
           return 0;
87
88
89
       int subquery(int u, int v, int t) {
90
           int uchain, vchain = cidx[v];
91
           int ret = 0;
92
           for (;;) {
```

```
93
                uchain = cidx[u];
94
                if (uchain == vchain) {
95
                    ret += query(flatpos[v], flatpos[u]);
96
                    break:
97
                }
98
99
                ret += query(flatpos[chead[uchain]], flatpos[u]);
100
                u = pa[0][chead[uchain]];
101
102
            return ret;
103
104
       inline int hldquery(int u, int v) {
105
106
            int p = lca(u, v);
107
            return subquery(u, p) + subquery(v, p) - query(flatpos[p], flatpos[p])
108
109 };
```

5.8 Bipartite Matching (Hopcroft-Karp)

```
1 // in: n, m, graph
2 // out: match, matched
3 // vertex cover: (reached[0][left_node] == 0) || (reached[1][right_node] == 1)
4 // O(E*sqrt(V))
5 struct BipartiteMatching {
6
       int n, m;
7
       vector<vector<int>> graph;
8
       vector<int> matched, match, edgeview, level;
9
       vector<int> reached[2];
10
       BipartiteMatching(int n, int m): n(n), m(m), graph(n), matched(m, -1),
         match(n, -1) {}
11
12
       bool assignLevel() {
13
           bool reachable = false;
14
           level.assign(n, -1);
15
           reached[0].assign(n, 0);
           reached[1].assign(m, 0);
16
17
           aueue<int> a:
18
           for (int i = 0; i < n; i++) {
19
               if (match[i] == -1) {
20
                   level[i] = 0;
21
                   reached[0][i] = 1;
22
                   q.push(i);
23
              }
24
25
           while (!q.empty()) {
26
               auto cur = q.front(); q.pop();
27
               for (auto adj : graph[cur]) {
28
                   reached[1][adj] = 1;
29
                   auto next = matched[adj];
30
                   if (next == -1) {
31
                       reachable = true;
32
33
                   else if (level[next] == -1) {
```

```
level[next] = level[cur] + 1;
                                                                                     19
                                                                                            };
35
                       reached[0][next] = 1;
                                                                                     20
                                                                                            int n;
36
                       q.push(next);
                                                                                     21
                                                                                            vector<vector<Edge>> graph;
37
                   }
                                                                                     22
                                                                                            vector<int> q, 1, start;
38
               }
                                                                                     23
                                                                                     24
39
                                                                                            void init(int n) {
40
           return reachable;
                                                                                     25
                                                                                                 n = n;
41
      }
                                                                                     26
                                                                                                 graph.resize(n);
42
                                                                                     27
                                                                                                 for (int i = 0; i < n; i++) graph[i].clear();</pre>
43
                                                                                     28
       int findpath(int nod) {
                                                                                     29
                                                                                            void add_edge(int s, int e, flow_t cap, flow_t caprev = 0) {
44
           for (int &i = edgeview[nod]; i < graph[nod].size(); i++) {</pre>
45
               int adj = graph[nod][i];
                                                                                     30
                                                                                                 Edge forward{ e, graph[e].size(), cap };
               int next = matched[adi];
                                                                                     31
                                                                                                 Edge reverse{ s, graph[s].size(), caprev };
46
47
               if (next >= 0 && level[next] != level[nod] + 1) continue;
                                                                                     32
                                                                                                 graph[s].push back(forward);
               if (next == -1 || findpath(next)) {
                                                                                     33
48
                                                                                                 graph[e].push back(reverse);
49
                   match[nod] = adj;
                                                                                     34
                   matched[adj] = nod;
                                                                                     35
                                                                                            bool assign level(int source, int sink) {
50
                                                                                     36
51
                   return 1;
                                                                                                 int t = 0;
52
              }
                                                                                     37
                                                                                                 memset(&1[0], 0, sizeof(1[0]) * 1.size());
                                                                                     38
53
           }
                                                                                                 1[source] = 1;
54
                                                                                     39
                                                                                                 q[t++] = source;
           return 0;
55
                                                                                     40
                                                                                                 for (int h = 0; h < t && !1[sink]; h++) {</pre>
      }
                                                                                     41
56
                                                                                                     int cur = q[h];
57
      int solve() {
                                                                                     42
                                                                                                     for (const auto& e : graph[cur]) {
58
           int ans = 0;
                                                                                     43
                                                                                                         if (1[e.next] || e.res == 0) continue;
59
           while (assignLevel()) {
                                                                                                        l[e.next] = l[cur] + 1;
60
               edgeview.assign(n, 0);
                                                                                     45
                                                                                                         q[t++] = e.next;
61
               for (int i = 0; i < n; i++)
                                                                                     46
                                                                                                     }
                                                                                     47
62
                   if (match[i] == -1)
63
                       ans += findpath(i);
                                                                                     48
                                                                                                 return l[sink] != 0;
64
           }
                                                                                     49
65
                                                                                     50
                                                                                            flow t block flow(int cur, int sink, flow t current) {
           return ans;
                                                                                     51
                                                                                                 if (cur == sink) return current;
67 };
                                                                                     52
                                                                                                 for (int& i = start[cur]; i < graph[cur].size(); i++) {</pre>
                                                                                     53
                                                                                                     auto& e = graph[cur][i];
                                                                                                     if (e.res == 0 || 1[e.next] != 1[cur] + 1) continue;
                                                                                     54
       Maximum Flow (Dinic)
                                                                                     55
                                                                                                     if (flow t res = block flow(e.next, sink, min(e.res, current))) {
                                                                                     56
                                                                                                         e.res -= res;
1 // usage:
                                                                                     57
                                                                                                         graph[e.next][e.inv].res += res;
2 // MaxFlowDinic::init(n):
                                                                                     58
                                                                                                         return res;
3 // MaxFlowDinic::add_edge(0, 1, 100, 100); // for bidirectional edge
                                                                                     59
                                                                                                     }
4 // MaxFlowDinic::add edge(1, 2, 100); // directional edge
                                                                                     60
5 // result = MaxFlowDinic::solve(0, 2); // source -> sink
                                                                                     61
                                                                                                 return 0;
6 // graph[i][edgeIndex].res -> residual
                                                                                     62
                                                                                     63
                                                                                            flow_t solve(int source, int sink) {
8 // in order to find out the minimum cut, use `l'.
                                                                                     64
                                                                                                q.resize(n);
9 // if L[i] == 0, i is unrechable.
                                                                                     65
                                                                                                 1.resize(n);
                                                                                                start.resize(n);
10 //
                                                                                     66
11 // O(V*V*E)
                                                                                     67
                                                                                                 flow t ans = 0;
12 // with unit capacities, O(\min(V^{(2/3)}, E^{(1/2)}) * E)
                                                                                     68
                                                                                                 while (assign level(source, sink)) {
13 struct MaxFlowDinic {
                                                                                     69
                                                                                                     memset(&start[0], 0, sizeof(start[0]) * n);
      typedef int flow_t;
                                                                                     70
                                                                                                     while (flow t flow = block flow(source, sink, numeric limits<
15
       struct Edge {
                                                                                                      flow_t>::max()))
16
           int next;
                                                                                     71
                                                                                                         ans += flow;
17
           int inv; /* inverse edge index */
                                                                                     72
                                                                                                }
18
           flow t res; /* residual */
```

