Actividad Práctica N°3: Diseño de controladores considerado la dinámica del error y la magnitud de la acción de control en sistemas no lineales multivariables

Caso de estudio 1. Sistema de tres variables de estado.

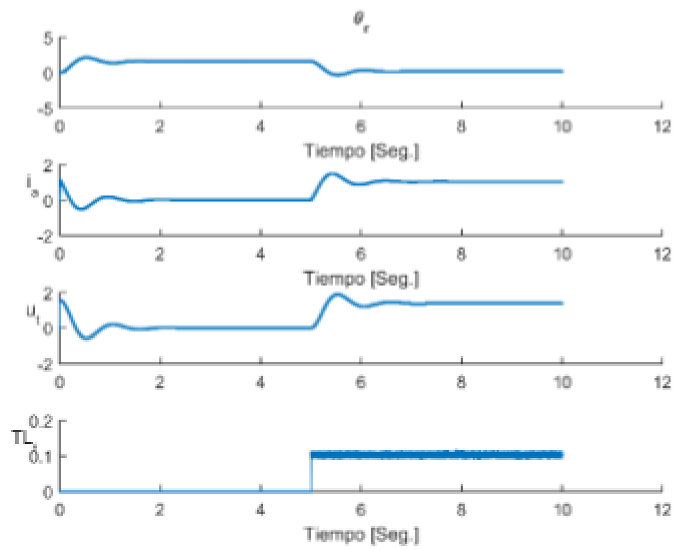


Fig. 1. Evolución del ángulo cuando el controlador tiene perturbaciones en su operación.

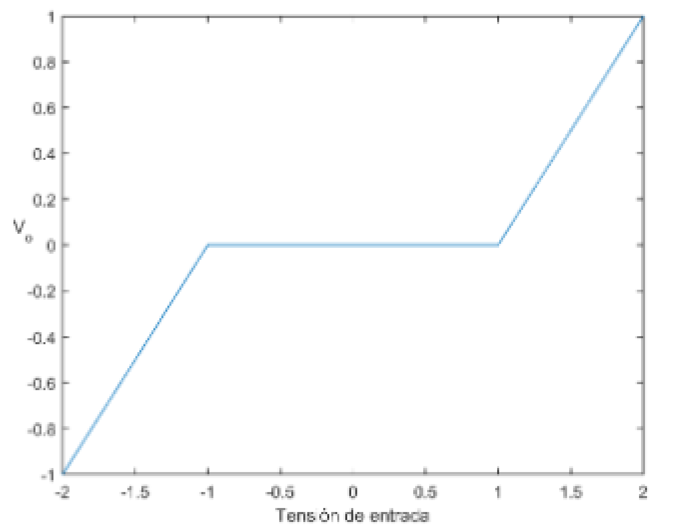


Fig. 2. Relación de entrada salida que tiene el actuador lineal con zona muerta de 1V.

Ítem [1] Implementar un sistema en variables de estado que controle el ángulo del motor para consignas de $\frac{\pi}{2}$ y $-\frac{\pi}{2}$ cambiando cada 5 segundos y que el $T_L = 0.115\,\mathrm{Nm}$ aparece sólo para $\frac{\pi}{2}$, para $-\frac{\pi}{2}$ es nulo. Hallar el valor de integración Euler adecuado.

Objetivo: acelerar la dinámica del controlador que muestra la figura 1 aunque ésta se muestre en otra escala de tiempo, verificando el resultado con las curvas del archivo xlsx adjunto.

- Evitando que la tensión supere los $24\ V$ en valor absoluto, especificar el tiempo de muestreo necesario para que el controlador cumpla el objetivo.
- Asumiento que no puede medirse directamente, proponer un controlador que logre el objetivo.
- Determinar el efecto de la no linealidad en la acción de control, descripta en la figura 2, y verificar cuál es el máximo valor admisible de esa no linealidad.

```
clear all;
close all;
clc;
```

Primero se abre el archivo de Excel con las curvas medidas, de forma de poder comparar posteriormente los resultados obtenidos.

```
file = '/Users/federicovillar/Documents/GitHub/TPsControl2/actividad3/motor.xls';
sheet = 'Hoja1';
[num, txt, raw] = xlsread(file, sheet);
timeExcel = num(:, 1);
angExcel = num(:, 2);
omegaExcel = num(:, 3);
iaExcel = num(:, 4);
vExcel = num(:, 5);
torqueExcel = num(:, 6);
```

Se declaran ahora los parametros del motor.

```
Laa = 0.56e-3;
J = 0.0019;
Ra = 1.35;
Bm = 0.000792;
Ki = 0.1;
Km = 0.1;
```

El modelado en espacio de estados para el sistema en tiempo continuo es:

```
A = [-Ra/Laa -Km/Laa 0; Ki/J -Bm/J 0; 0 1 0]
```

```
A = 3×3
10^3 ×
   -2.4107   -0.1786         0
    0.0526   -0.0004         0
         0    0.0010         0
```

```
B = [1/Laa; 0; 0]
```

```
B = 3×1
10^3 ×
    1.7857
         0
         0
```

```
C = [0 0 1; 0 1 0]
```

```
C = 2×3
     0     0     1
     0     1     0
```

```
D = [0]
```

```
D = 0
```

```
sys = ss(A,B,C,D)
```

```
sys =

  A =
            x1        x2        x3
```

```
   x1    -2411   -178.6        0
   x2    52.63  -0.4168        0
   x3        0        1        0

  B =
         u1
   x1  1786
   x2     0
   x3     0

  C =
       x1  x2  x3
   y1   0   0   1
   y2   0   1   0

  D =
       u1
   y1   0
   y2   0

Continuous-time state-space model.
Model Properties
```

```
pole(sys)
```

```
ans = 3×1
10^3 ×
         0
   -0.0043
   -2.4068
```

Los tiempos de integración y muestreo son:

```
Ts = 1e-4;
h = 1e-5;
```

Se discretiza el sistema.

```
dSys = c2d(sys,Ts,'zoh')
```

```
dSys =

  A =
              x1          x2          x3
   x1     0.7857    -0.01587           0
   x2   0.004677      0.9999           0
   x3  2.432e-07      0.0001           1

  B =
              u1
   x1     0.1587
   x2  0.0004343
   x3  1.476e-08

  C =
       x1  x2  x3
   y1   0   0   1
   y2   0   1   0

  D =
       u1
```

```
   y1   0
   y2   0

Sample time: 0.0001 seconds
Discrete-time state-space model.
Model Properties
```

```
A = dSys.A;
B = dSys.B;
C = dSys.C;
```

Para eliminar el error en estado estable se amplian las matrices y se implementa un integrador.

```
Aamp = [A,zeros(3,1);-C(1,:)*A, 1]
```

```
Aamp = 4×4
    0.7857   -0.0159         0         0
    0.0047    0.9999         0         0
    0.0000    0.0001    1.0000         0
   -0.0000   -0.0001   -1.0000    1.0000
```

```
Bamp = [B;-C(1,:)*B]
```

```
Bamp = 4×1
    0.1587
    0.0004
    0.0000
   -0.0000
```

```
Camp = [C(1,:) 0]
```

```
Camp = 1×4
     0     0     1     0
```

El controlador DLQR queda declarado como a continuacion:

```
Q = diag([1 1 1 0.1]);
R = 0.001;
Kamp = dlqr(Aamp,Bamp,Q,R)
```

```
Kamp = 1×4
    4.8058   14.3855  746.2666   -1.9207
```

```
K = Kamp(1:3)
```

```
K = 1×3
    4.8058   14.3855  746.2666
```

```
Ki = -Kamp(4)
```

```
Ki = 1.9207
```

Como no puede medirse la corriente, se implementa un observador para controlarla.

```
Ao = A';
Bo = C';
Co = B';
Qo = 1*diag([10 10 10]);
Ro = 0.1;
```

```
Ko = dlqr(Ao,Bo,Qo,Ro)
```

```
Ko = 2×3
     0.0000    0.0000    0.9902
    -0.0084    0.9902    0.0001
```

Para poder simular el sistema, los parametros temporales son:

```
T = 10;
h = 1e-5;
Kmax = T/Ts;
t = 0:h:(T);
```

La referencia y el torque aplicado, son respectivamente:

```
thetaRef = (pi/2)*square(2*pi*(1/10)*t);
torqueRef = ((1.5)/2)*square(2*pi*(1/10)*t)+((1.5)/2);
```

La alinealidad del controlador como zona muerta es:

```
deadZone = 1.2
```

```
deadZone = 1.2000
```

Luego, vector de estados y vector observado.

```
x(1,1) = 0;
x(2,1) = 0;
x(3,1) = 0;
x(4,1) = 0;
xObs(1,1) = 0;
xObs(2,1) = 0;
xObs(3,1) = 0;
```

Para simular:

```
xTs = x((1:3),1);
vTs = x(4,1);
z = 1;
for i=1:1:Kmax
    xK = xTs;
    vK = vTs;
    u = -K(1:3)*xObs(1:3)+Ki*vK;
    uPrev = u;
```

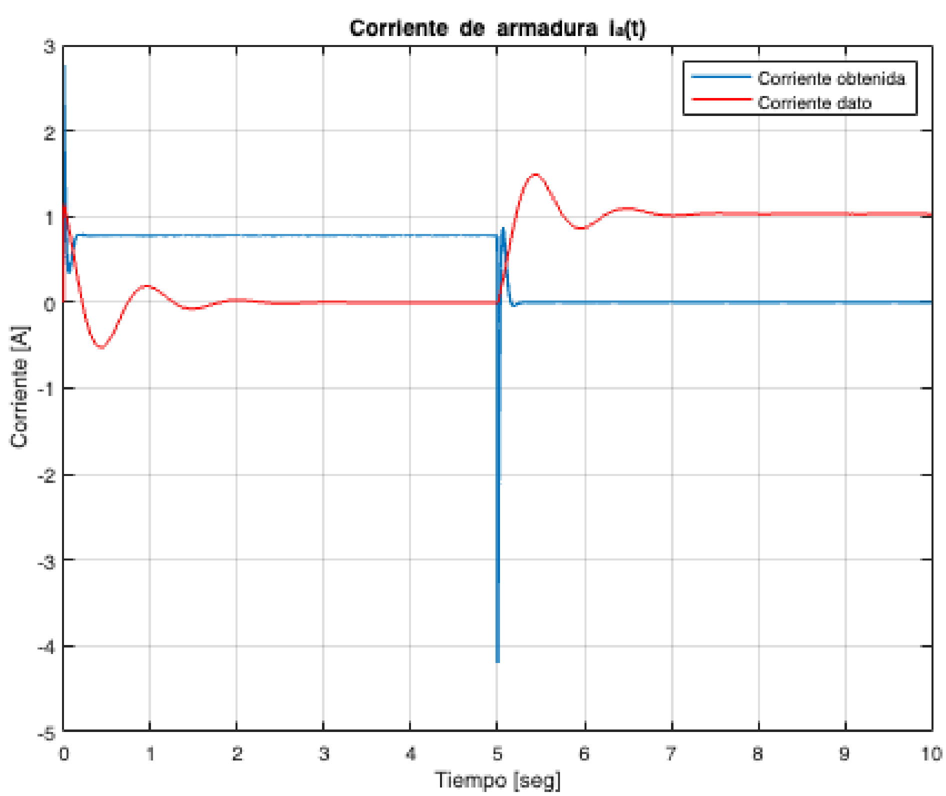
El actuador llega a la alinealidad.

```
    if abs(u)<deadZone
        u = 0;
    else
        u = sign(u)*(abs(u)-deadZone);
    end
    yS = C*x(1:3,z);
    for j = 1:1:Ts/h
        prevU(z) = uPrev;
```

```
        x1P = -Ra*x(1,z)/Laa-Km*x(2,z)/Laa+u/Laa;
        x2P = Ki*x(1,z)/J-Bm*x(2,z)/J-torqueRef(z)/J;
        x3P = x(2,z);
        xP = [x1P; x2P; x3P];
        x((1:3),z+1) = x((1:3),z)+h*xP;
        z = z+1;
    end
    yHat = C*xObs;
    xObs = A*xObs+B*u+Ko'*(yS-yHat);
    vTs = vTs+thetaRef(z)-C(1,:)*xTs;
    xTs = x((1:3),z);
end
```

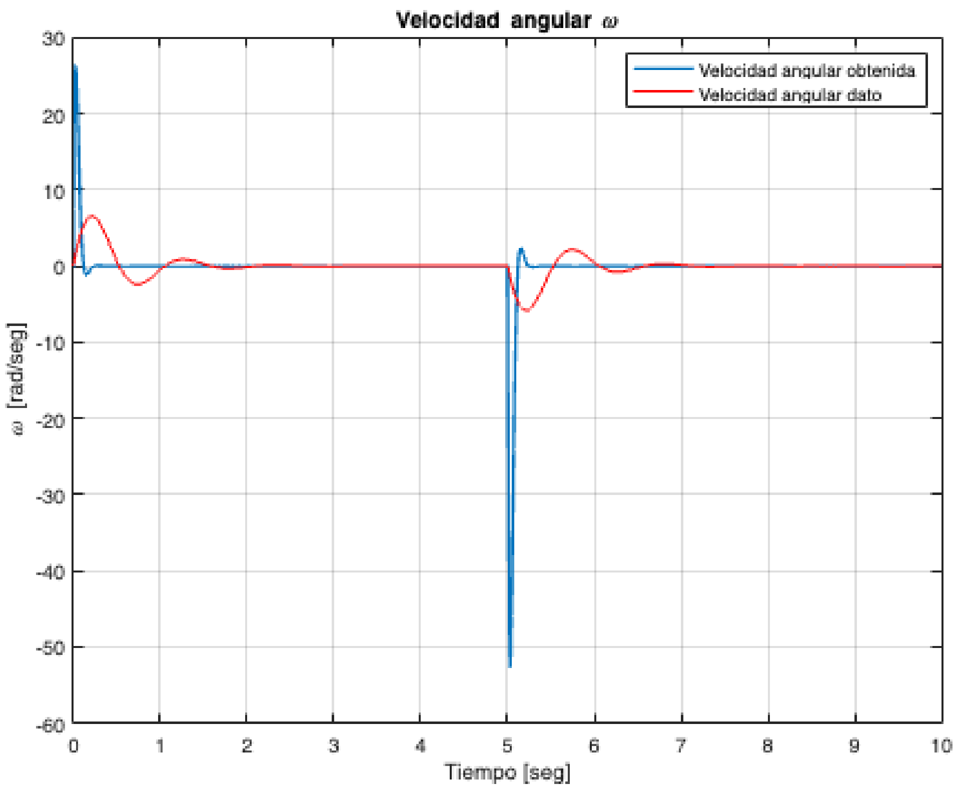
Para comparar, se obtienen las graficas:

```
plot(t(1:length(x(1,:))),x(1,:),'LineWidth',1.5);
hold on;
plot(timeExcel,iaExcel,'r');
hold off
grid;
xlim([0 T]);
legend('Corriente obtenida','Corriente dato');
title('Corriente de armadura i_a(t)');
xlabel('Tiempo [seg]');
ylabel('Corriente [A]');
```

