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1 Setting

1.1 vimrc

```
set ts=4 sts=4 sw=4
set ai si nu
```

2 Math

2.1 Basic Arithmetic

```
typedef long long ll;
typedef unsigned long long ull;

// calculate ceil(a/b)
// |a|, |b| <= (2^63)-1 (does not cover -2^63)
ll ceildiv(ll a, ll b) {
    if (b < 0) return ceildiv(-a, -b);
    if (a < 0) return (-a) / b;
    return ((ull)a + (ull)b - 1ull) / b;
}

// calculate floor(a/b)
// |a|, |b| <= (2^63)-1 (does not cover -2^63)
ll floordiv(ll a, ll b) {
    if (b < 0) return floordiv(-a, -b);
    if (a >= 0) return a / b;
    return -(ll)((ull)(-a) + b - 1) / b;
}

// calculate a*b % m
// x86-64 only
ll large_mod_mul(ll a, ll b, ll m)
{

```

```

    return ll((__int128)a*(__int128)b%m);
}

// calculate a*b % m
// |m| < 2^62, x86 available
// O(logb)
ll large_mod_mul(ll a, ll b, ll m)
{
    a %= m; b %= m; ll r = 0, v = a;
    while (b) {
        if (b&1) r = (r + v) % m;
        b >>= 1;
        v = (v << 1) % m;
    }
    return r;
}

// calculate n^k % m
ll modpow(ll n, ll k, ll m) {
    ll ret = 1;
    n %= m;
    while (k) {
        if (k & 1) ret = large_mod_mul(ret, n, m);
        n = large_mod_mul(n, n, m);
        k /= 2;
    }
    return ret;
}

// calculate gcd(a, b)
ll gcd(ll a, ll b) {
    return b == 0 ? a : gcd(b, a % b);
}

// find a pair (c, d) s.t. ac + bd = gcd(a, b)
pair<ll, ll> extended_gcd(ll a, ll b) {
    if (b == 0) return { 1, 0 };
    auto t = extended_gcd(b, a % b);
    return { t.second, t.first - t.second * (a / b) };
}

// find x in [0,m) s.t. ax === gcd(a, m) (mod m)
ll modinverse(ll a, ll m) {
    return (extended_gcd(a, m).first % m + m) % m;
}

// calculate modular inverse for 1 ~ n
void calc_range_modinv(int n, int mod, int ret[]) {
    ret[1] = 1;
    for (int i = 2; i <= n; ++i)
        ret[i] = (ll)(mod - mod/i) * ret[mod%i] % mod;
}

```

2.2 Sieve Methods : Prime, Divisor, Euler phi

```

// find prime numbers in 1 ~ n
// ret[x] = false -> x is prime
// O(n*loglogn)
void sieve(int n, bool ret[]) {
    for (int i = 2; i * i <= n; ++i)
        if (!ret[i])
            for (int j = i * i; j <= n; j += i)
                ret[j] = true;
}

// calculate number of divisors for 1 ~ n
// when you need to calculate sum, change += 1 to += i
// O(n*logn)
void num_of_divisors(int n, int ret[]) {
    for (int i = 1; i <= n; ++i)
        for (int j = i; j <= n; j += i)
            ret[j] += 1;
}

// calculate euler totient function for 1 ~ n
// phi(n) = number of x s.t. 0 < x < n && gcd(n, x) = 1
// O(n*loglogn)
void euler_phi(int n, int ret[]) {
    for (int i = 1; i <= n; ++i) ret[i] = i;
    for (int i = 2; i <= n; ++i)
        if (ret[i] == i)
            for (int j = i; j <= n; j += i)
                ret[j] -= ret[j] / i;
}

```

2.3 Primality Test

```

bool test_witness(ull a, ull n, ull s) {
    if (a >= n) a %= n;
    if (a <= 1) return true;
    ull d = n >> s;
    ull x = modpow(a, d, n);
    if (x == 1 || x == n-1) return true;
    while (s-- > 1) {
        x = large_mod_mul(x, x, n);
        x = x * x % n;
        if (x == 1) return false;
        if (x == n-1) return true;
    }
    return false;
}

// test whether n is prime
// based on miller-rabin test
// O(logn*logn)
bool is_prime(ull n) {
    if (n == 2) return true;
    if (n < 2 || n % 2 == 0) return false;

    ull d = n >> 1, s = 1;