```
return ans:
                                                                                    48
                                                                                                    v[cur] = 0; q.pop();
74
                                                                                    49
                                                                                                    if (++relax count[cur] >= n) return false;
      }
75 };
                                                                                    50
                                                                                                   for (const auto &e : graph[cur]) {
                                                                                    51
                                                                                                        if (iszerocap(e.residual capacity)) continue;
                                                                                    52
                                                                                                        auto next = e.target;
  5.10 Min-cost Maximum Flow
                                                                                    53
                                                                                                        auto ncost = dist[cur] + e.cost;
                                                                                    54
                                                                                                       if (dist[next] > ncost) {
1 // precondition: there is no negative cycle.
                                                                                    55
                                                                                                            dist[next] = ncost;
2 // usage:
                                                                                    56
                                                                                                            if (v[next]) continue;
3 // MinCostFlow mcf(n);
                                                                                    57
                                                                                                            v[next] = 1; q.push(next);
4 // for(each edges) mcf.addEdge(from, to, cost, capacity);
                                                                                    58
5 // mcf.solve(source, sink); // min cost max flow
                                                                                    59
                                                                                                   }
6 // mcf.solve(source, sink, 0); // min cost flow
                                                                                    60
7 // mcf.solve(source, sink, goal flow); // min cost flow with total flow >=
                                                                                    61
                                                                                                for (int i = 0; i < n; i++) pi[i] = dist[i];</pre>
    goal flow if possible
                                                                                    62
                                                                                                return true:
8 struct MinCostFlow
                                                                                    63
                                                                                           }
9 {
                                                                                    64
10
       typedef int cap t;
                                                                                    65
                                                                                           pair<cost_t, cap_t> AugmentShortest(int s, int e, cap_t flow_limit) {
11
       typedef int cost t;
                                                                                                auto infinite cost = numeric limits<cost t>::max();
                                                                                    66
12
                                                                                    67
                                                                                                auto infinite flow = numeric limits<cap t>::max();
13
       bool iszerocap(cap t cap) { return cap == 0; }
                                                                                    68
                                                                                                typedef pair<cost_t, int> pq_t;
14
                                                                                    69
                                                                                                priority_queue<pq_t, vector<pq_t>, greater<pq_t>> pq;
15
       struct edge {
                                                                                    70
                                                                                                vector<pair<cost t, cap t>> dist(n, make pair(infinite cost, 0));
16
          int target;
                                                                                    71
                                                                                                vector<int> from(n, -1), v(n);
17
          cost t cost;
                                                                                    72
          cap t residual capacity;
                                                                                    73
18
                                                                                                if (needNormalize || (ranbefore && lastStart != s))
19
                                                                                    74
          cap_t orig_capacity;
                                                                                                    normalize(s):
                                                                                               ranbefore = true;
20
           size_t revid;
                                                                                    75
21
      };
                                                                                    76
                                                                                                lastStart = s;
22
                                                                                    77
23
                                                                                    78
                                                                                                dist[s] = pair<cost_t, cap_t>(0, infinite_flow);
24
      vector<vector<edge>> graph;
                                                                                    79
                                                                                                pq.emplace(dist[s].first, s);
25
       vector<cost t> pi;
                                                                                    80
                                                                                                while(!pq.empty()) {
26
       bool needNormalize, ranbefore;
                                                                                    81
                                                                                                    auto cur = pq.top().second; pq.pop();
27
       int lastStart;
                                                                                    82
                                                                                                   if (v[cur]) continue;
28
                                                                                    83
                                                                                                   v[cur] = 1;
      MinCostFlow(int n) : graph(n), n(n), pi(n, 0), needNormalize(false),
29
                                                                                                   if (cur == e) continue;
                                                                                    84
        ranbefore(false) {}
                                                                                    85
                                                                                                   for (const auto &e : graph[cur]) {
30
       void addEdge(int s, int e, cost t cost, cap t cap)
                                                                                    86
                                                                                                        auto next = e.target;
31
                                                                                    87
                                                                                                        if (v[next]) continue;
32
          if (s == e) return;
                                                                                    88
                                                                                                        if (iszerocap(e.residual capacity)) continue;
          edge forward={e, cost, cap, cap, graph[e].size()};
                                                                                                        auto ncost = dist[cur].first + e.cost - pi[next] + pi[cur];
33
                                                                                    89
34
           edge backward={s, -cost, 0, 0, graph[s].size()};
                                                                                    90
                                                                                                        auto nflow = min(dist[cur].second, e.residual capacity);
35
          if (cost < 0 || ranbefore) needNormalize = true;</pre>
                                                                                    91
                                                                                                        if (dist[next].first <= ncost) continue;</pre>
           graph[s].emplace back(forward);
36
                                                                                    92
                                                                                                        dist[next] = make pair(ncost, nflow);
37
           graph[e].emplace back(backward);
                                                                                    93
                                                                                                        from[next] = e.revid;
38
                                                                                    94
                                                                                                        pq.emplace(dist[next].first, next);
39
       bool normalize(int s) {
                                                                                    95
                                                                                                   }
40
          auto infinite cost = numeric limits<cost t>::max();
                                                                                    96
41
          vector<cost t> dist(n, infinite cost);
                                                                                    97
                                                                                                /** augment the shortest path **/
42
          dist[s] = 0;
                                                                                    98
                                                                                                auto p = e;
43
           queue<int> q;
                                                                                    99
                                                                                                auto pathcost = dist[p].first + pi[p] - pi[s];
44
          vector<int> v(n), relax count(n);
                                                                                    100
                                                                                                auto flow = dist[p].second;
45
          v[s] = 1; q.push(s);
                                                                                   101
                                                                                                if (iszerocap(flow)|| (flow limit <= 0 && pathcost >= 0)) return pair
46
          while(!q.empty()) {
                                                                                                 cost t, cap t>(0, 0);
47
               int cur = q.front();
```