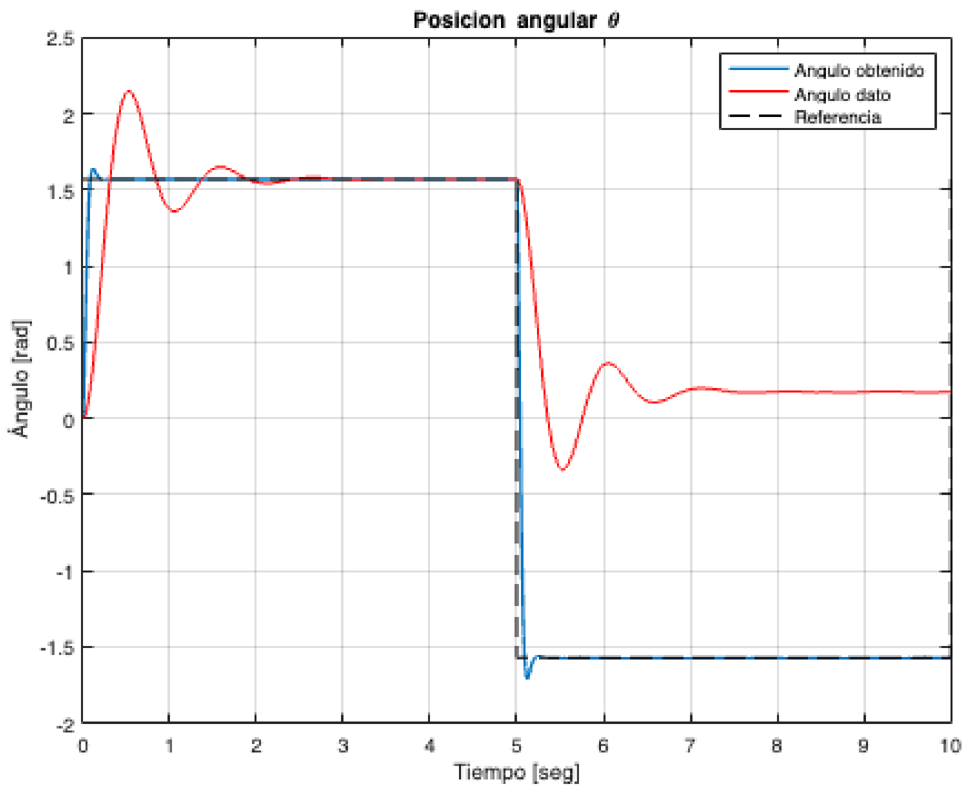



```
plot(t(1:length(x(2,:))),x(2,:),'LineWidth',1.5);
hold on;
```

```
plot(timeExcel,omegaExcel,'r');
hold off;
grid;
xlim([0 T]);
legend('Velocidad angular obtenida','Velocidad angular dato');
title('Velocidad angular \omega');
xlabel('Tiempo [seg]');
ylabel('\omega [rad/seg]');
```

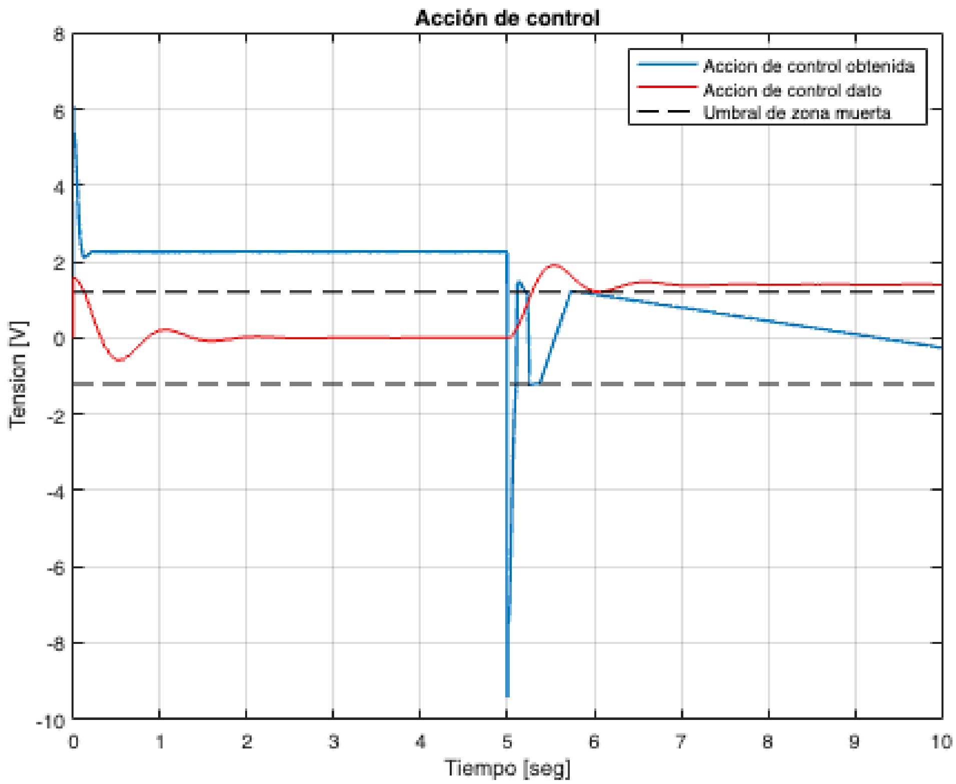



```
plot(t(1:length(x(3,:))),x(3,:),'LineWidth',1.5);
hold on;
plot(timeExcel,angExcel,'r');
plot(t(1:length(thetaRef)),thetaRef,'k--','LineWidth',0.5);
hold off;
grid;
xlim([0 T]);
legend('Angulo obtenido','Angulo dato','Referencia');
title('Posicion angular \theta');
xlabel('Tiempo [seg]');
ylabel('Ángulo [rad]');
```



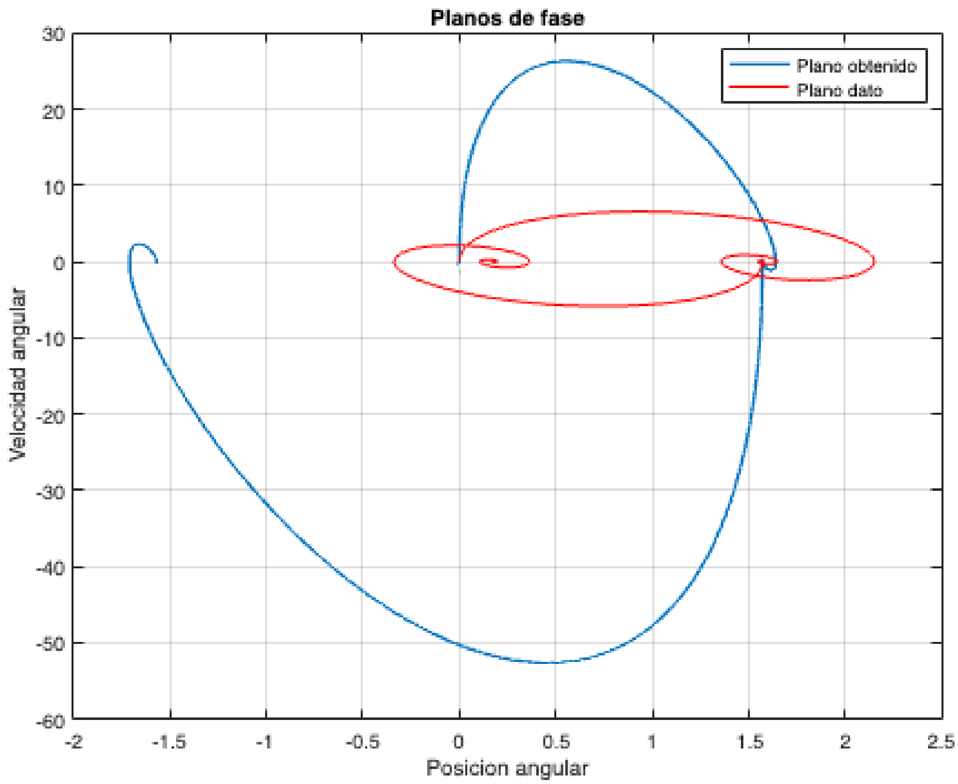
Finalmente, se compara la accion de control.

```
dead = ones(2,length(prevU));
dead(1,:) = dead(1,:)* deadZone;
dead(2,:) = dead(2,:)* (-1)*deadZone;
plot(t(1:length(prevU)),prevU,'LineWidth',1.5);
hold on;
plot(timeExcel,vExcel,'r');
plot(t(1:length(prevU)),dead,'k--');
hold off
xlim([0 T]);
grid;
title('Acción de control');
xlabel('Tiempo [seg]');
ylabel('Tension [V]');
legend('Accion de control obtenida','Accion de control dato','Umbral de zona muerta');
```



El plano de fases es el siguiente:

```
plot(x(3,:),x(2,:),'LineWidth',1.5);
hold on;
plot(angExcel,omegaExcel,'r');
hold off;
grid
xlabel('Posicion angular');
ylabel('Velocidad angular');
title('Planos de fase');
legend('Plano obtenido', 'Plano dato')
```



Caso de estudio 2. Sistema lineal de cuatro variables de estado.

```
clear all;
close all;
clc
```

Primero se declaran los parámetros del sistema.

```
a = 0.07;
b = 5;
c = 150;
w = 9;
```

Su modelado en el espacio de estados es el siguiente:

```
A = [-a a 0 0; 0 0 1 0; w^2 -w^2 0 0; c 0 0 0]
```

```
A = 4×4
   -0.0700    0.0700         0         0
         0         0    1.0000         0
   81.0000  -81.0000         0         0
  150.0000         0         0         0
```

```
B = [0; 0; b*w^2; 0]
```

```
B = 4×1
     0
     0
```

```
   405
     0
```

```
C = [0 0 0 1;0 1 0 0]
```

```
C = 2×4
     0     0     0     1
     0     1     0     0
```

```
D = [0]
```

```
D = 0
```

La consigna pide unos determinados polos a lazo cerrado:

```
p1 = -15+15i;
p2 = -15-15i;
p3 = -0.5+0.5i;
p4 = -0.5-0.5i;
poles = [p1 p2 p3 p4];
K = acker(A,B,poles)
```

```
K = 1×4
   15.4210    0.9811    0.0764    0.0529
```

```
Ap = A-B*K
```

```
Ap = 4×4
10^3 ×
   -0.0001    0.0001         0         0
         0         0    0.0010         0
   -6.1645   -0.4783   -0.0309   -0.0214
    0.1500         0         0         0
```

```
sysCL = ss(Ap,B,C,D)
```

```
sysCL =
 
  A = 
            x1       x2       x3       x4
   x1    -0.07     0.07        0        0
   x2        0        0        1        0
   x3    -6165   -478.3   -30.93   -21.43
   x4      150        0        0        0
 
  B = 
        u1
   x1    0
   x2    0
   x3  405
   x4    0
 
  C = 
       x1  x2  x3  x4
   y1   0   0   0   1
   y2   0   1   0   0
 
  D = 
       u1
   y1   0
   y2   0
```

```
Continuous-time state-space model.
Model Properties
```

```
pole(sysCL)
```

```
ans = 4×1 complex
 -15.0000 +15.0000i
 -15.0000 -15.0000i
  -0.5000 + 0.5000i
  -0.5000 - 0.5000i
```

La ganancia de preamplificación:

```
G = -inv(C(1,:)*inv(Ap)*B)
```

```
G = 0.0529
```

Ahora, se plantea el siguiente observador:

```
Ao = A'
```

```
Ao = 4×4
   -0.0700         0   81.0000  150.0000
    0.0700         0  -81.0000         0
         0    1.0000         0         0
         0         0         0         0
```

```
Bo = C'
```

```
Bo = 4×2
     0     0
     0     1
     0     0
     1     0
```

```
Co = B'
```

```
Co = 1×4
     0     0   405     0
```

```
Qo = diag([100 10000 100 100000]);
Ro = 1000000;
Ko = lqr(Ao,Bo,Qo,Ro)
```

```
Ko = 2×4
    0.0100    0.0036   -0.0118    1.7615
    0.0001    0.0524   -0.0036    0.0036
```

Lo siguiente es para poder simular:

```
T = 100;
h = 1e-4;
t = 0:h:(T-h);
ref = 100;
alpha(1) = 0;
phi(1) = 0;
phiP(1) = 0;
high(1) = 500;
```

```
u(1) = 0;
uu(1) = 0;
stateVector = [alpha(1);phi(1);phiP(1);high(1)];
xOp = [0;0;0;0];
xOp = [alpha(1);phi(1);phiP(1);high(1)];
alphaO(1) = 0;
phiO(1) = 0;
phiPO(1) = 0;
highO(1) = 400;
xObs = [alphaO(1);phiO(1);phiPO(1); highO(1)];
obsStateVector = [alphaO(1);phiO(1);phiPO(1);highO(1)];
deadZone = 0.1
```

```
deadZone = 0.1000
```

Ahora, la simulación con observador:

```
for i=1:T/h
    u(i) = -K*obsStateVector+G*ref;
    if (abs(u(i)) < deadZone)
        uu(i)=0;
    else
        uu(i)=sign(u(i))*(abs(u(i)));
    end
    alpha(i) = xOp(1);
    phi(i) = xOp(2);
    phiP(i) = xOp(3);
    high(i) = xOp(4);
    xP = A*xOp+B*uu(i);
    xOp = xOp+h*xP;
    alphaO(i)= xObs(1);
    phiO(i)= xObs(2);
    phiPO(i)= xObs(3);
    highO(i)= xObs(4);
    yOutObs = C*obsStateVector;
    yOut    = C*stateVector;
    xPrevP = A*xObs+B*uu(i)+Ko'*(yOut-yOutObs);
    xObs = xObs + xPrevP*h;
    stateVector=[alpha(i);phi(i);phiP(i);high(i)];
    obsStateVector=[alphaO(i);phiO(i);phiPO(i);highO(i)];
end
```

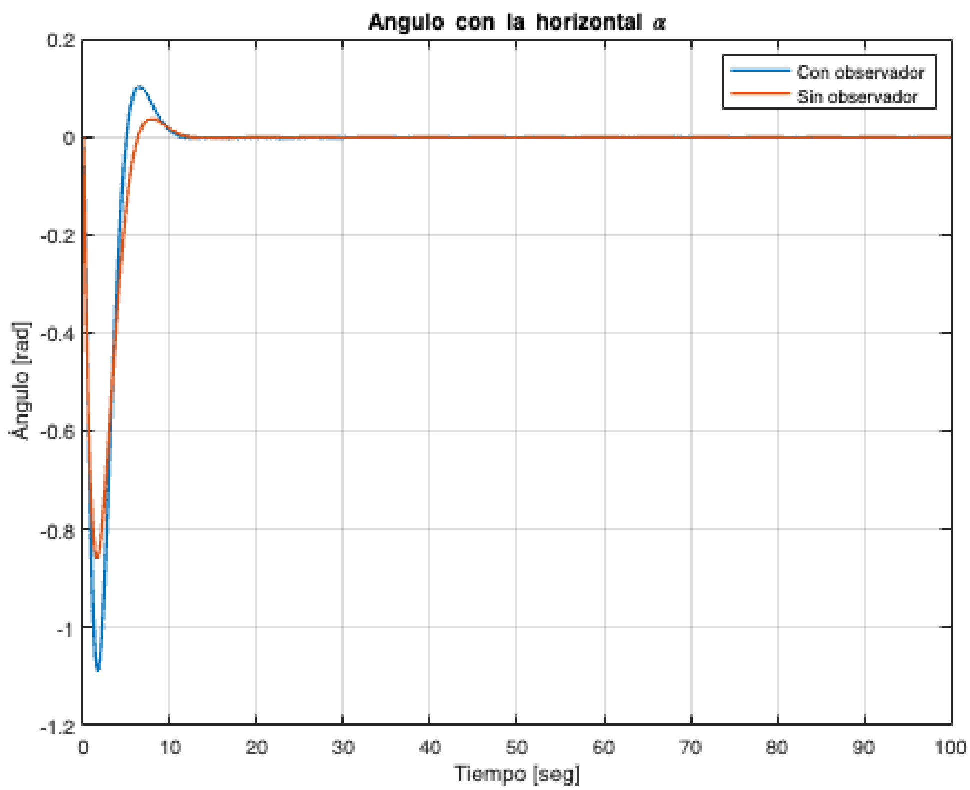
La simulación sin observador:

```
uSO(1)=0;
stateVectorSO=[alpha(1);phi(1);phiP(1);high(1)];
xSO=[alpha(1);phi(1);phiP(1); high(1)];
for i=1:T/h
    uSO(i) = -K*stateVectorSO+G*ref;
    alphaSO(i) = xSO(1);
    phiSO(i) = xSO(2);
    phiPSO(i) = xSO(3);
    highSO(i) = xSO(4);
    xp_so = A*xSO+B*uSO(i);
```