```

```

    for(; (d&1) == 0; s++) d >>= 1;

#define T(a) test_witness(a##ull, n, s)
    if (n < 4759123141ull) return T(2) && T(7) && T(61);
    return T(2) && T(325) && T(9375) && T(28178)
        && T(450775) && T(9780504) && T(1795265022);
#undef T
}

```

2.4 Chinese Remainder Theorem

```

// find x s.t. x === a[0] (mod n[0])
//              === a[1] (mod n[1])
//              ...
// assumption: gcd(n[i], n[j]) = 1
ll chinese_remainder(ll* a, ll* n, int size) {
    if (size == 1) return *a;
    ll tmp = modinverse(n[0], n[1]);
    ll tmp2 = (tmp * (a[1] - a[0]) % n[1] + n[1]) % n[1];
    ll ora = a[1];
    ll tgcd = gcd(n[0], n[1]);
    a[1] = a[0] + n[0] / tgcd * tmp2;
    n[1] *= n[0] / tgcd;
    ll ret = chinese_remainder(a + 1, n + 1, size - 1);
    n[1] /= n[0] / tgcd;
    a[1] = ora;
    return ret;
}

```

2.5 Burnside's Lemma

경우의 수를 세는데, 특정 transform operation(회전, 반사, ..)해서 같은 경우들은 하나로 친다. 전체 경우의 수는?

- 각 operation마다 이 operation을 했을 때 변하지 않는 경우의 수를 센다 (단, “아무것도 하지 않는다”라는 operation도 있어야 함!)

- 전체 경우의 수를 더한 후, operation의 수로 나눈다. (답이 맞다면 항상 나누어 떨어져야 한다)

2.6 Kirchoff's Theorem

그래프의 스패닝 트리의 개수를 구하는 정리.

무향 그래프의 Laplacian matrix L 를 만든다. 이것은 (정점의 차수 대각 행렬) - (인접행렬)이다. L 에서 행과 열을 하나씩 제거한 것을 L' 라 하자. 어느 행/열이든 관계 없다. 그래프의 스패닝 트리의 개수는 $\det(L')$ 이다.

2.7 Fast Fourier Transform

```

void fft(int sign, int n, double *real, double *imag) {
    double theta = sign * 2 * pi / n;
    for (int m = n; m >= 2; m >>= 1, theta *= 2) {
        double wr = 1, wi = 0, c = cos(theta), s = sin(theta);
        for (int i = 0, mh = m >> 1; i < mh; ++i) {
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                double xr = real[j] - real[k], xi = imag[j] - imag[k];
                real[j] += real[k], imag[j] += imag[k];
                real[k] = wr * xr - wi * xi, imag[k] = wr * xi + wi * xr;
            }
            double _wr = wr * c - wi * s, _wi = wr * s + wi * c;
            wr = _wr, wi = _wi;
        }
    }
    for (int i = 1, j = 0; i < n; ++i) {
        for (int k = n >> 1; k > (j ^= k); k >>= 1);
        if (j < i) swap(real[i], real[j]), swap(imag[i], imag[j]);
    }
}

// Compute Poly(a)*Poly(b), write to r; Indexed from 0
// O(n*logn)
int mult(int *a, int n, int *b, int m, int *r) {
    const int maxn = 100;
    static double ra[maxn], rb[maxn], ia[maxn], ib[maxn];
    int fn = 1;
    while (fn < n + m) fn <= 1; // n + m: interested length
    for (int i = 0; i < n; ++i) ra[i] = a[i], ia[i] = 0;
    for (int i = n; i < fn; ++i) ra[i] = ia[i] = 0;
    for (int i = 0; i < m; ++i) rb[i] = b[i], ib[i] = 0;
    for (int i = m; i < fn; ++i) rb[i] = ib[i] = 0;
    fft(1, fn, ra, ia);
    fft(1, fn, rb, ib);
    for (int i = 0; i < fn; ++i) {
        double real = ra[i] * rb[i] - ia[i] * ib[i];
        double imag = ra[i] * ib[i] + rb[i] * ia[i];
        ra[i] = real, ia[i] = imag;
    }
    fft(-1, fn, ra, ia);
    for (int i = 0; i < fn; ++i) r[i] = (int)floor(ra[i] / fn + 0.5);
    return fn;
}

```

2.8 Matrix Operations

2.9 Gaussian Elimination

2.10 Simplex Algorithm

3 Data Structure

3.1 Order statistic tree

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>
#include <functional>
#include <iostream>
using namespace __gnu_pbds;
using namespace std;