```
102
            if (flow limit > 0) flow = min(flow, flow limit);
                                                                                      23
                                                                                             }
103
            /* update potential */
                                                                                      24
104
            for (int i = 0; i < n; i++) {
                                                                                      25
                                                                                             pair<cap_t, pair<int, int>> stMinCut(vector<int> &active) {
105
                if (iszerocap(dist[i].second)) continue;
                                                                                      26
                                                                                                  vector<cap t> key(n);
106
                pi[i] += dist[i].first;
                                                                                      27
                                                                                                  vector<int> v(n);
                                                                                      28
                                                                                                  int s = -1, t = -1;
107
108
            while (from[p] != -1) {
                                                                                      29
                                                                                                  for (int i = 0; i < active.size(); i++) {</pre>
109
                auto nedge = from[p];
                                                                                      30
                                                                                                      cap t maxv = -1;
110
                auto np = graph[p][nedge].target;
                                                                                      31
                                                                                                      int cur = -1;
111
                                                                                      32
                auto fedge = graph[p][nedge].revid;
                                                                                                      for (auto j : active) {
112
                                                                                      33
                graph[p][nedge].residual_capacity += flow;
                                                                                                          if (v[j] == 0 \&\& maxv < key[j]) {
113
                                                                                      34
                graph[np][fedge].residual capacity -= flow;
                                                                                                              maxv = key[j];
                                                                                      35
114
                p = np;
                                                                                                              cur = j;
115
            }
                                                                                      36
                                                                                                         }
116
            return make pair(pathcost * flow, flow);
                                                                                      37
                                                                                                      }
117
                                                                                      38
                                                                                                      t = s; s = cur;
                                                                                      39
118
                                                                                                     v[cur] = 1;
119
        pair<cost_t,cap_t> solve(int s, int e, cap_t flow_minimum = numeric_limits
                                                                                                      for (auto j : active) key[j] += graph[cur][j];
          <cap t>::max()) {
                                                                                      41
120
            cost t total cost = 0;
                                                                                      42
                                                                                                  return make pair(key[s], make pair(s, t));
121
            cap_t total_flow = 0;
                                                                                      43
                                                                                             }
122
            for(;;) {
                                                                                      44
123
                auto res = AugmentShortest(s, e, flow minimum - total flow);
                                                                                      45
                                                                                             vector<int> cut;
124
                if (res.second <= 0) break;</pre>
                                                                                      46
125
                total cost += res.first;
                                                                                      47
                                                                                             cap t solve() {
126
                total flow += res.second;
                                                                                      48
                                                                                                  cap_t res = numeric_limits<cap_t>::max();
127
                                                                                      49
                                                                                                  vector<vector<int>> grps;
128
            return make pair(total cost, total flow);
                                                                                      50
                                                                                                  vector<int> active;
129
                                                                                      51
                                                                                                  cut.resize(n);
130 };
                                                                                      52
                                                                                                  for (int i = 0; i < n; i++) grps.emplace back(1, i);
                                                                                      53
                                                                                                  for (int i = 0; i < n; i++) active.push back(i);</pre>
                                                                                      54
                                                                                                  while (active.size() >= 2) {
   5.11 General Min-cut (Stoer-Wagner)
                                                                                      55
                                                                                                      auto stcut = stMinCut(active);
                                                                                      56
                                                                                                      if (stcut.first < res) {</pre>
 1 // implementation of Stoer-Wagner algorithm
                                                                                      57
                                                                                                          res = stcut.first;
 2 // O(V^3)
                                                                                      58
                                                                                                          fill(cut.begin(), cut.end(), 0);
 3 //usage
                                                                                      59
                                                                                                          for (auto v : grps[stcut.second.first]) cut[v] = 1;
 4 // MinCut mc;
                                                                                      60
                                                                                                     }
 5 // mc.init(n);
                                                                                      61
 6 // for (each edge) mc.addEdge(a,b,weight);
                                                                                      62
                                                                                                      int s = stcut.second.first, t = stcut.second.second;
                                                                                      63
 7 // mincut = mc.solve();
                                                                                                      if (grps[s].size() < grps[t].size()) swap(s, t);</pre>
 8 // mc.cut = \{0,1\}^n describing which side the vertex belongs to.
                                                                                      64
 9 struct MinCutMatrix
                                                                                      65
                                                                                                      active.erase(find(active.begin(), active.end(), t));
 10 {
                                                                                      66
                                                                                                      grps[s].insert(grps[s].end(), grps[t].begin(), grps[t].end());
 11
        typedef int cap t;
                                                                                      67
                                                                                                      for (int i = 0; i < n; i++) { graph[i][s] += graph[i][t]; graph[i</pre>
 12
        int n;
                                                                                                       ][t] = 0; }
 13
        vector<vector<cap t>> graph;
                                                                                      68
                                                                                                      for (int i = 0; i < n; i++) { graph[s][i] += graph[t][i]; graph[t
 14
                                                                                                       ][i] = 0; }
 15
        void init(int _n) {
                                                                                      69
                                                                                                      graph[s][s] = 0;
 16
                                                                                      70
 17
            graph = vector<vector<cap_t>>(n, vector<cap_t>(n, 0));
                                                                                      71
                                                                                                  return res;
 18
                                                                                      72
 19
        void addEdge(int a, int b, cap t w) {
                                                                                      73 };
 20
           if (a == b) return;
 21
            graph[a][b] += w;
 22
            graph[b][a] += w;
```

6 Geometry

6.1 Basic Operations