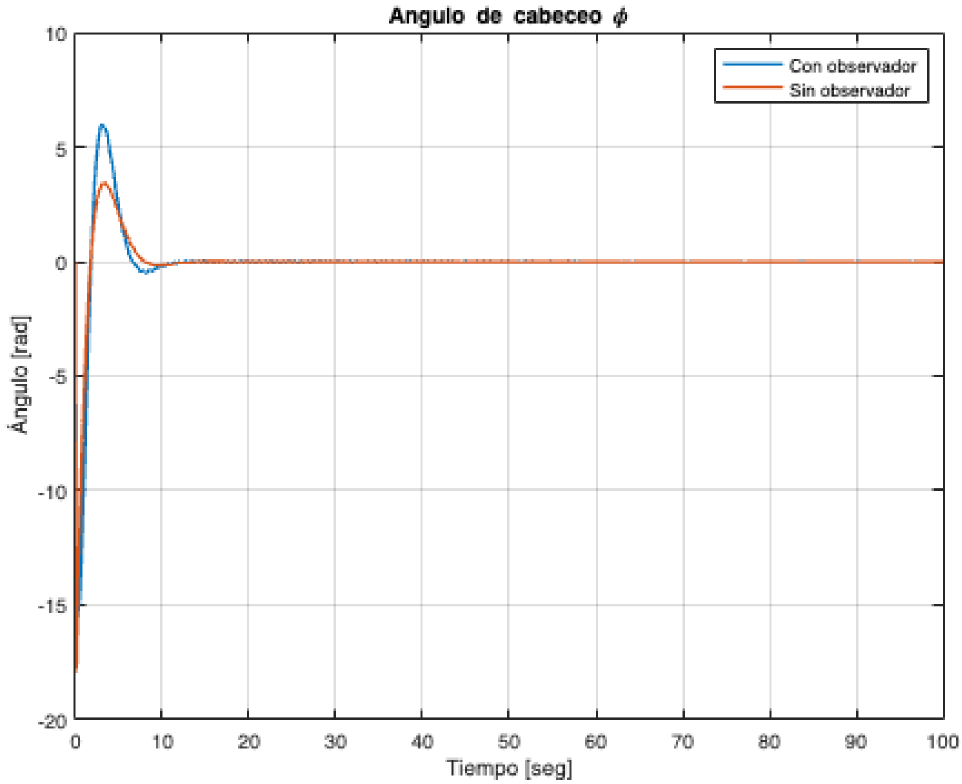
```
    xSO = xSO+h*xp_so;
    stateVectorSO = [alphaSO(i);phiSO(i);phiPSO(i);highSO(i)];
end
```

Se obtienen las siguientes gráficas:

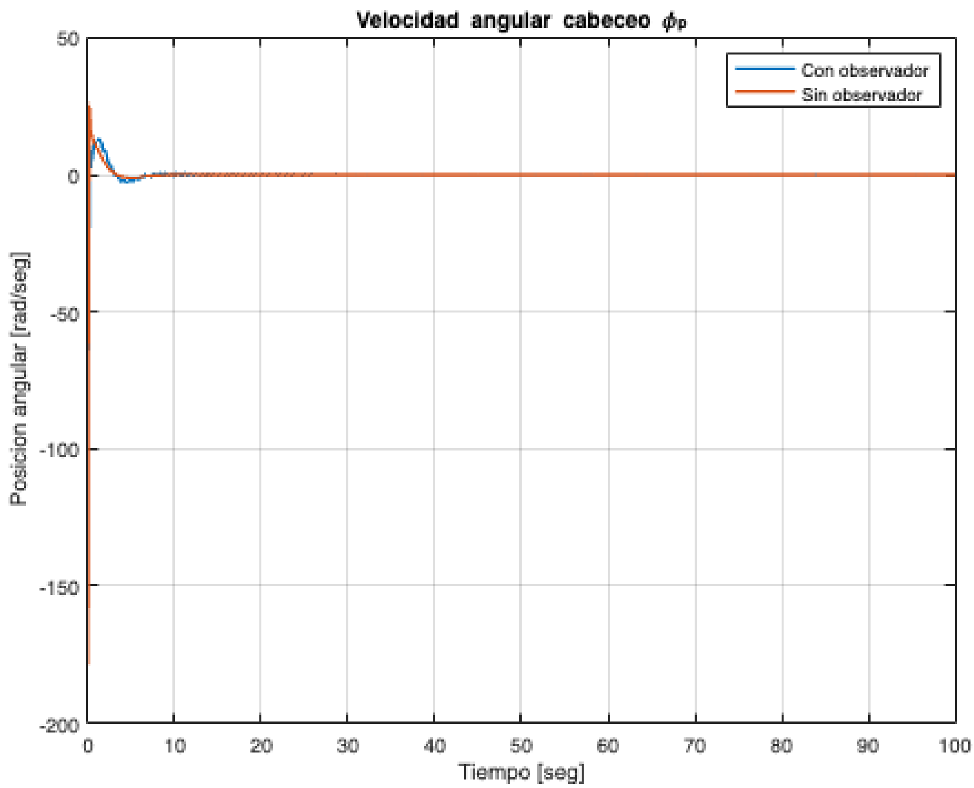
```
plot(t,alpha,'LineWidth',1.5);
hold on;
plot(t,alphaSO,'LineWidth',1.5);
hold off
title('Angulo con la horizontal \alpha');
legend('Con observador','Sin observador')
xlabel('Tiempo [seg]');
ylabel('Ángulo [rad]');
grid;
```



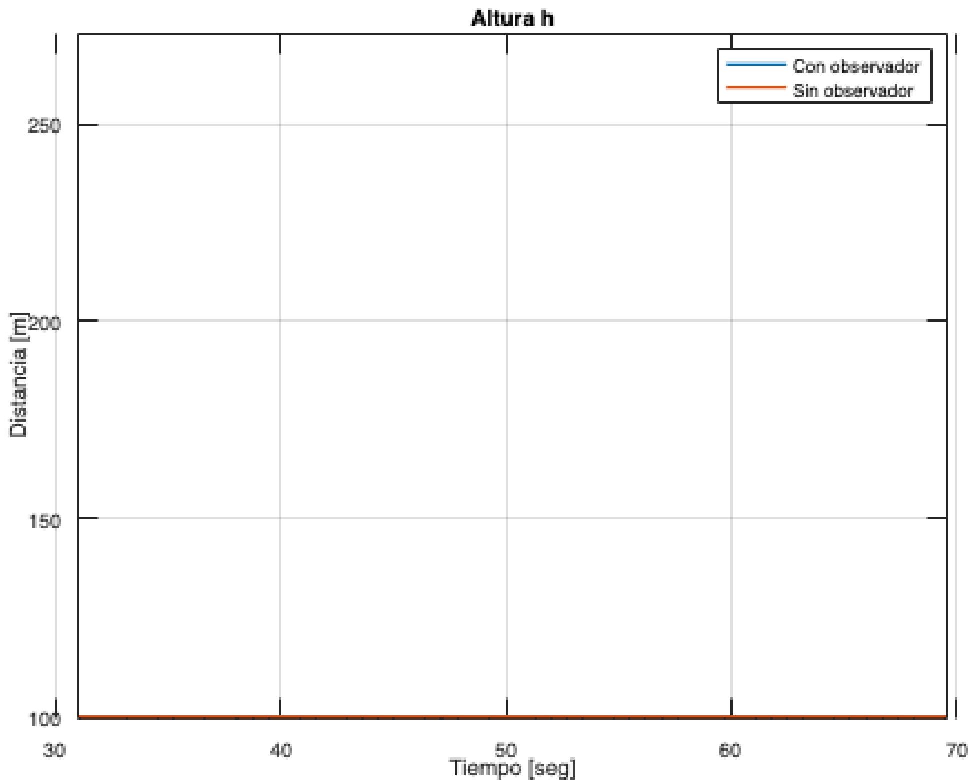

```
plot(t,phi,'LineWidth',1.5);
hold on;
plot(t,phiSO,'LineWidth',1.5);
hold off;
title('Angulo de cabeceo \phi');
legend('Con observador','Sin observador')
xlabel('Tiempo [seg]');
ylabel('Ángulo [rad]');
grid;
```



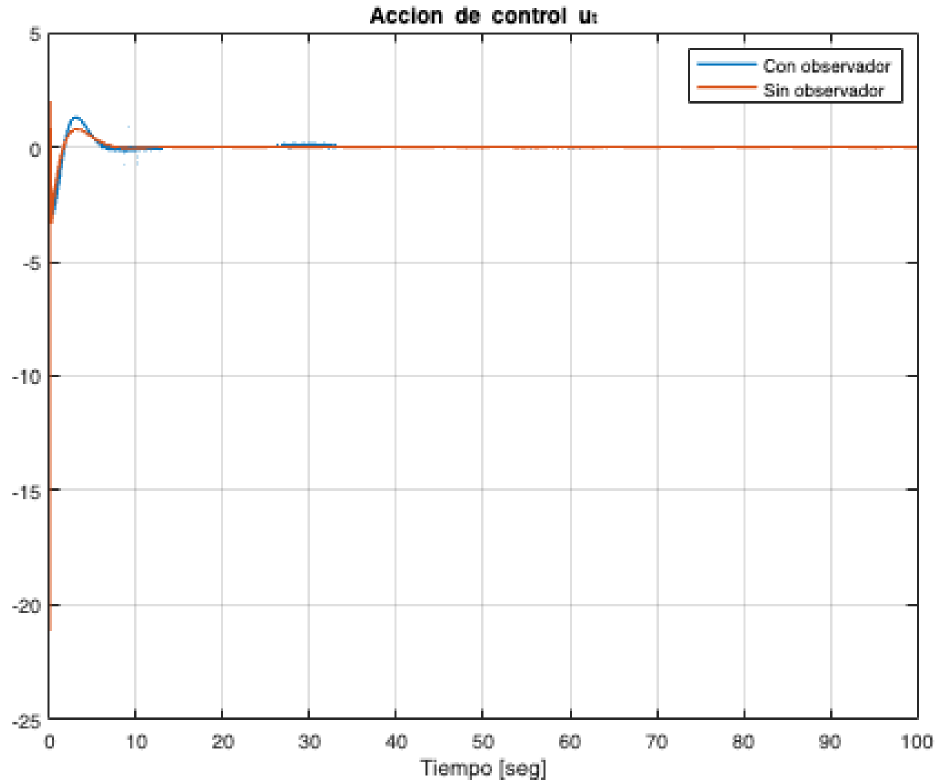

```
plot(t,phiP,'LineWidth',1.5);
hold on;
plot(t,phiPSO,'LineWidth',1.5);
hold off
title('Velocidad angular cabeceo \phi_p');
legend('Con observador','Sin observador')
xlabel('Tiempo [seg]');
ylabel('Posicion angular [rad/seg]');
grid;
```




```
plot(t,high,'LineWidth',1.5);
hold on;
plot(t,highSO,'LineWidth',1.5);
hold off
title('Altura h');
legend('Con observador','Sin observador')
xlabel('Tiempo [seg]');
ylabel('Distancia [m]');
grid;
```




```
plot(t,uu,'LineWidth',1.5);
hold on;
plot(t,uSO,'LineWidth',1.5);
hold off
title('Accion de control u_t');
legend('Con observador','Sin observador')
xlabel('Tiempo [seg]');
grid;
```



Se aprecia en las figuras que se cumple el requerimiento de la magnitud en los ángulos, pero no en la acción de control, por lo que se implementará un controlador del tipo LQR.

Ahora, las simulaciones con el nuevo controlador:

```
clear all;
close all;
clc
```

Primero se declaran los parámetros del sistema.

```
a = 0.07;
b = 5;
c = 150;
w = 9;
```

Su modelado en el espacio de estados es el siguiente:

```
A = [-a a 0 0; 0 0 1 0; w^2 -w^2 0 0; c 0 0 0]
```

```
A = 4×4
   -0.0700    0.0700         0         0
         0         0    1.0000         0
   81.0000  -81.0000         0         0
  150.0000         0         0         0
```

```
B = [0; 0; b*w^2; 0]
```

```
B = 4×1
     0
     0
   405
     0
```

```
C = [0 0 0 1;0 1 0 0]
```

```
C = 2×4
     0     0     0     1
     0     1     0     0
```

```
D = [0];
```

Diseño del LQR:

```
Q = diag([1 1000000 1 1]);
R = 1000000;
K = lqr(A,B,Q,R)
```

```
K = 1×4
    1.5013    0.8255    0.0639    0.0010
```

La ganancia de preamplificación:

```
G = -inv(C(1,:)*inv(A-B*K)*B)
```

```
G = 1.0000e-03
```

Ahora, se plantea el siguiente observador:

```
Ao = A'
```

```
Ao = 4×4
   -0.0700         0   81.0000  150.0000
    0.0700         0  -81.0000         0
         0    1.0000         0         0
         0         0         0         0
```

```
Bo = C'
```

```
Bo = 4×2
     0     0
     0     1
     0     0
     1     0
```

```
Co = B'
```

```
Co = 1×4
     0     0   405     0
```

```
Qo = diag([100 1000 100 1000]);
Ro = 5000000;
Ko = lqr(Ao,Bo,Qo,Ro);
```

Lo siguiente es para poder simular:

```
T = 100;
```

```
h = 1e-4;
t = 0:h:(T-h);
ref = 100;
alpha(1) = 0;
phi(1) = 0;
phiP(1) = 0;
high(1) = 500;
u(1) = 0;
uu(1) = 0;
stateVector = [alpha(1);phi(1);phiP(1);high(1)];
xOp = [0;0;0;0];
xOp = [alpha(1);phi(1);phiP(1);high(1)];
alphaO(1) = 0;
phiO(1) = 0;
phiPO(1) = 0;
highO(1) = 500;
xObs = [alphaO(1);phiO(1);phiPO(1); highO(1)];
obsStateVector = [alphaO(1);phiO(1);phiPO(1);highO(1)];
deadZone = 0.1;
```

Ahora, la simulación con observador:

```
for i=1:T/h
    u(i) = -K*obsStateVector+G*ref;
    if (abs(u(i)) < deadZone)
        uu(i)=0;
    else
        uu(i)=sign(u(i))*(abs(u(i)));
    end
    alpha(i) = xOp(1);
    phi(i) = xOp(2);
    phiP(i) = xOp(3);
    high(i) = xOp(4);
    xP = A*xOp+B*uu(i);
    xOp = xOp+h*xP;
    alphaO(i)= xObs(1);
    phiO(i)= xObs(2);
    phiPO(i)= xObs(3);
    highO(i)= xObs(4);
    yOutObs = C*obsStateVector;
    yOut   = C*stateVector;
    xPrevP = A*xObs+B*uu(i)+Ko'*(yOut-yOutObs);
    xObs = xObs + xPrevP*h;
    stateVector=[alpha(i);phi(i);phiP(i);high(i)];
    obsStateVector=[alphaO(i);phiO(i);phiPO(i);highO(i)];
end
```