// tree<key_type, value_type(set if null), comparator, ...>
using ordered_set = tree<int, null_type, less<int>, rb_tree_tag,
    tree_order_statistics_node_update>;

int main()
{
    ordered_set X;
    for (int i = 1; i < 10; i += 2) X.insert(i); // 1 3 5 7 9
    cout << boolalpha;
    cout << *X.find_by_order(2) << endl; // 5
    cout << *X.find_by_order(4) << endl; // 9
    cout << (X.end() == X.find_by_order(5)) << endl; // true

    cout << X.order_of_key(-1) << endl; // 0
    cout << X.order_of_key(1) << endl; // 0
    cout << X.order_of_key(4) << endl; // 2
    X.erase(3);
    cout << X.order_of_key(4) << endl; // 1
    for (int t : X) printf("%d ", t); // 1 5 7 9
}
```

3.2 Fenwick Tree

```
const int TSIZE = 100000;
int tree[TSIZE + 1];

// Returns the sum from index 1 to p, inclusive
int query(int p) {
    int ret = 0;
    for (; p > 0; p -= p & -p) ret += tree[p];
    return ret;
}
```

```
// Adds val to element with index pos
void add(int p, int val) {
    for (; p <= TSIZE; p += p & -p) tree[p] += val;
}
```

3.3 Segment Tree with Lazy Propagation

3.4 Persistent Segment Tree

3.5 Link/Cut Tree

4 DP

4.1 Convex Hull Optimization

$O(n^2) \rightarrow O(n \log n)$

조건 1) DP 점화식 꼴

$$D[i] = \min_{j < i} (D[j] + b[j] * a[i])$$

조건 2) $b[j] \leq b[j + 1]$

특수조건) $a[i] \leq a[i + 1]$ 도 만족하는 경우, 마지막 쿼리의 위치를 저장해두면 이분검색이 필요없어지기 때문에 amortized $O(n)$ 에 해결할 수 있음

4.2 Divide & Conquer Optimization

$O(kn^2) \rightarrow O(kn \log n)$

조건 1) DP 점화식 꼴

$$D[t][i] = \min_{j < i} (D[t - 1][j] + C[j][i])$$

조건 2) $A[t][i]$ 는 $D[t][i]$ 의 답이 되는 최소의 j 라 할 때, 아래의 부등식을 만족해야 함

$$A[t][i] \leq A[t][i + 1]$$

조건 2-1) 비용 C 가 다음의 사각부등식을 만족하는 경우도 조건 2) 를 만족하게 됨

$$C[a][c] + C[b][d] \leq C[a][d] + C[b][c] \quad (a \leq b \leq c \leq d)$$

4.3 Knuth Optimization

$O(n^3) \rightarrow O(n^2)$

조건 1) DP 점화식 꼴

$$D[i][j] = \min_{i < k < j} (D[i][k] + D[k][j]) + C[i][j]$$

조건 2) 사각 부등식

$$C[a][c] + C[b][d] \leq C[a][d] + C[b][c] \quad (a \leq b \leq c \leq d)$$

조건 3) 단조성

$$C[b][c] \leq C[a][d] \quad (a \leq b \leq c \leq d)$$

결론) 조건 2, 3을 만족한다면 $A[i][j]$ 를 $D[i][j]$ 의 답이 되는 최소의 k 라 할 때, 아래의 부등식을 만족하게 됨

$$A[i][j-1] \leq A[i][j] \leq A[i+1][j]$$

3중 루프를 돌릴 때 위 조건을 이용하면 최종적으로 시간복잡도가 $O(n^2)$ 이 됨

5 Graph

5.1 SCC (Tarjan)

5.2 SCC (Kosaraju)

```
const int MAXN = 100;
vector<int> graph[MAXN], grev[MAXN];
int visit[MAXN], vcnt;
int scc_idx[MAXN], scc_cnt;
vector<int> emit;

void dfs(int nod, vector<int> graph[]) {
    visit[nod] = vcnt;
    for (int next : graph[nod]) {
        if (visit[next] == vcnt) continue;
        dfs(next, graph);
    }
    emit.push_back(nod);
}