```
1 const double eps = 1e-9;
3 inline int diff(double lhs, double rhs) {
       if (lhs - eps < rhs && rhs < lhs + eps) return 0;</pre>
       return (lhs < rhs) ? -1 : 1:
6 }
8 inline bool is between(double check, double a, double b) {
       if (a < b)
           return (a - eps < check && check < b + eps);</pre>
10
11
       else
12
           return (b - eps < check && check < a + eps);</pre>
13 }
14
15 struct Point {
       double x, y;
17
       bool operator==(const Point& rhs) const {
           return diff(x, rhs.x) == 0 && diff(y, rhs.y) == 0;
18
19
20
       Point operator+(const Point& rhs) const {
21
           return Point{ x + rhs.x, y + rhs.y };
22
23
       Point operator-(const Point& rhs) const {
24
           return Point{ x - rhs.x, y - rhs.y };
25
26
       Point operator*(double t) const {
27
           return Point{ x * t, y * t };
28
29 };
30
31 struct Circle {
       Point center;
       double r;
34 };
35
36 struct Line {
       Point pos, dir;
38 };
40 inline double inner(const Point& a, const Point& b) {
       return a.x * b.x + a.y * b.y;
42 }
44 inline double outer(const Point& a, const Point& b) {
45
       return a.x * b.y - a.y * b.x;
46 }
47
48 inline int ccw_line(const Line& line, const Point& point) {
       return diff(outer(line.dir, point - line.pos), 0);
50 }
```