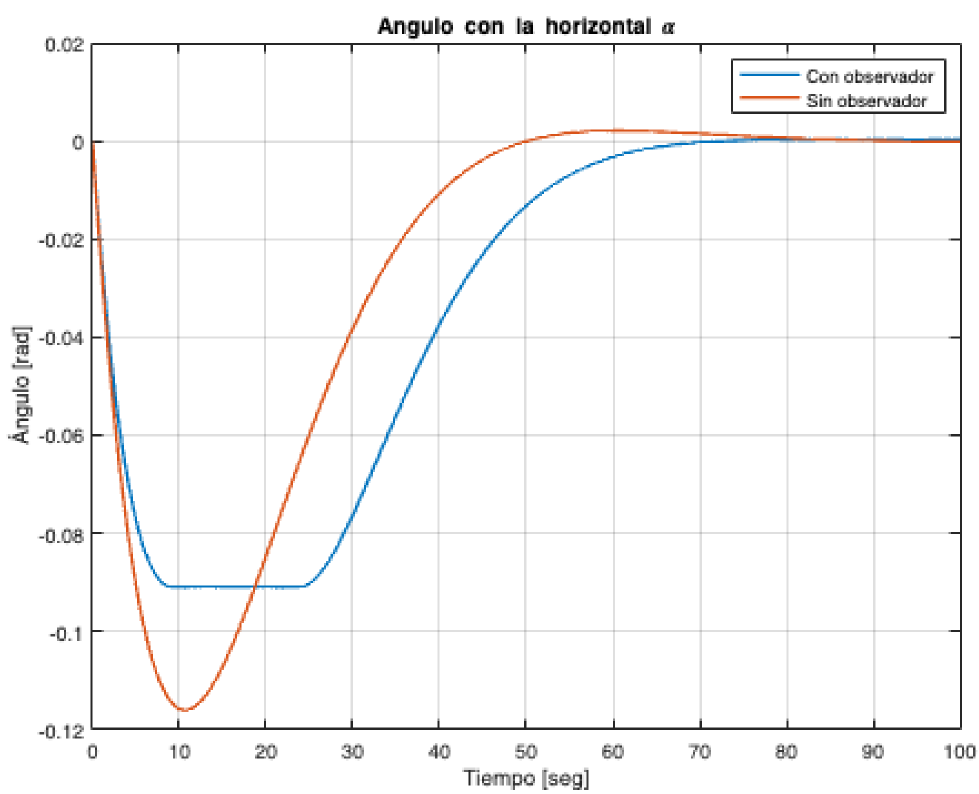
La simulación sin observador:

```
uSO(1)=0;
stateVectorSO=[alpha(1);phi(1);phiP(1);high(1)];
xSO=[alpha(1);phi(1);phiP(1); high(1)];
for i=1:T/h
    uSO(i) = -K*stateVectorSO+G*ref;
```

```
    alphaSO(i) = xSO(1);
    phiSO(i) = xSO(2);
    phiPSO(i) = xSO(3);
    highSO(i) = xSO(4);
    xp_so = A*xSO+B*uSO(i);
    xSO = xSO+h*xp_so;
    stateVectorSO = [alphaSO(i);phiSO(i);phiPSO(i);highSO(i)];
end
```

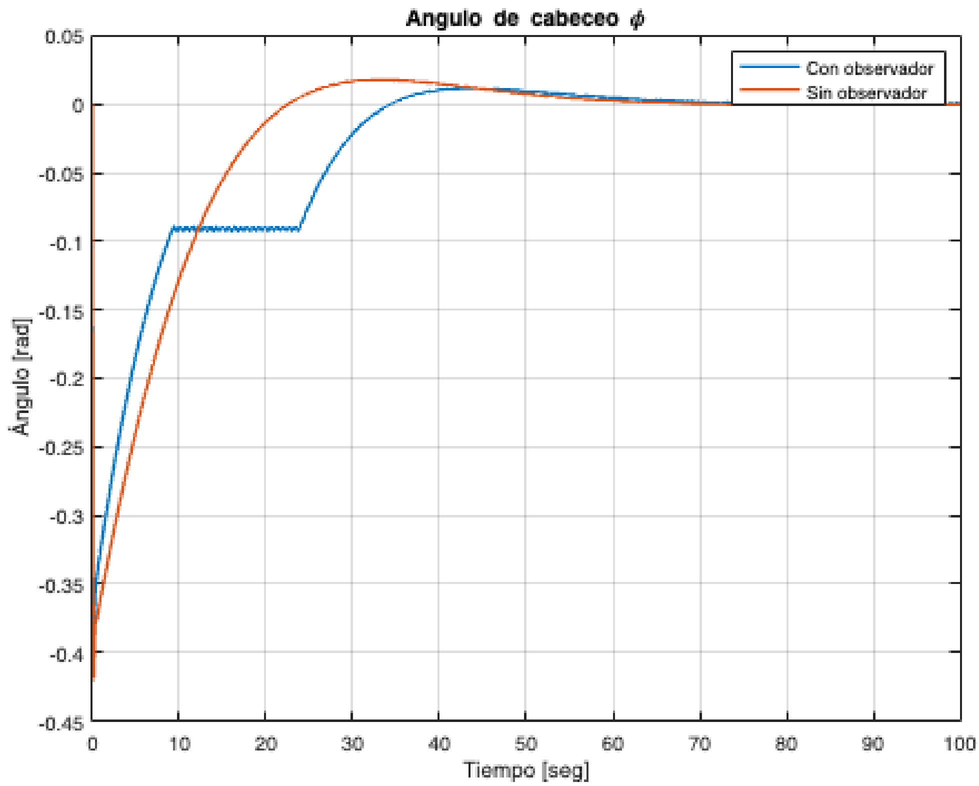
Se obtienen las siguientes gráficas:

```
plot(t,alpha,'LineWidth',1.5);
hold on;
plot(t,alphaSO,'LineWidth',1.5);
hold off
title('Angulo con la horizontal \alpha');
legend('Con observador','Sin observador')
xlabel('Tiempo [seg]');
ylabel('Ángulo [rad]');
grid;
```

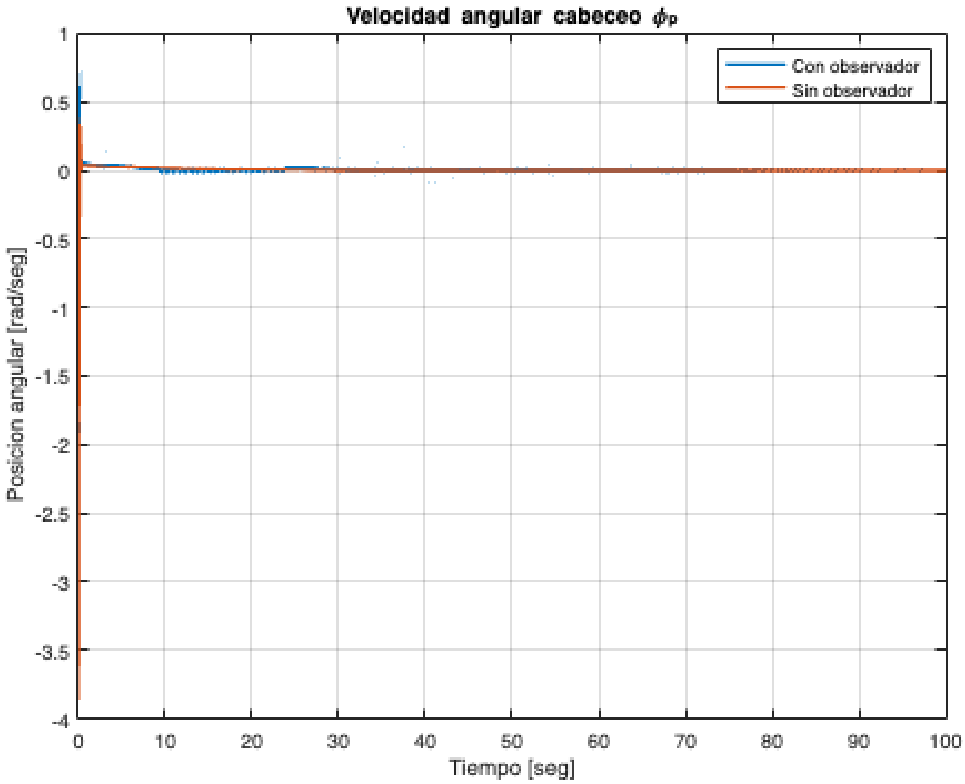



```
plot(t,phi,'LineWidth',1.5);
hold on;
plot(t,phiSO,'LineWidth',1.5);
hold off;
title('Angulo de cabeceo \phi');
legend('Con observador','Sin observador')
xlabel('Tiempo [seg]');
```

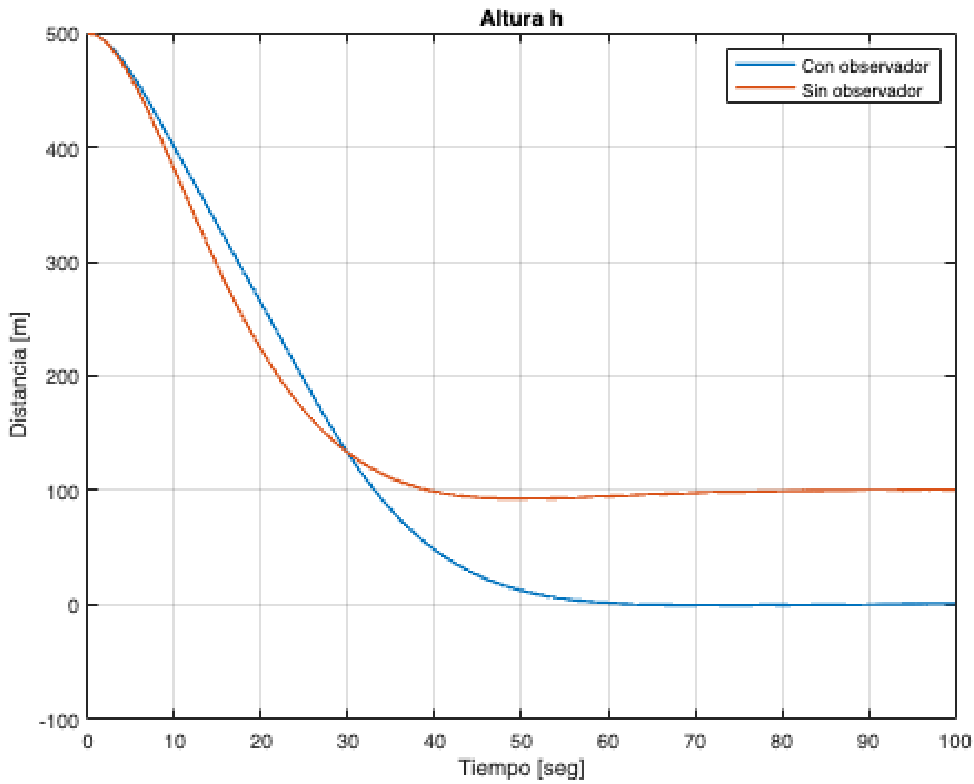
```
ylabel('Ángulo [rad]');
grid;
```




```
plot(t,phiP,'LineWidth',1.5);
hold on;
plot(t,phiPSO,'LineWidth',1.5);
hold off
title('Velocidad angular cabeceo \phi_p');
legend('Con observador','Sin observador')
xlabel('Tiempo [seg]');
ylabel('Posicion angular [rad/seg]');
grid;
```




```
plot(t,high,'LineWidth',1.5);
hold on;
plot(t,highSO,'LineWidth',1.5);
hold off
title('Altura h');
legend('Con observador','Sin observador')
xlabel('Tiempo [seg]');
ylabel('Distancia [m]');
grid;
```




```
plot(t,uu,'LineWidth',0.5);
hold on;
plot(t,uSO,'LineWidth',1.5);
hold off
title('Accion de control u_t');
legend('Con observador','Sin observador')
xlabel('Tiempo [seg]');
grid;
```

Ahora mejora la dinámica, sin embargo, existe un error en estado estacionario para el observador de la altura. Sin embargo, esto no es problema por el hecho de que la altura es medible, no observada. Las variables observadas en la realidad son α y ϕ .

Se procede con estos controladores entonces, a simular las otras 3 situaciones de referencia y altura inicial.

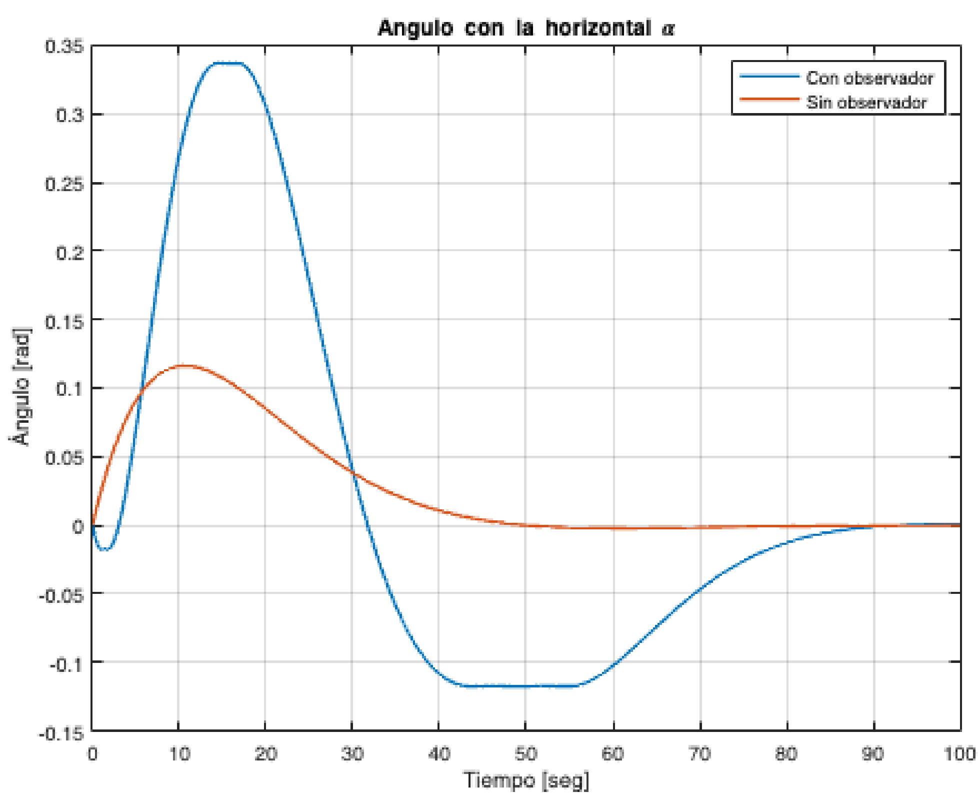
```
clear all;
close all;
clc
a = 0.07;
b = 5;
c = 150;
w = 9;
A = [-a a 0 0; 0 0 1 0; w^2 -w^2 0 0; c 0 0 0];
B = [0; 0; b*w^2; 0];
C = [0 0 0 1;0 1 0 0];
D = [0];
Q = diag([1 1000000 1 1]);
```

```
R = 1000000;
K = lqr(A,B,Q,R);
G = -inv(C(1,:)*inv(A-B*K)*B);
Ao = A';
Bo = C';
Co = B';
Qo = diag([100 1000 100 1000]);
Ro = 10000000000;
Ko = lqr(Ao,Bo,Qo,Ro);
T = 100;
h = 1e-4;
t = 0:h:(T-h);
ref = -100;
alpha(1) = 0;
phi(1) = 0;
phiP(1) = 0;
high(1) = -500;
u(1) = 0;
uu(1) = 0;
stateVector = [alpha(1);phi(1);phiP(1);high(1)];
xOp = [0;0;0;0];
xOp = [alpha(1);phi(1);phiP(1);high(1)];
alphaO(1) = 0;
phiO(1) = 0;
phiPO(1) = 0;
highO(1) = 400;
xObs = [alphaO(1);phiO(1);phiPO(1); highO(1)];
obsStateVector = [alphaO(1);phiO(1);phiPO(1);highO(1)];
deadZone = 0.1;
for i=1:T/h
    u(i) = -K*obsStateVector+G*ref;
    if (abs(u(i)) < deadZone)
        uu(i)=0;
    else
        uu(i)=sign(u(i))*(abs(u(i)));
    end
    alpha(i) = xOp(1);
    phi(i) = xOp(2);
    phiP(i) = xOp(3);
    high(i) = xOp(4);
    xP = A*xOp+B*uu(i);
    xOp = xOp+h*xP;
    alphaO(i)= xObs(1);
    phiO(i)= xObs(2);
    phiPO(i)= xObs(3);
    highO(i)= xObs(4);
    yOutObs = C*obsStateVector;
    yOut   = C*stateVector;
    xPrevP = A*xObs+B*uu(i)+Ko'*(yOut-yOutObs);
    xObs = xObs + xPrevP*h;
    stateVector=[alpha(i);phi(i);phiP(i);high(i)];
    obsStateVector=[alphaO(i);phiO(i);phiPO(i);highO(i)];
end
uSO(1)=0;
```

```
stateVectorSO=[alpha(1);phi(1);phiP(1);high(1)];
xSO=[alpha(1);phi(1);phiP(1); high(1)];
for i=1:T/h
    uSO(i) = -K*stateVectorSO+G*ref;
    alphaSO(i) = xSO(1);
    phiSO(i) = xSO(2);
    phiPSO(i) = xSO(3);
    highSO(i) = xSO(4);
    xp_so = A*xSO+B*uSO(i);
    xSO = xSO+h*xp_so;
    stateVectorSO = [alphaSO(i);phiSO(i);phiPSO(i);highSO(i)];
end
```