// find SCCs in given graph
// O(V+E)
void get_scc() {
    scc_cnt = 0;
    vcnt = 1;
    emit.clear();
    memset(visit, 0, sizeof(visit));

    for (int i = 0; i < n; i++) {
        if (visit[i] == vcnt) continue;
        dfs(i, graph);
    }

    ++vcnt;
    for (auto st : vector<int>(emit.rbegin(), emit.rend())) {
        if (visit[st] == vcnt) continue;
```

```
        emit.clear();
        dfs(st, grev);
        ++scc_cnt;
        for (auto node : emit)
            scc_idx[node] = scc_cnt;
    }
}
```

5.3 2-SAT

5.4 BCC, Cut vertex, Bridge

5.5 Heavy-Light Decomposition

5.6 Bipartite Matching (Hopcroft-Karp)

```
// in: n, m, graph
// out: match, matched
// vertex cover: (reached[0][left_node] == 0) || (reached[1][right_node] == 1)
// O(E*sqrt(V))
struct BipartiteMatching {
    int n, m;
    vector<vector<int>> graph;
    vector<int> matched, match, edgeview, level;
    vector<int> reached[2];
    BipartiteMatching(int n, int m) : n(n), m(m), graph(n), matched(m, -1),
        match(n, -1) {}

    bool assignLevel() {
        bool reachable = false;
        level.assign(n, -1);
        reached[0].assign(n, 0);
        reached[1].assign(m, 0);
        queue<int> q;
        for (int i = 0; i < n; i++) {
            if (match[i] == -1) {
                level[i] = 0;
                reached[0][i] = 1;
                q.push(i);
            }
        }
        while (!q.empty()) {
            auto cur = q.front(); q.pop();
            for (auto adj : graph[cur]) {
                reached[1][adj] = 1;
                auto next = matched[adj];
                if (next == -1) {
                    reachable = true;
                }
                else if (level[next] == -1) {
                    level[next] = level[cur] + 1;
                    reached[0][next] = 1;
```

```

        q.push(next);
    }
}
return reachable;
}

int findpath(int nod) {
    for (int &i = edgeview[nod]; i < graph[nod].size(); i++) {
        int adj = graph[nod][i];
        int next = matched[adj];
        if (next >= 0 && level[next] != level[nod] + 1) continue;
        if (next == -1 || findpath(next)) {
            match[nod] = adj;
            matched[adj] = nod;
            return 1;
        }
    }
    return 0;
}

int solve() {
    int ans = 0;
    while (assignLevel()) {
        edgeview.assign(n, 0);
        for (int i = 0; i < n; i++)
            if (match[i] == -1)
                ans += findpath(i);
    }
    return ans;
}
};

```

5.7 Maximum Flow (Edmonds-Karp)

5.8 Maximum Flow (Dinic)

5.9 Min-cost Maximum Flow

6 Geometry

6.1 Basic Operations

6.2 Compare angles

6.3 Convex Hull

```

// find convex hull
// O(n*logn)

```

```

vector<Point> convex_hull(vector<Point>& dat) {
    if (dat.size() <= 3) return dat;
    vector<Point> upper, lower;
    sort(dat.begin(), dat.end(), [](const Point& a, const Point& b) {
        return (a.x == b.x) ? a.y < b.y : a.x < b.x;
    });
    for (const auto& p : dat) {
        while (upper.size() >= 2 && ccw(++upper.rbegin(), *upper.rbegin(), p)
            >= 0) upper.pop_back();
        while (lower.size() >= 2 && ccw(++lower.rbegin(), *lower.rbegin(), p)
            <= 0) lower.pop_back();
        upper.emplace_back(p);
        lower.emplace_back(p);
    }
    upper.insert(upper.end(), ++lower.rbegin(), --lower.rend());
    return upper;
}

```

6.4 Polygon Cut

6.5 Pick's theorem

격자점으로 구성된 simple polygon이 주어짐. i 는 polygon 내부의 격자점 수, b 는 polygon 선분 위 격자점 수, A 는 polygon의 넓이라고 할 때, 다음과 같은 식이 성립한다.

$$A = i + \frac{b}{2} - 1$$

7 String

7.1 KMP

```

typedef vector<int> seq_t;

void calculate_pi(vector<int>& pi, const seq_t& str) {
    pi[0] = -1;
    int j = -1;
    for (int i = 1; i < str.size(); i++) {
        while (j >= 0 && str[i] != str[j + 1]) j = pi[j];
        if (str[i] == str[j + 1])
            pi[i] = ++j;
        else
            pi[i] = -1;
    }
}