```
51
52 inline int ccw(const Point& a, const Point& b, const Point& c) {
       return diff(outer(b - a, c - a), 0);
54 }
55
 56 inline double dist(const Point& a, const Point& b) {
       return sqrt(inner(a - b, a - b));
58 }
 60 inline double dist2(const Point &a, const Point &b) {
       return inner(a - b, a - b);
 62 }
 63
64 inline double dist(const Line& line, const Point& point, bool segment = false)
65
       double c1 = inner(point - line.pos, line.dir);
 66
       if (segment && diff(c1, 0) <= 0) return dist(line.pos, point);</pre>
       double c2 = inner(line.dir, line.dir);
       if (segment && diff(c2, c1) <= 0) return dist(line.pos + line.dir, point);</pre>
       return dist(line.pos + line.dir * (c1 / c2), point);
 69
70 }
71
72 bool get cross(const Line& a, const Line& b, Point& ret) {
73
       double mdet = outer(b.dir, a.dir);
74
       if (diff(mdet, 0) == 0) return false;
75
       double t2 = outer(a.dir, b.pos - a.pos) / mdet;
76
       ret = b.pos + b.dir * t2;
77
       return true;
78 }
79
 80 bool get segment cross(const Line& a, const Line& b, Point& ret) {
       double mdet = outer(b.dir, a.dir);
82
       if (diff(mdet, 0) == 0) return false;
       double t1 = -outer(b.pos - a.pos, b.dir) / mdet;
       double t2 = outer(a.dir, b.pos - a.pos) / mdet;
       if (!is between(t1, 0, 1) || !is between(t2, 0, 1)) return false;
86
       ret = b.pos + b.dir * t2;
87
       return true;
 88 }
 90 Point inner center(const Point &a, const Point &b, const Point &c) {
       double wa = dist(b, c), wb = dist(c, a), wc = dist(a, b);
92
       double w = wa + wb + wc;
       return Point{ (wa * a.x + wb * b.x + wc * c.x) / w, (wa * a.v + wb * b.v +
          wc * c.y) / w };
94 }
95
 96 Point outer_center(const Point &a, const Point &b, const Point &c) {
       Point d1 = b - a, d2 = c - a;
98
       double area = outer(d1, d2);
99
       double dx = d1.x * d1.x * d2.y - d2.x * d2.x * d1.y
            + d1.y * d2.y * (d1.y - d2.y);
100
       double dy = d1.y * d1.y * d2.x - d2.y * d2.y * d1.x
101
102
            + d1.x * d2.x * (d1.x - d2.y);
       return Point{ a.x + dx / area / 2.0, a.y - dy / area / 2.0 };
103
```