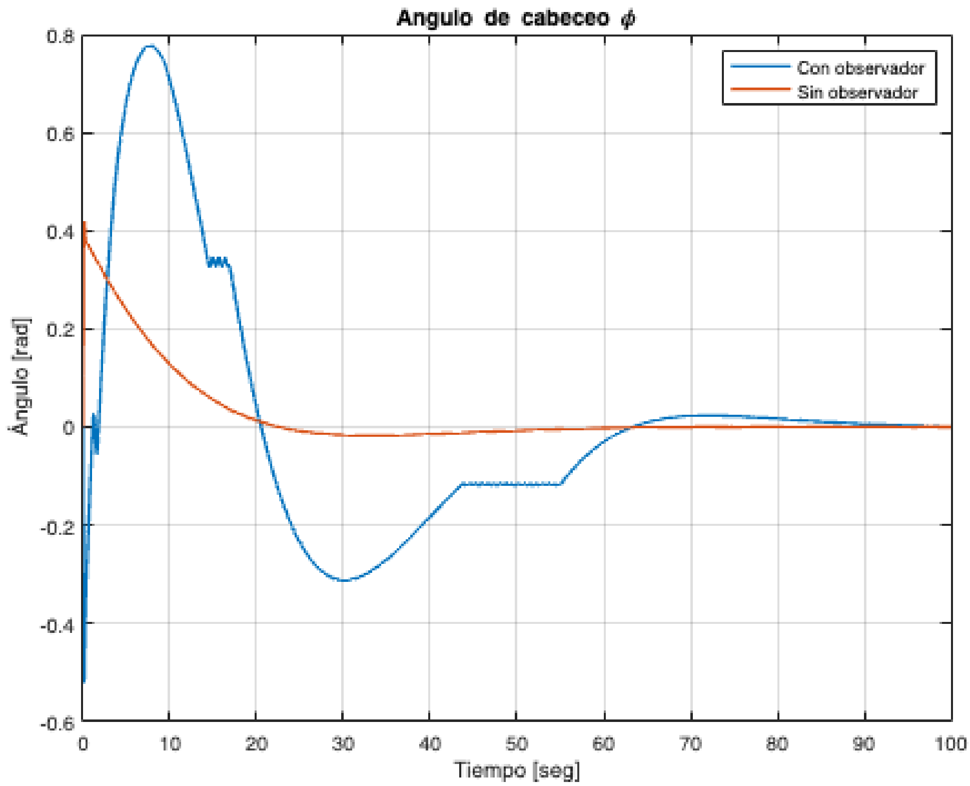
Se obtienen las siguientes gráficas:

```
plot(t,alpha,'LineWidth',1.5);
hold on;
plot(t,alphaSO,'LineWidth',1.5);
hold off
title('Angulo con la horizontal \alpha');
legend('Con observador','Sin observador')
xlabel('Tiempo [seg]');
ylabel('Ángulo [rad]');
grid;
```

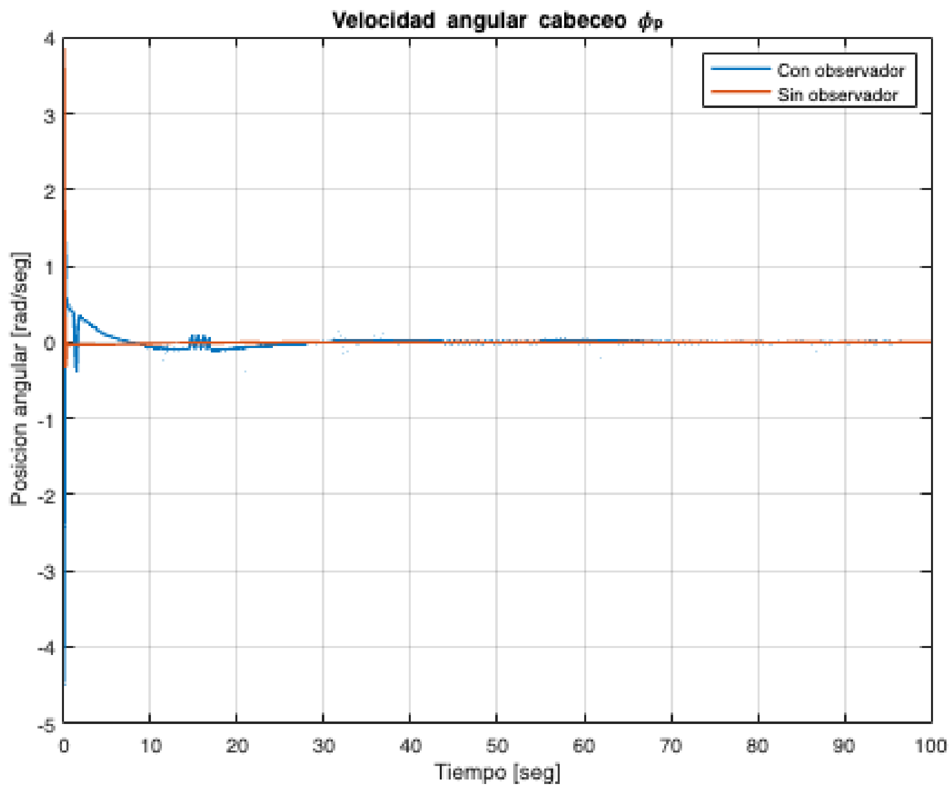



```
plot(t,phi,'LineWidth',1.5);
hold on;
plot(t,phiSO,'LineWidth',1.5);
```

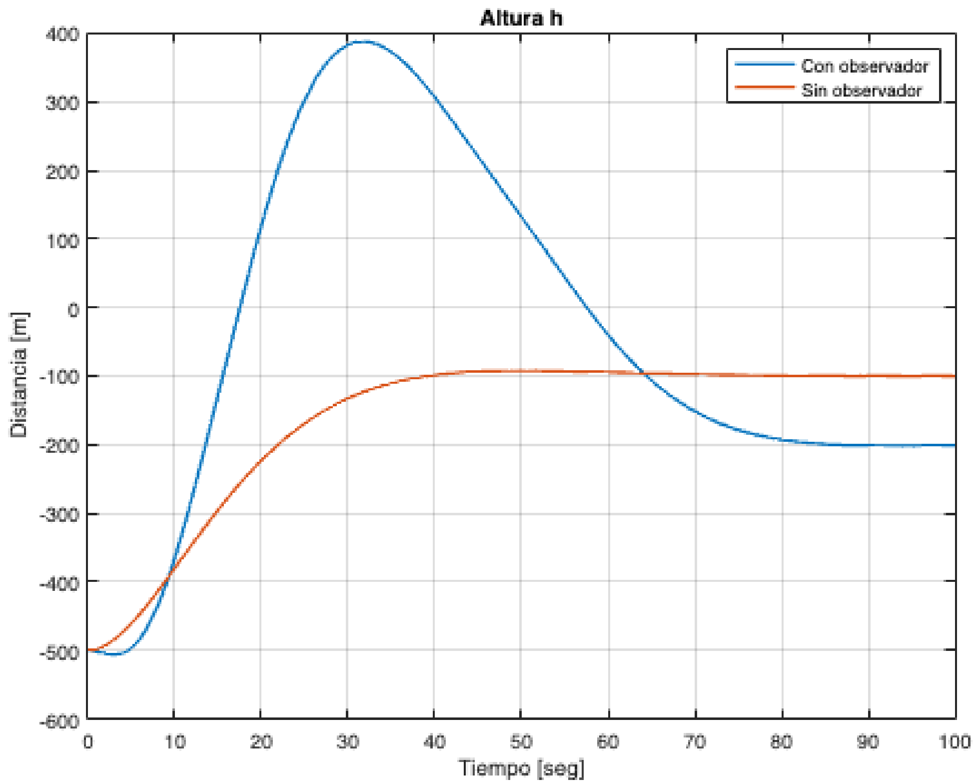
```
hold off;
title('Angulo de cabeceo \phi');
legend('Con observador','Sin observador')
xlabel('Tiempo [seg]');
ylabel('Ángulo [rad]');
grid;
```




```
plot(t,phiP,'LineWidth',1.5);
hold on;
plot(t,phiPSO,'LineWidth',1.5);
hold off
title('Velocidad angular cabeceo \phi_p');
legend('Con observador','Sin observador')
xlabel('Tiempo [seg]');
ylabel('Posicion angular [rad/seg]');
grid;
```




```
plot(t,high,'LineWidth',1.5);
hold on;
plot(t,highSO,'LineWidth',1.5);
hold off
title('Altura h');
legend('Con observador','Sin observador')
xlabel('Tiempo [seg]');
ylabel('Distancia [m]');
grid;
```




```
plot(t,uu,'LineWidth',1.5);
hold on;
plot(t,uSO,'LineWidth',1.5);
hold off
title('Accion de control u_t');
legend('Con observador','Sin observador')
xlabel('Tiempo [seg]');
grid;
```

```
clear all;
close all;
clc
a = 0.07;
b = 5;
c = 150;
w = 9;
A = [-a a 0 0; 0 0 1 0; w^2 -w^2 0 0; c 0 0 0];
B = [0; 0; b*w^2; 0];
C = [0 0 0 1;0 1 0 0];
D = [0];
Q = diag([1 1000000 1 1]);
R = 1000000
```

```
R = 1000000
```

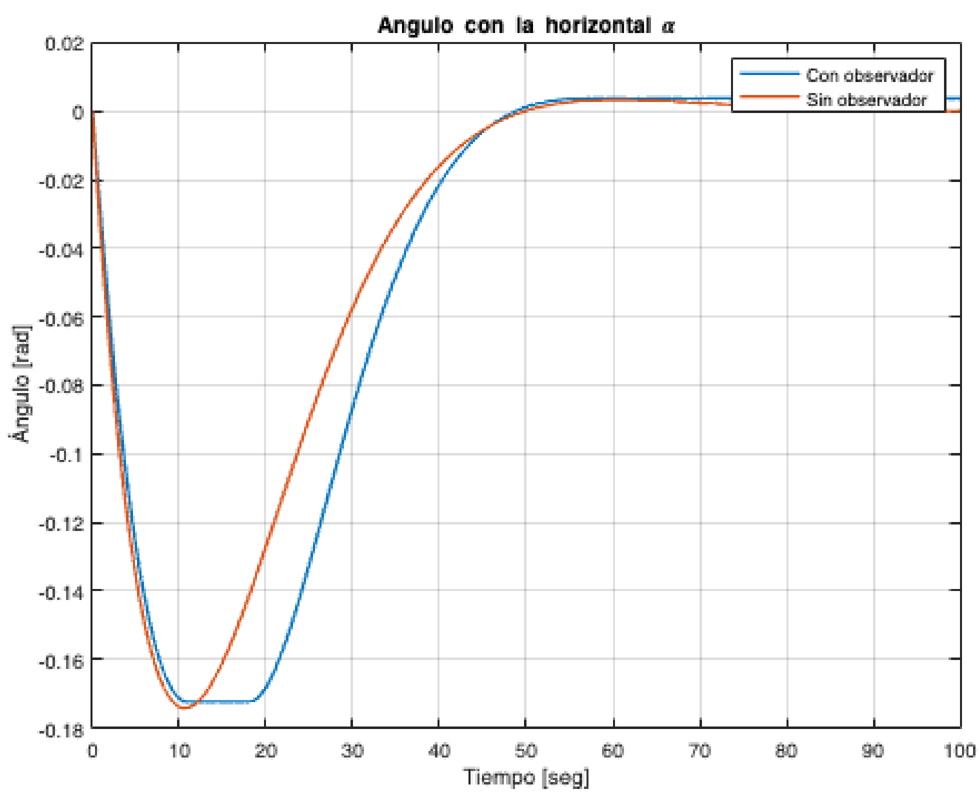
```
K = lqr(A,B,Q,R);
```

```
G = -inv(C(1,:)*inv(A-B*K)*B);
Ao = A';
Bo = C';
Co = B';
Qo = diag([100 1000 100 1000]);
Ro = 10000000000;
Ko = lqr(Ao,Bo,Qo,Ro);
T = 100;
h = 1e-4;
t = 0:h:(T-h);
ref = -100;
alpha(1) = 0;
phi(1) = 0;
phiP(1) = 0;
high(1) = 500;
u(1) = 0;
uu(1) = 0;
stateVector = [alpha(1);phi(1);phiP(1);high(1)];
xOp = [0;0;0;0];
xOp = [alpha(1);phi(1);phiP(1);high(1)];
alphaO(1) = 0;
phiO(1) = 0;
phiPO(1) = 0;
highO(1) = 400;
xObs = [alphaO(1);phiO(1);phiPO(1); highO(1)];
obsStateVector = [alphaO(1);phiO(1);phiPO(1);highO(1)];
deadZone = 0.1;
for i=1:T/h
    u(i) = -K*obsStateVector+G*ref;
    if (abs(u(i)) < deadZone)
        uu(i)=0;
    else
        uu(i)=sign(u(i))*(abs(u(i)));
    end
    alpha(i) = xOp(1);
    phi(i) = xOp(2);
    phiP(i) = xOp(3);
    high(i) = xOp(4);
    xP = A*xOp+B*uu(i);
    xOp = xOp+h*xP;
    alphaO(i)= xObs(1);
    phiO(i)= xObs(2);
    phiPO(i)= xObs(3);
    highO(i)= xObs(4);
    yOutObs = C*obsStateVector;
    yOut   = C*stateVector;
    xPrevP = A*xObs+B*uu(i)+Ko'*(yOut-yOutObs);
    xObs = xObs + xPrevP*h;
    stateVector=[alpha(i);phi(i);phiP(i);high(i)];
    obsStateVector=[alphaO(i);phiO(i);phiPO(i);highO(i)];
end
uSO(1)=0;
stateVectorSO=[alpha(1);phi(1);phiP(1);high(1)];
xSO=[alpha(1);phi(1);phiP(1); high(1)];
```

```
for i=1:T/h
    uSO(i) = -K*stateVectorSO+G*ref;
    alphaSO(i) = xSO(1);
    phiSO(i) = xSO(2);
    phiPSO(i) = xSO(3);
    highSO(i) = xSO(4);
    xp_so = A*xSO+B*uSO(i);
    xSO = xSO+h*xp_so;
    stateVectorSO = [alphaSO(i);phiSO(i);phiPSO(i);highSO(i)];
end
```