// returns all positions matched
// O(|text|+|pattern|)
vector<int> kmp(seq_t& text, seq_t& pattern) {
    vector<int> pi(pattern.size());
    calculate_pi(pi, pattern);
    vector<int> ans;
}

```

```

if (pattern.size() == 0) return ans;
calculate_pi(pi, pattern);
int j = -1;
for (int i = 0; i < text.size(); i++) {
    while (j >= 0 && text[i] != pattern[j + 1]) j = pi[j];
    if (text[i] == pattern[j + 1]) {
        j++;
        if (j + 1 == pattern.size()) {
            ans.push_back(i - j);
            j = pi[j];
        }
    }
}
return ans;
}
}

```

7.2 Aho-Corasick

```

#include <algorithm>
#include <vector>
#include <queue>
using namespace std;

struct AhoCorasick
{
    const int alphabet;
    struct node {
        node() {}
        explicit node(int alphabet) : next(alphabet) {}
        vector<int> next, report;
        int back = 0, output_link = 0;
    };
    int maxid = 0;
    vector<node> dfa;
    explicit AhoCorasick(int alphabet) : alphabet(alphabet), dfa(1, node(
        alphabet)) {}
    template<typename InIt, typename Fn> void add(int id, InIt first, InIt
        last, Fn func) {
        int cur = 0;
        for (; first != last; ++first) {
            auto s = func(*first);
            if (auto next = dfa[cur].next[s]) cur = next;
            else {
                cur = dfa[cur].next[s] = (int)dfa.size();
                dfa.emplace_back(alphabet);
            }
        }
        dfa[cur].report.push_back(id);
        maxid = max(maxid, id);
    }
    void build() {
        queue<int> q;
        vector<char> visit(dfa.size());
        visit[0] = 1;
        q.push(0);

```

```

        while(!q.empty()) {
            auto cur = q.front(); q.pop();
            dfa[cur].output_link = dfa[cur].back;
            if (dfa[dfa[cur].back].report.empty())
                dfa[cur].output_link = dfa[dfa[cur].back].output_link;
            for (int s = 0; s < alphabet; s++) {
                auto &next = dfa[cur].next[s];
                if (next == 0) next = dfa[dfa[cur].back].next[s];
                if (visit[next]) continue;
                if (cur) dfa[next].back = dfa[dfa[cur].back].next[s];
                visit[next] = 1;
                q.push(next);
            }
        }
    }
    template<typename InIt, typename Fn> vector<int> countMatch(InIt first,
        InIt last, Fn func) {
        int cur = 0;
        vector<int> ret(maxid+1);
        for (; first != last; ++first) {
            cur = dfa[cur].next[func(*first)];
            for (int p = cur; p; p = dfa[p].output_link)
                for (auto id : dfa[p].report) ret[id]++;
        }
        return ret;
    }
};

```

7.3 Suffix Array with LCP

7.4 Suffix Tree

7.5 Manacher's Algorithm

```

// find longest palindromic span for each element in str
// O(|str|)
void manacher(const string& str, int plen[]) {
    int r = -1, p = -1;
    for (int i = 0; i < str.length(); ++i) {
        if (i <= r)
            plen[i] = min((2 * p - i >= 0) ? plen[2 * p - i] : 0, r - i);
        else
            plen[i] = 0;
        while (i - plen[i] - 1 >= 0 && i + plen[i] + 1 < str.length()
            && str[i - plen[i] - 1] == str[i + plen[i] + 1]) {
            plen[i] += 1;
        }
        if (i + plen[i] > r) {
            r = i + plen[i];
            p = i;
        }
    }
}

```

8 Miscellaneous

8.1 Fast I/O

```
namespace fio {
    const int BSIZE = 524288;
    char buffer[BSIZE];
    int p = BSIZE;
    inline char readChar() {
        if(p == BSIZE) {
            fread(buffer, 1, BSIZE, stdin);
            p = 0;
        }
        return buffer[p++];
    }
    int readInt() {
        char c = readChar();
        while ((c < '0' || c > '9') && c != '-') {
            c = readChar();
        }
        int ret = 0; bool neg = c == '-';
        if (neg) c = readChar();
        while (c >= '0' && c <= '9') {
            ret = ret * 10 + c - '0';
            c = readChar();
        }
        return neg ? -ret : ret;
    }
}
```

8.2 Magic Numbers