```
104 }
                                                                                    157
105
                                                                                    158 Circle circle from 2pts rad(const Point& a, const Point& b, double r) {
106 vector<Point> circle_line(const Circle& circle, const Line& line) {
                                                                                    159
                                                                                           double det = r * r / dist2(a, b) - 0.25;
        vector<Point> result:
107
                                                                                    160
                                                                                           Circle circle:
108
        double a = 2 * inner(line.dir, line.dir);
                                                                                    161
                                                                                           if (det < 0)
109
        double b = 2 * (line.dir.x * (line.pos.x - circle.center.x)
                                                                                    162
                                                                                                circle.r = -1;
110
            + line.dir.y * (line.pos.y - circle.center.y));
                                                                                    163
                                                                                           else {
111
        double c = inner(line.pos - circle.center, line.pos - circle.center)
                                                                                    164
                                                                                                double h = sqrt(det);
112
            - circle.r * circle.r;
                                                                                    165
                                                                                                // center is to the left of a->b
                                                                                                circle.center = (a + b) * 0.5 + Point{ a.y - b.y, b.x - a.x } * h;
113
        double det = b * b - 2 * a * c;
                                                                                    166
114
        int pred = diff(det, 0);
                                                                                    167
                                                                                                circle.r = r;
115
        if (pred == 0)
                                                                                    168
                                                                                           }
            result.push back(line.pos + line.dir * (-b / a));
                                                                                    169
                                                                                           return circle;
116
117
        else if (pred > 0) {
                                                                                   170 }
118
            det = sart(det);
119
            result.push_back(line.pos + line.dir * ((-b + det) / a));
                                                                                       6.2 Compare angles
120
            result.push back(line.pos + line.dir * ((-b - det) / a));
121
122
        return result;
                                                                                       6.3 Convex Hull
123 }
124
                                                                                     1 // find convex hull
125 vector<Point> circle circle(const Circle& a, const Circle& b) {
                                                                                     2 // O(n*Logn)
        vector<Point> result:
                                                                                     3 vector<Point> convex hull(vector<Point>& dat) {
127
        int pred = diff(dist(a.center, b.center), a.r + b.r);
                                                                                           if (dat.size() <= 3) return dat;</pre>
128
        if (pred > 0) return result;
                                                                                           vector<Point> upper. lower:
129
        if (pred == 0) {
                                                                                           sort(dat.begin(), dat.end(), [](const Point& a, const Point& b) {
130
            result.push_back((a.center * b.r + b.center * a.r) * (1 / (a.r + b.r))
                                                                                                return (a.x == b.x)? a.y < b.y: a.x < b.x;
             );
                                                                                           });
            return result;
131
                                                                                     9
                                                                                           for (const auto& p : dat) {
132
                                                                                                while (upper.size() >= 2 && ccw(*++upper.rbegin(), *upper.rbegin(), p)
133
        double aa = a.center.x * a.center.x + a.center.y * a.center.y - a.r * a.r;
                                                                                                   >= 0) upper.pop back();
134
        double bb = b.center.x * b.center.x + b.center.y * b.center.y - b.r * b.r;
                                                                                     11
                                                                                                while (lower.size() >= 2 && ccw(*++lower.rbegin(), *lower.rbegin(), p)
135
        double tmp = (bb - aa) / 2.0;
                                                                                                   <= 0) lower.pop back();
136
        Point cdiff = b.center - a.center;
                                                                                    12
                                                                                                upper.emplace back(p);
137
        if (diff(cdiff.x, 0) == 0) {
                                                                                    13
                                                                                                lower.emplace back(p);
138
            if (diff(cdiff.y, 0) == 0)
                                                                                    14
                return result; // if (diff(a.r, b.r) == 0): same circle
139
                                                                                    15
                                                                                           upper.insert(upper.end(), ++lower.rbegin(), --lower.rend());
140
            return circle_line(a, Line{ Point{ 0, tmp / cdiff.y }, Point{ 1, 0 }
                                                                                     16
                                                                                           return upper;
             });
                                                                                    17 }
141
142
        return circle line(a,
            Line{ Point{ tmp / cdiff.x, 0 }, Point{ -cdiff.y, cdiff.x } });
143
                                                                                       6.4 Polygon Cut
144 }
146 Circle circle_from_3pts(const Point& a, const Point& b, const Point& c) {
                                                                                     1 // Left side of a->b
        Point ba = b - a, cb = c - b:
                                                                                      2 vector<Point> cut polygon(const vector<Point>& polygon, Line line) {
147
148
        Line p{ (a + b) * 0.5, Point{ ba.y, -ba.x } };
                                                                                           if (!polygon.size()) return polygon;
                                                                                           typedef vector<Point>::const iterator piter;
149
        Line q\{(b + c) * 0.5, Point\{cb.y, -cb.x\}\};
150
        Circle circle;
                                                                                           piter la, lan, fi, fip, i, j;
        if (!get_cross(p, q, circle.center))
151
                                                                                           la = lan = fi = fip = polygon.end();
152
            circle.r = -1;
                                                                                           i = polygon.end() - 1;
                                                                                           bool lastin = diff(ccw_line(line, polygon[polygon.size() - 1]), 0) > 0;
153
        else
                                                                                           for (i = polygon.begin(); j != polygon.end(); j++) {
154
                                                                                     9
            circle.r = dist(circle.center, a);
155
                                                                                    10
                                                                                                bool thisin = diff(ccw_line(line, *j), 0) > 0;
        return circle;
                                                                                    11
                                                                                                if (lastin && !thisin) {
156 }
                                                                                    12
                                                                                                    la = i;
```

```
13
               lan = j;
14
15
          if (!lastin && thisin) {
16
               fi = j;
17
               fip = i;
18
19
          i = j;
20
           lastin = thisin;
21
22
      if (fi == polygon.end()) {
23
           if (!lastin) return vector<Point>();
24
           return polygon;
25
26
       vector<Point> result;
27
       for (i = fi ; i != lan ; i++) {
28
          if (i == polygon.end()) {
29
               i = polygon.begin();
30
               if (i == lan) break;
31
32
           result.push back(*i);
33
34
       Point lc. fc:
       get cross(Line{ *la, *lan - *la }, line, lc);
36
       get_cross(Line{ *fip, *fi - *fip }, line, fc);
37
       result.push back(lc);
38
       if (diff(dist2(lc, fc), 0) != 0) result.push_back(fc);
39
       return result;
40 }
```

6.5 Pick's theorem

격자점으로 구성된 simple polygon이 주어짐. i는 polygon 내부의 격자점 수, b는 polygon 선분 위 격자점 수, A는 polygon의 넓이라고 할 때, 다음과 같은 식이 성립한다.