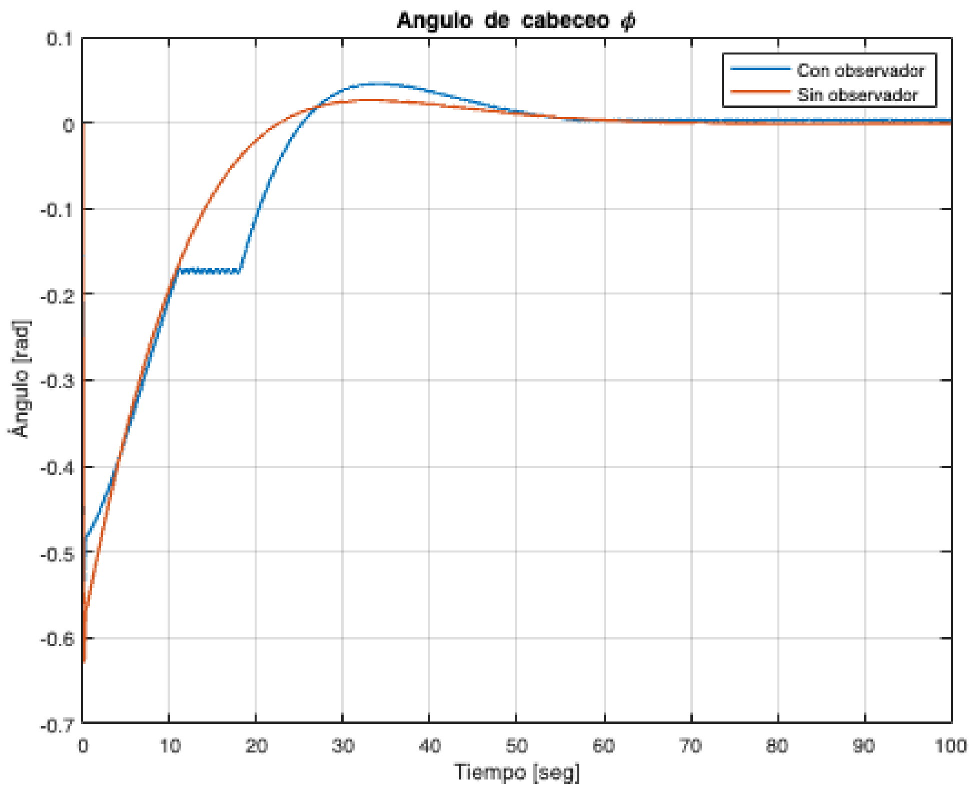
Se obtienen las siguientes gráficas:

```
plot(t,alpha,'LineWidth',1.5);
hold on;
plot(t,alphaSO,'LineWidth',1.5);
hold off
title('Angulo con la horizontal \alpha');
legend('Con observador','Sin observador')
xlabel('Tiempo [seg]');
ylabel('Ángulo [rad]');
grid;
```

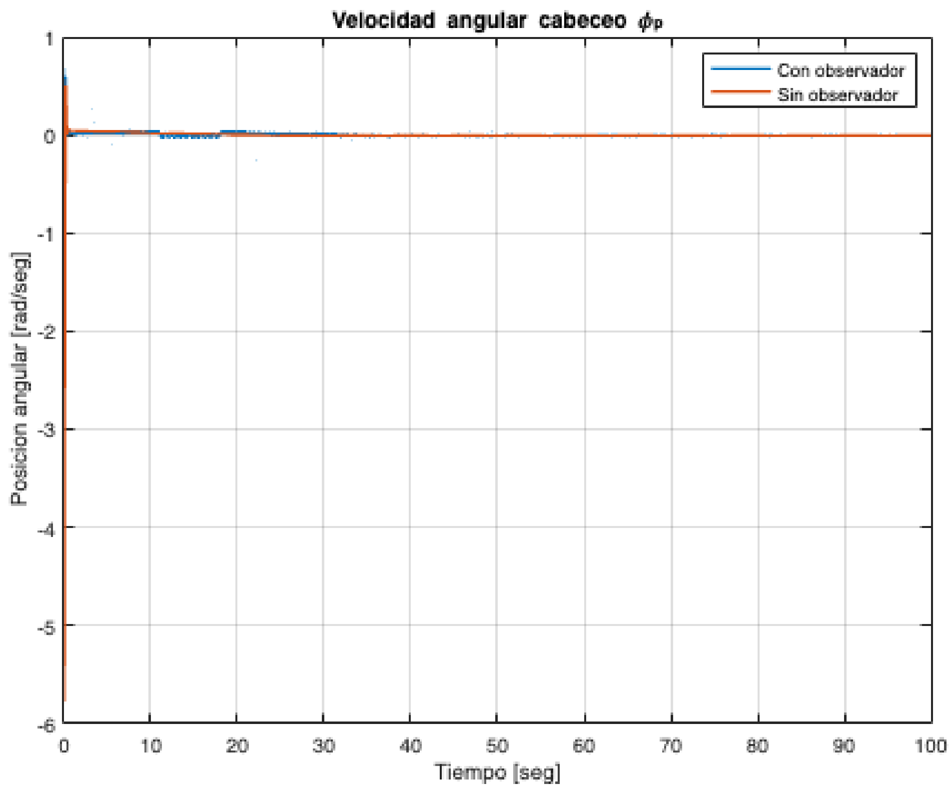



```
plot(t,phi,'LineWidth',1.5);
hold on;
plot(t,phiSO,'LineWidth',1.5);
hold off;
title('Angulo de cabeceo \phi');
```

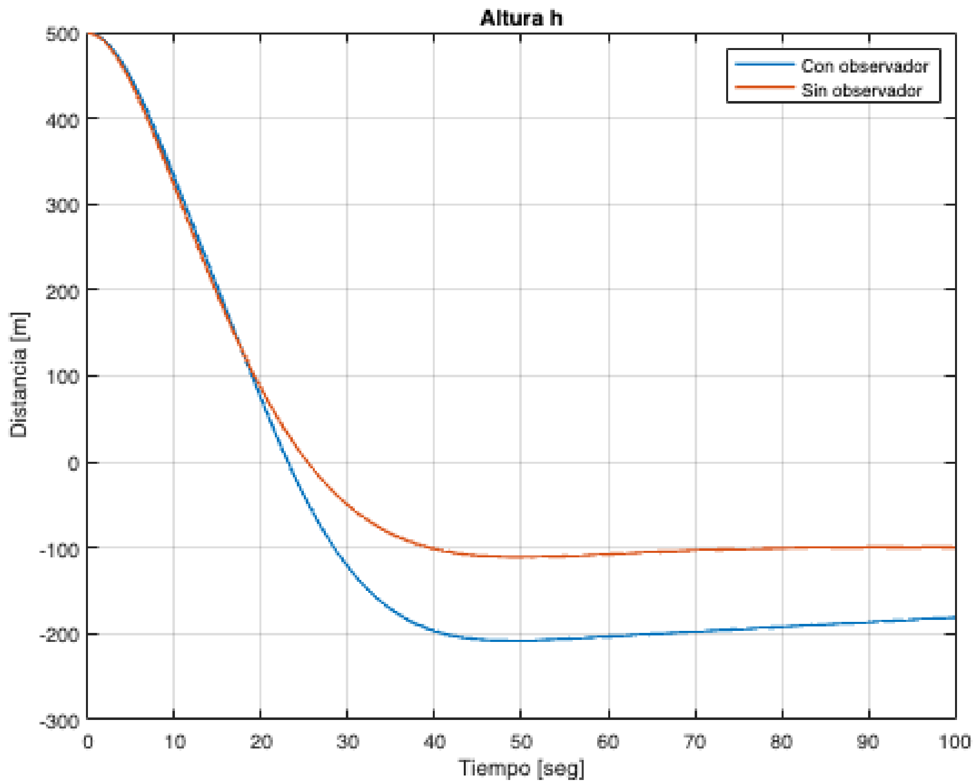
```
legend('Con observador','Sin observador')
xlabel('Tiempo [seg]');
ylabel('Ángulo [rad]');
grid;
```




```
plot(t,phiP,'LineWidth',1.5);
hold on;
plot(t,phiPSO,'LineWidth',1.5);
hold off
title('Velocidad angular cabeceo \phi_p');
legend('Con observador','Sin observador')
xlabel('Tiempo [seg]');
ylabel('Posicion angular [rad/seg]');
grid;
```




```
plot(t,high,'LineWidth',1.5);
hold on;
plot(t,highSO,'LineWidth',1.5);
hold off
title('Altura h');
legend('Con observador','Sin observador')
xlabel('Tiempo [seg]');
ylabel('Distancia [m]');
grid;
```




```
plot(t,uu,'LineWidth',1.5);
hold on;
plot(t,uSO,'LineWidth',1.5);
hold off
title('Accion de control u_t');
legend('Con observador','Sin observador')
xlabel('Tiempo [seg]');
grid;
clear all;
close all;
clc
a = 0.07;
b = 5;
c = 150;
w = 9;
A = [-a a 0 0; 0 0 1 0; w^2 -w^2 0 0; c 0 0 0];
B = [0; 0; b*w^2; 0];
C = [0 0 0 1;0 1 0 0];
D = [0];
Q = diag([1 1000000 1 1]);
R = 1000000
```

```
R = 1000000
```

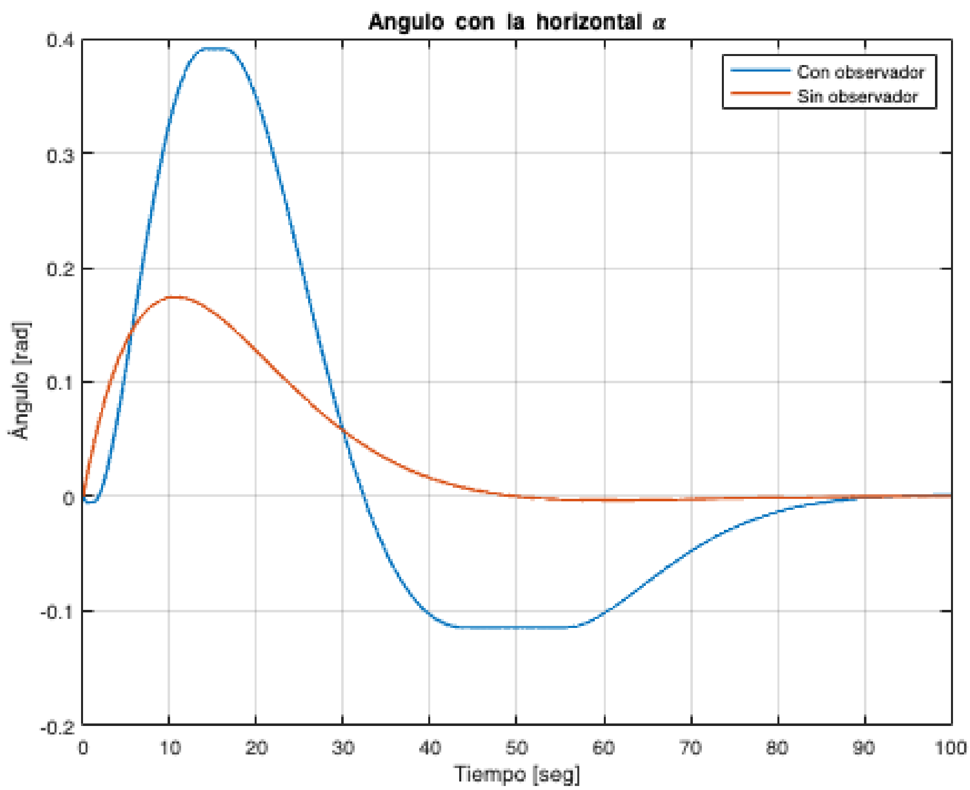
```
K = lqr(A,B,Q,R);
G = -inv(C(1,:)*inv(A-B*K)*B);
Ao = A';
Bo = C';
```

```
Co = B';
Qo = diag([100 1000 100 1000]);
Ro = 10000000000;
Ko = lqr(Ao,Bo,Qo,Ro);
T = 100;
h = 1e-4;
t = 0:h:(T-h);
ref = 100;
alpha(1) = 0;
phi(1) = 0;
phiP(1) = 0;
high(1) = -500;
u(1) = 0;
uu(1) = 0;
stateVector = [alpha(1);phi(1);phiP(1);high(1)];
xOp = [0;0;0;0];
xOp = [alpha(1);phi(1);phiP(1);high(1)];
alphaO(1) = 0;
phiO(1) = 0;
phiPO(1) = 0;
highO(1) = 400;
xObs = [alphaO(1);phiO(1);phiPO(1); highO(1)];
obsStateVector = [alphaO(1);phiO(1);phiPO(1);highO(1)];
deadZone = 0.1;
for i=1:T/h
    u(i) = -K*obsStateVector+G*ref;
    if (abs(u(i)) < deadZone)
        uu(i)=0;
    else
        uu(i)=sign(u(i))*(abs(u(i)));
    end
    alpha(i) = xOp(1);
    phi(i) = xOp(2);
    phiP(i) = xOp(3);
    high(i) = xOp(4);
    xP = A*xOp+B*uu(i);
    xOp = xOp+h*xP;
    alphaO(i)= xObs(1);
    phiO(i)= xObs(2);
    phiPO(i)= xObs(3);
    highO(i)= xObs(4);
    yOutObs = C*obsStateVector;
    yOut    = C*stateVector;
    xPrevP = A*xObs+B*uu(i)+Ko'*(yOut-yOutObs);
    xObs = xObs + xPrevP*h;
    stateVector=[alpha(i);phi(i);phiP(i);high(i)];
    obsStateVector=[alphaO(i);phiO(i);phiPO(i);highO(i)];
end
uSO(1)=0;
stateVectorSO=[alpha(1);phi(1);phiP(1);high(1)];
xSO=[alpha(1);phi(1);phiP(1); high(1)];
for i=1:T/h
    uSO(i) = -K*stateVectorSO+G*ref;
    alphaSO(i) = xSO(1);
```

```
    phiSO(i) = xSO(2);
    phiPSO(i) = xSO(3);
    highSO(i) = xSO(4);
    xp_so = A*xSO+B*uSO(i);
    xSO = xSO+h*xp_so;
    stateVectorSO = [alphaSO(i);phiSO(i);phiPSO(i);highSO(i)];
end
```

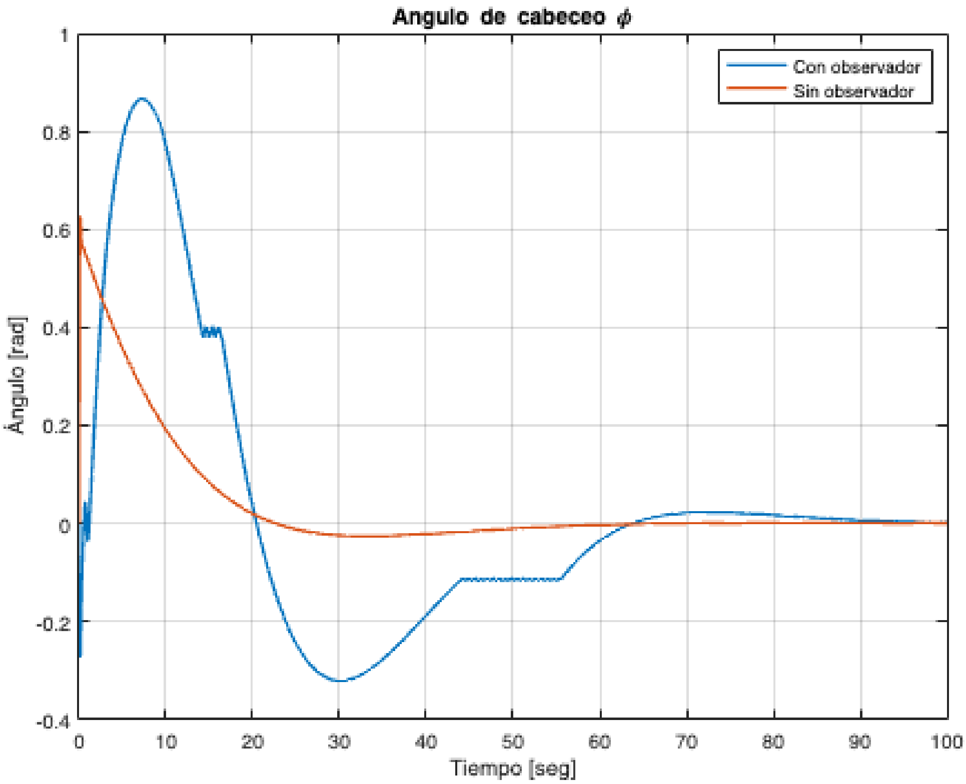
Se obtienen las siguientes gráficas:

```
plot(t,alpha,'LineWidth',1.5);
hold on;
plot(t,alphaSO,'LineWidth',1.5);
hold off
title('Angulo con la horizontal \alpha');
legend('Con observador','Sin observador')
xlabel('Tiempo [seg]');
ylabel('Ángulo [rad]');
grid;
```

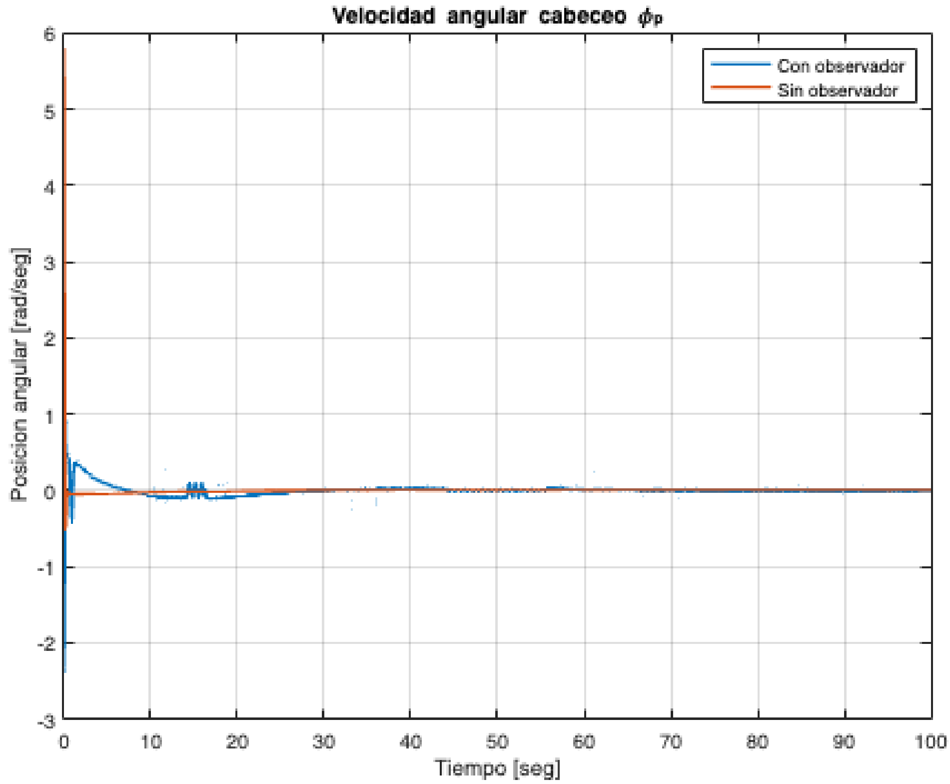



```
plot(t,phi,'LineWidth',1.5);
hold on;
plot(t,phiSO,'LineWidth',1.5);
hold off;
title('Angulo de cabeceo \phi');
legend('Con observador','Sin observador')
xlabel('Tiempo [seg]');
ylabel('Ángulo [rad]');
```

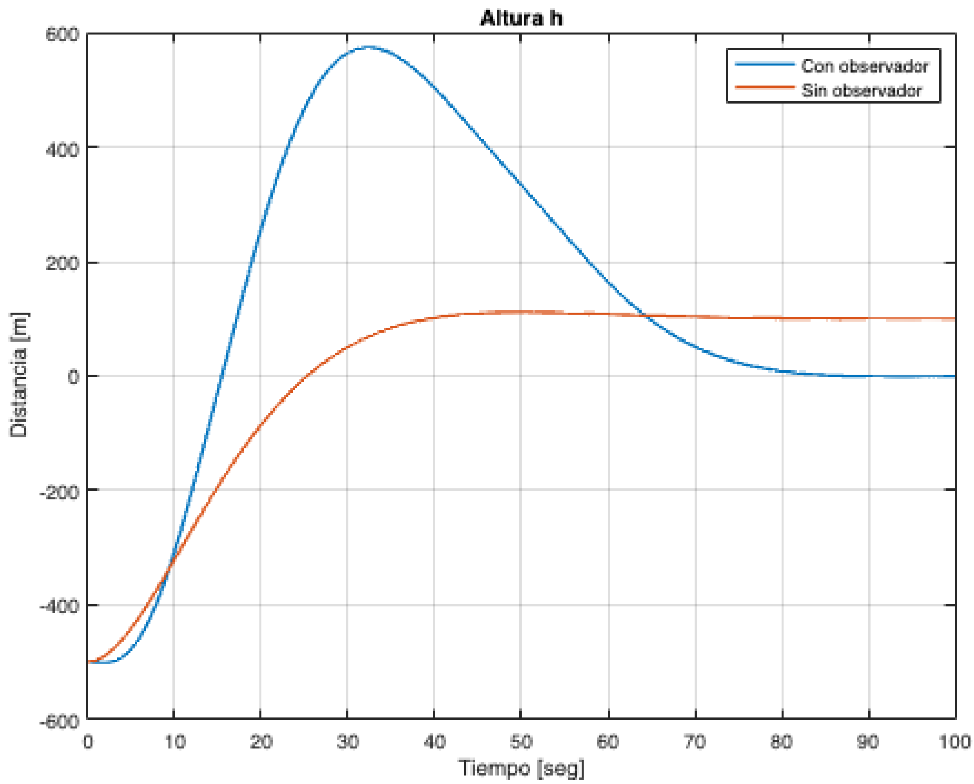
```
grid;
```



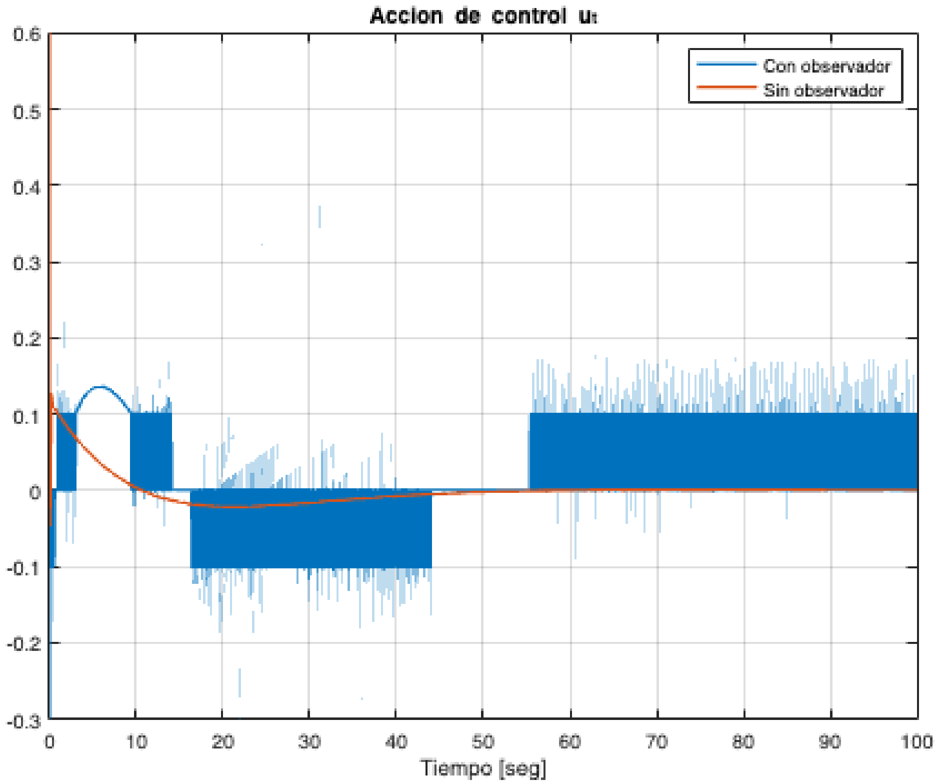

```
plot(t,phiP,'LineWidth',1.5);
hold on;
plot(t,phiPSO,'LineWidth',1.5);
hold off
title('Velocidad angular cabeceo \phi_p');
legend('Con observador','Sin observador')
xlabel('Tiempo [seg]');
ylabel('Posicion angular [rad/seg]');
grid;
```




```
plot(t,high,'LineWidth',1.5);
hold on;
plot(t,highSO,'LineWidth',1.5);
hold off
title('Altura h');
legend('Con observador','Sin observador')
xlabel('Tiempo [seg]');
ylabel('Distancia [m]');
grid;
```




```
plot(t,uu,'LineWidth',1.5);
hold on;
plot(t,uSO,'LineWidth',1.5);
hold off
title('Accion de control u_t');
legend('Con observador','Sin observador')
xlabel('Tiempo [seg]');
grid;
```



Caso de estudio 3. Sistema no lineal de cuatro variables de estado.

```
clear all;
close all;
clc;
```

Se definen los parámetros del sistema:

```
m = 0.1;
F = 0.1;
l = 1.6;
g = 9.8;
M = 1.5;
```

Las matrices del sistema en tiempo continuo son:

```
Ac = [0 1 0 0; 0 -F/M -m*g/M 0; 0 0 0 1; 0 -F/(l*M) -g*(m+M)/(l*M) 0]
```

```
Ac = 4×4
          0    1.0000         0         0
          0   -0.0667   -0.6533         0
          0         0         0    1.0000
          0   -0.0417   -6.5333         0
```

```
Bc = [0; 1/M; 0; 1/(l*M)]
```

```
Bc = 4×1
          0
```

```
    0.6667
         0
    0.4167
```

```
Cc = [1 0 0 0; 0 0 1 0];
Dc = [0];
```

Los tiempos que se manejarán son:

```
Ts = 1e-2;
T = 25;
At = 1e-4;
Kmax = T/Ts;
```

Se obtiene el sistema de tiempo continuo para luego discretizar.

```
sys1 = ss(Ac, Bc, Cc, Dc)
```

```
sys1 =

  A = 
             x1        x2        x3        x4
   x1         0         1         0         0
   x2         0  -0.06667   -0.6533         0
   x3         0         0         0         1
   x4         0  -0.04167    -6.533         0

  B = 
           u1
   x1       0
   x2  0.6667
   x3       0
   x4  0.4167

  C = 
       x1  x2  x3  x4
   y1   1   0   0   0
   y2   0   0   1   0

  D = 
       u1
   y1   0
   y2   0

Continuous-time state-space model.
Model Properties
```

```
dSys1 = c2d(sys1,Ts,'zoh')
```

```
dSys1 =

  A = 
                x1           x2           x3           x4
   x1            1     0.009997   -3.266e-05   -1.089e-07
   x2            0       0.9993     -0.00653   -3.266e-05
   x3            0   -2.083e-06       0.9997     0.009999
   x4            0   -0.0004165     -0.06532       0.9997

  B = 
              u1
   x1  3.333e-05
```

```
   x2   0.006664
   x3  2.083e-05
   x4   0.004165

  C =
       x1  x2  x3  x4
   y1   1   0   0   0
   y2   0   0   1   0

  D =
       u1
   y1   0
   y2   0

Sample time: 0.01 seconds
Discrete-time state-space model.
Model Properties
```

```
A = dSys1.A
```

```
A = 4×4
    1.0000    0.0100   -0.0000   -0.0000
         0    0.9993   -0.0065   -0.0000
         0   -0.0000    0.9997    0.0100
         0   -0.0004   -0.0653    0.9997
```

```
B = dSys1.B
```

```
B = 4×1
    0.0000
    0.0067
    0.0000
    0.0042
```

Se implementa un integrador para medir el desplazamiento:

```
Cref = Cc(1,:);
Aamp1 = [A,zeros(4,1);-Cref*A,eye(1)]
```

```
Aamp1 = 5×5
    1.0000    0.0100   -0.0000   -0.0000         0
         0    0.9993   -0.0065   -0.0000         0
         0   -0.0000    0.9997    0.0100         0
         0   -0.0004   -0.0653    0.9997         0
   -1.0000   -0.0100    0.0000    0.0000    1.0000
```

```
Bamp1 = [B;-Cref*B]
```

```
Bamp1 = 5×1
    0.0000
    0.0067
    0.0000
    0.0042
   -0.0000
```

```
Q1 = diag([.1 1e-2 1 .1 0.0000093])
```

```
Q1 = 5×5
    0.1000         0         0         0         0
         0    0.0100         0         0         0
         0         0    1.0000         0         0
         0         0         0    0.1000         0
```

```
        0         0         0         0    0.0000
```

```
R1 = 1.9e-4;
K1 = dlqr(Aamp1,Bamp1,Q1,R1)
```

```
K1 = 1×5
   40.7242   29.6109   78.8271  -12.0997   -0.2047
```

```
Kp1 = K1(1:4)
```

```
Kp1 = 1×4
   40.7242   29.6109   78.8271  -12.0997
```

```
Kint1 = -K1(5)
```

```
Kint1 = 0.2047
```

Ahora, para cuando la masa aumente en 10 veces, se plantea otro controlador.

```
m1 = 10*m;
Ac2 = [0 1 0 0; 0 -F/M -m1*g/M 0; 0 0 0 1; 0 -F/(l*M) -g*(m1+M)/(l*M) 0]
```

```
Ac2 = 4×4
        0    1.0000         0         0
        0   -0.0667   -6.5333         0
        0         0         0    1.0000
        0   -0.0417  -10.2083         0
```

```
sys2 = ss(Ac2,Bc,Cc,Dc)
```

```
sys2 =

  A = 
             x1        x2        x3        x4
   x1         0         1         0         0
   x2         0  -0.06667    -6.533         0
   x3         0         0         0         1
   x4         0  -0.04167    -10.21         0

  B = 
           u1
   x1       0
   x2  0.6667
   x3       0
   x4  0.4167

  C = 
       x1  x2  x3  x4
   y1   1   0   0   0
   y2   0   0   1   0

  D = 
       u1
   y1   0
   y2   0

Continuous-time state-space model.
Model Properties
```

```
dSys2 = c2d(sys2,Ts,'zoh')
```

```
dSys2 =

  A = 
              x1          x2          x3          x4
   x1          1    0.009997  -0.0003266  -1.089e-06
   x2          0      0.9993     -0.0653  -0.0003266
   x3          0  -2.083e-06      0.9995    0.009998
   x4          0  -0.0004165     -0.1021      0.9995

  B = 
              u1
   x1  3.332e-05
   x2   0.006664
   x3  2.083e-05
   x4   0.004165

  C = 
       x1  x2  x3  x4
   y1   1   0   0   0
   y2   0   0   1   0

  D = 
       u1
   y1   0
   y2   0

Sample time: 0.01 seconds
Discrete-time state-space model.
Model Properties
```

```
A2 = dSys2.A
```

```
A2 = 4×4
    1.0000    0.0100   -0.0003   -0.0000
         0    0.9993   -0.0653   -0.0003
         0   -0.0000    0.9995    0.0100
         0   -0.0004   -0.1021    0.9995
```

```
B2 = dSys2.B
```

```
B2 = 4×1
    0.0000
    0.0067
    0.0000
    0.0042
```

```
Aamp2 = [A2,zeros(4,1);-Cref*A2,eye(1)]
```

```
Aamp2 = 5×5
    1.0000    0.0100   -0.0003   -0.0000         0
         0    0.9993   -0.0653   -0.0003         0
         0   -0.0000    0.9995    0.0100         0
         0   -0.0004   -0.1021    0.9995         0
   -1.0000   -0.0100    0.0003    0.0000    1.0000
```

```
Q2 = diag([.1 1e-2 1 .1 0.0000093])
```

```
Q2 = 5×5
    0.1000         0         0         0         0
         0    0.0100         0         0         0
         0         0    1.0000         0         0
         0         0         0    0.1000         0
         0         0         0         0    0.0000
```

```
R2 = 0.0005;
K2 = dlqr(Aamp2,Bamp1,Q2,R2)
```

```
K2 = 1×5
   26.2505   19.7487   42.3199   -6.6219   -0.1292
```

```
Kp2 = K2(1:4)
```

```
Kp2 = 1×4
   26.2505   19.7487   42.3199   -6.6219
```

```
Kint2 = -K2(5)
```

```
Kint2 = 0.1292
```