```
A = i + \frac{b}{2} - 1
```

7 String

7.1 KMP

```
1 typedef vector<int> seq_t;
3 void calculate pi(vector<int>& pi, const seq t& str) {
       pi[0] = -1;
5
       for (int i = 1, j = -1; i < str.size(); i++) {
6
           while (j \ge 0 \&\& str[i] != str[j + 1]) j = pi[j];
7
          if (str[i] == str[j + 1])
8
               pi[i] = ++j;
9
          else
10
               pi[i] = -1;
11
      }
```

```
12 }
13
14 // returns all positions matched
15 // O(|text|+|pattern|)
16 vector<int> kmp(const seq_t& text, const seq_t& pattern) {
17
       vector<int> pi(pattern.size()), ans;
18
       if (pattern.size() == 0) return ans;
19
       calculate_pi(pi, pattern);
20
       for (int i = 0, j = -1; i < text.size(); i++) {
21
           while (j >= 0 && text[i] != pattern[j + 1]) j = pi[j];
22
           if (text[i] == pattern[j + 1]) {
23
               j++;
24
               if (j + 1 == pattern.size()) {
25
                   ans.push_back(i - j);
26
                   j = pi[j];
27
28
29
       }
30
       return ans;
31 }
```

7.2 Aho-Corasick

```
1 #include <algorithm>
 2 #include <vector>
 3 #include <queue>
 4 using namespace std;
6 struct AhoCorasick
7 {
       const int alphabet;
       struct node {
           node() {}
11
           explicit node(int alphabet) : next(alphabet) {}
12
           vector<int> next, report;
13
           int back = 0, output link = 0;
14
       };
15
       int maxid = 0;
16
       vector<node> dfa:
17
       explicit AhoCorasick(int alphabet) : alphabet(alphabet), dfa(1, node(
         alphabet)) { }
18
       template<typename InIt, typename Fn> void add(int id, InIt first, InIt
         last, Fn func) {
19
           int cur = 0;
20
           for ( ; first != last; ++first) {
21
               auto s = func(*first);
22
               if (auto next = dfa[cur].next[s]) cur = next;
23
24
                   cur = dfa[cur].next[s] = (int)dfa.size();
25
                   dfa.emplace_back(alphabet);
26
27
28
           dfa[cur].report.push_back(id);
29
           maxid = max(maxid, id);
30
       }
```

```
31
       void build() {
                                                                                  21
                                                                                             c = 0:
32
                                                                                  22
          queue<int> q;
                                                                                             for (int i = 0; i + 1 < n; i++) {
33
          vector<char> visit(dfa.size());
                                                                                  23
                                                                                                 int a = (bckt[i] != bckt[i + 1]) || (temp[i] >= n - h)
                                                                                  24
34
                                                                                                         visit[0] = 1;
35
          q.push(0);
                                                                                  25
                                                                                                 bckt[i] = c;
36
                                                                                  26
          while(!q.empty()) {
                                                                                                 c += a;
37
              auto cur = q.front(); q.pop();
                                                                                  27
38
              dfa[cur].output_link = dfa[cur].back;
                                                                                  28
                                                                                             bckt[n - 1] = c++;
39
              if (dfa[dfa[cur].back].report.empty())
                                                                                  29
                                                                                             temp.swap(out);
40
                                                                                  30
                  dfa[cur].output_link = dfa[dfa[cur].back].output_link;
                                                                                         }
41
              for (int s = 0; s < alphabet; s++) {</pre>
                                                                                  31
                                                                                         return out;
                  auto &next = dfa[cur].next[s];
                                                                                  32 }
42
                  if (next == 0) next = dfa[dfa[cur].back].next[s];
43
                                                                                  33
44
                  if (visit[next]) continue;
                                                                                  34 // calculates lcp array. it needs suffix array & original sequence.
45
                  if (cur) dfa[next].back = dfa[dfa[cur].back].next[s];
                                                                                  35 // O(n)
                  visit[next] = 1;
                                                                                  36 vector<int> lcp(const vector<T>& in, const vector<int>& sa) {
46
47
                  q.push(next);
                                                                                  37
                                                                                         int n = (int)in.size();
48
              }
                                                                                  38
                                                                                         if (n == 0) return vector<int>();
          }
49
                                                                                  39
                                                                                         vector<int> rank(n), height(n - 1);
50
                                                                                  40
                                                                                         for (int i = 0; i < n; i++) rank[sa[i]] = i;</pre>
                                                                                  41
                                                                                         for (int i = 0, h = 0; i < n; i++) {
51
      template<typename InIt, typename Fn> vector<int> countMatch(InIt first,
        InIt last, Fn func) {
                                                                                  42
                                                                                             if (rank[i] == 0) continue;
                                                                                  43
52
          int cur = 0;
                                                                                             int j = sa[rank[i] - 1];
53
          vector<int> ret(maxid+1);
                                                                                  44
                                                                                             while (i + h < n \& i + h < n \& in[i + h] == in[i + h]) h++;
54
          for (; first != last; ++first) {
                                                                                  45
                                                                                             height[rank[i] - 1] = h;
55
              cur = dfa[cur].next[func(*first)];
                                                                                  46
                                                                                             if (h > 0) h--;
56
              for (int p = cur; p; p = dfa[p].output_link)
                                                                                  47
57
                                                                                  48
                  for (auto id : dfa[p].report) ret[id]++;
                                                                                         return height;
                                                                                  49 }
58
59
          return ret;
60
                                                                                     7.4 Suffix Tree
61 };
```

7.5 Manacher's Algorithm

```
1 typedef char T;
2
3 // calculates suffix array.
4 // O(n*Loan)
5 vector<int> suffix_array(const vector<T>& in) {
       int n = (int)in.size(), c = 0;
7
       vector<int> temp(n), pos2bckt(n), bckt(n), bpos(n), out(n);
       for (int i = 0; i < n; i++) out[i] = i;
9
       sort(out.begin(), out.end(), [&](int a, int b) { return in[a] < in[b]; });</pre>
       for (int i = 0; i < n; i++) {
10
11
          bckt[i] = c;
12
          if (i + 1 == n || in[out[i]] != in[out[i + 1]]) c++;
13
      for (int h = 1; h < n && c < n; h <<= 1) {
14
15
           for (int i = 0; i < n; i++) pos2bckt[out[i]] = bckt[i];</pre>
           for (int i = n - 1; i >= 0; i--) bpos[bckt[i]] = i;
16
17
           for (int i = 0; i < n; i++)
18
               if (out[i] >= n - h) temp[bpos[bckt[i]]++] = out[i];
19
          for (int i = 0; i < n; i++)
20
               if (out[i] >= h) temp[bpos[pos2bckt[out[i] - h]]++] = out[i] - h;
```

Suffix Array with LCP

```
1 // find longest palindromic span for each element in str
2 // O(|str|)
3 void manacher(const string& str, int plen[]) {
       int r = -1, p = -1;
       for (int i = 0; i < str.length(); ++i) {</pre>
           if (i <= r)
               plen[i] = min((2 * p - i >= 0) ? plen[2 * p - i] : 0, r - i);
           else
9
               plen[i] = 0;
10
           while (i - plen[i] - 1 >= 0 \&\& i + plen[i] + 1 < str.length()
11
                   && str[i - plen[i] - 1] == str[i + plen[i] + 1]) {
12
               plen[i] += 1;
13
14
           if (i + plen[i] > r) {
15
               r = i + plen[i];
16
               p = i;
17
18
       }
19 }
```

8 Miscellaneous

```
8.1 Fast I/O
```

```
1 namespace fio {
       const int BSIZE = 524288;
       char buffer[BSIZE];
       int p = BSIZE;
       inline char readChar() {
          if(p == BSIZE) {
              fread(buffer, 1, BSIZE, stdin);
              p = 0;
8
9
          }
10
          return buffer[p++];
11
12
      int readInt() {
13
          char c = readChar();
          while ((c < '0' | c > '9') \&\& c != '-') {
14
15
              c = readChar();
16
17
          int ret = 0; bool neg = c == '-';
          if (neg) c = readChar();
18
19
          while (c >= '0' && c <= '9') {
20
              ret = ret * 10 + c - '0';
21
              c = readChar();
22
          }
23
           return neg ? -ret : ret;
24
25 }
```

8.2 Magic Numbers

소수: 10007, 10009, 10111, 31567, 70001, 1000003, 1000033, 4000037, 1000000007, 10000000009

8.3 Java Examples

```
1 import java.util.Scanner;
3 public class example
4 {
5
       public static void main(String[] args)
6
          Scanner in = new Scanner(System.in);
8
          int T = in.nextInt();
9
          while (T --> 0)
10
11
              String str = in.next();
12
              if (str.matches("[A-F]?A+F+C+[A-F]?"))
13
                   System.out.println("Infected!");
14
              else
```

15

16 17

18 }

System.out.println("Good");