El observador, a continuacion:

```
Ao = A'
```

```
Ao = 4×4
    1.0000         0         0         0
    0.0100    0.9993   -0.0000   -0.0004
   -0.0000   -0.0065    0.9997   -0.0653
   -0.0000   -0.0000    0.0100    0.9997
```

```
Bo = Cc'
```

```
Bo = 4×2
     1     0
     0     0
     0     1
     0     0
```

```
Co = B'
```

```
Co = 1×4
    0.0000    0.0067    0.0000    0.0042
```

```
Qo = diag([0.001 1000 0.5 0.0001])
```

Qo = 4×4

10^3 ×

```
    0.0000         0         0         0
         0    1.0000         0         0
         0         0    0.0005         0
         0         0         0    0.0000
```

```
Ro = diag([80 10000])
```

```
Ro = 2×2
          80           0
           0       10000
```

```
Ko = dlqr(Ao,Bo,Qo,Ro);
Ko = Ko'
```

```
Ko = 4×2
    0.2641   -0.0000
    3.0782   -0.0004
   -0.0018    0.0070
```

```
-0.0102   -0.0005
```

Para poder simular, se plantean las siguientes variables:

```
phi(1) = pi;
x = [0;0;phi(1);0];
delta = x(1);
deltaP = x(2);
phi = x(3);
omega = x(4);
phiPP(1) = 0;
h = Ts/20;
i = 1;
deltaRef = 10;
bool = 0;
v(1) = 0;
xHat = [0;0;pi;0];
xOp=[0 0 pi 0]';
reference(1) = 10;
```

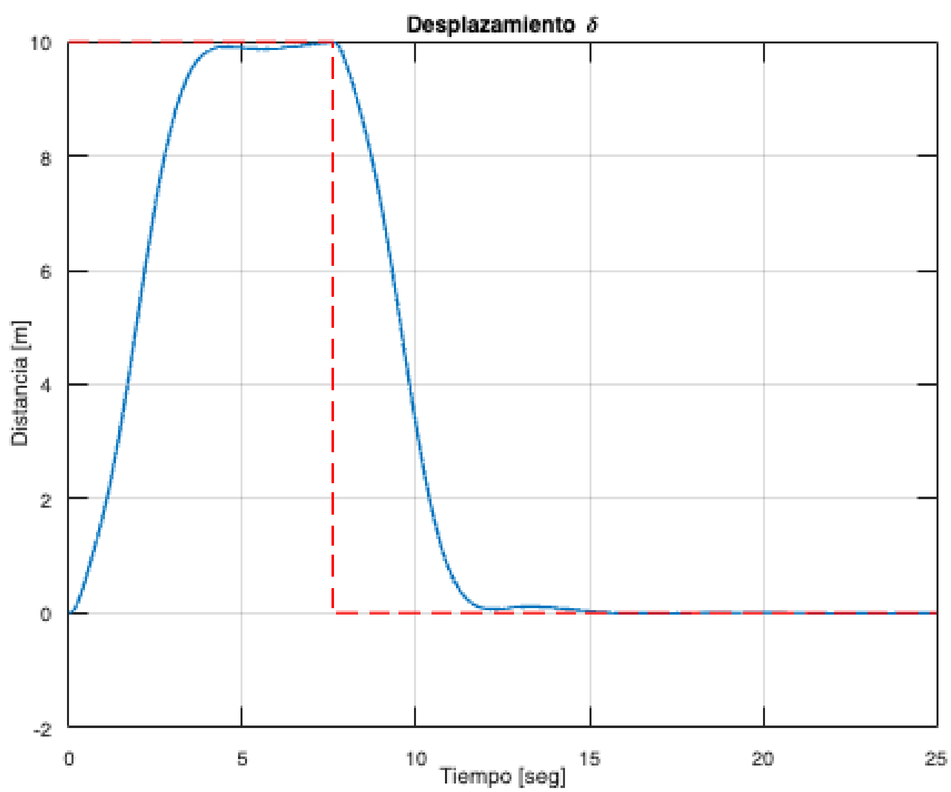
Simulacion:

```
K = Kp1;
KI = Kint1;
for index=1:Kmax
    yOut = Cc*x;
    yOutObs = Cc*(xHat-xOp);
    v(index+1) = v(index)+deltaRef-yOut(1);
    u1(index) = -K*(x-xOp)+KI*v(index+1);
    deadZone = 0.1;
    if(abs(u1(index)) < deadZone)
        u1(index) = 0;
    else
        u1(index)=sign(u1(index))*(abs(u1(index))-deadZone);
    end
    for j=1:Ts/h
        u(i) = u1(index);
        p_pp = (1/(M+m))*(u(i)-m*l*phiPP*cos(phi(i))+m*l*omega(i)^2*sin(phi(i))-F*deltaP(i));
        phiPP = (1/l)*(g*sin(phi(i))-p_pp*cos(phi(i)));
        deltaP(i+1) = deltaP(i)+h*p_pp;
        delta(i+1) = delta(i)+h*deltaP(i);
        omega(i+1) = omega(i)+h*phiPP;
        phi(i+1) = phi(i)+h*omega(i);
        if(delta(i) >= 9.99)
            if(bool == 0)
                deltaRef = 0;
                m = m*10;
                bool = 1;
                K = Kp2;
                KI = Kint2;
            end
        end
        i = i+1;
        reference(i) = deltaRef;
```

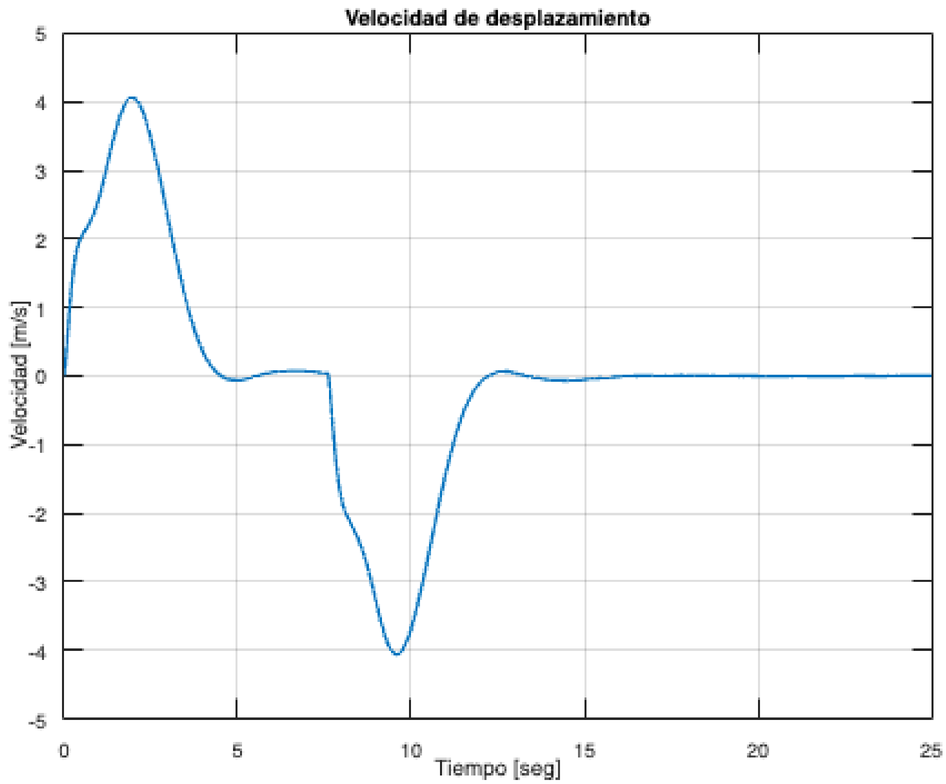
```
    end
    x = [delta(i-1); deltaP(i-1); phi(i-1); omega(i-1)];
    xHat = A*xHat+B*u1(index)+Ko*(yOut-yOutObs)+xOp;
end
u(i) = u1(index);
t = 0:h:T;
```

Se muestran las gráficas obtenidas:

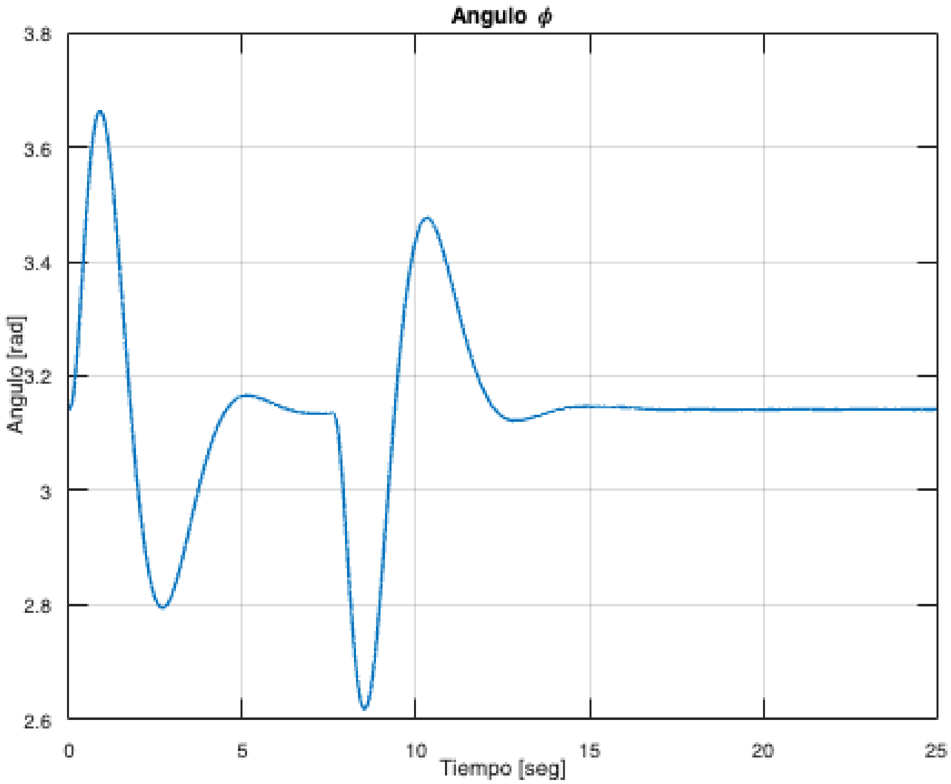
```
plot(t,delta,'LineWidth',1.5);
hold on;
plot(t,reference,'r--')
hold off;
grid;
title('Desplazamiento \delta');
xlabel('Tiempo [seg]')
ylabel('Distancia [m]')
```



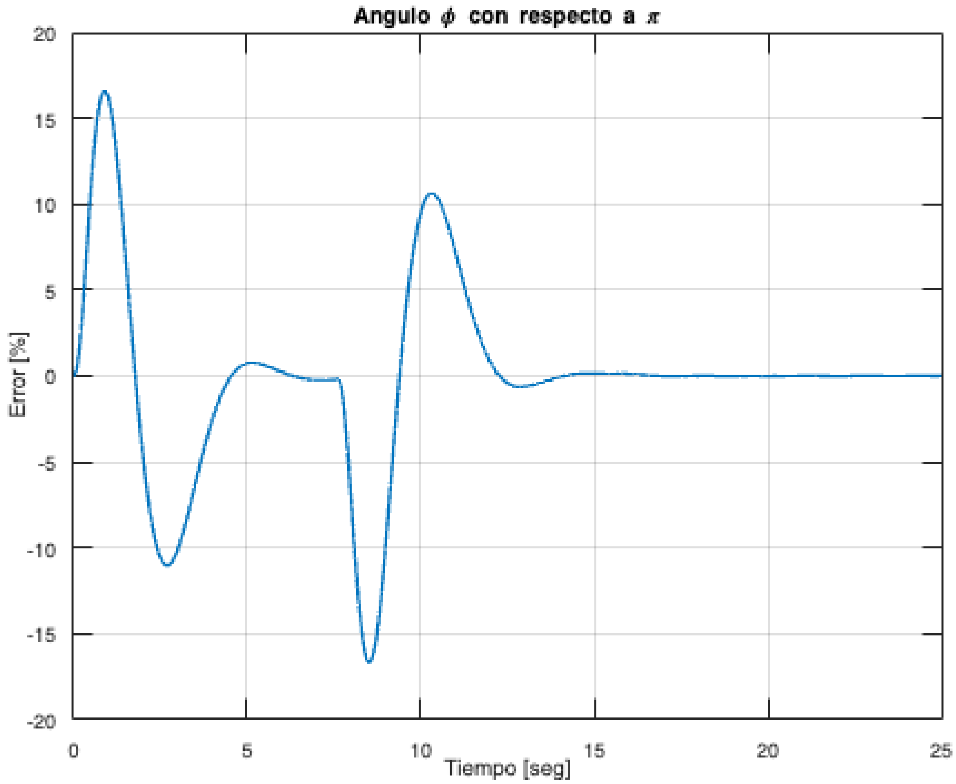

```
plot(t,deltaP,'LineWidth',1.5);
grid;
title('Velocidad de desplazamiento')
xlabel('Tiempo [seg]')
ylabel('Velocidad [m/s]')
```



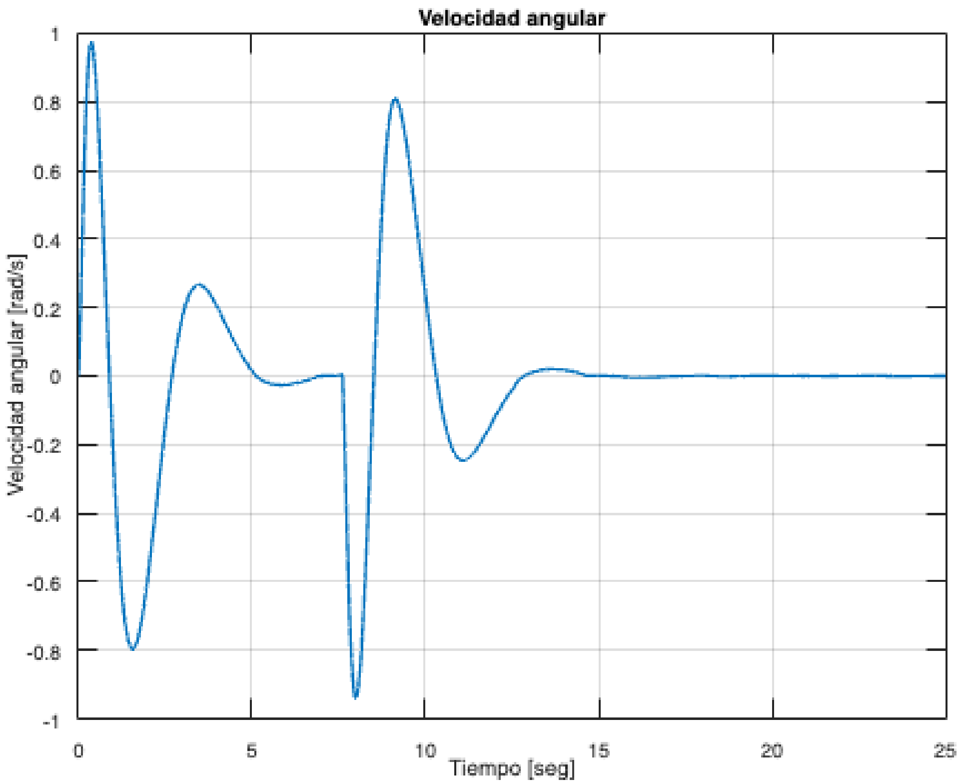

```
plot(t,phi,'LineWidth',1.5);
grid;
title('Angulo \phi')
xlabel('Tiempo [seg]')
ylabel('Angulo [rad]')
```




```
plot(t,phi/pi*100-100,'LineWidth',1.5);
grid;
title('Angulo \phi con respecto a \pi')
xlabel('Tiempo [seg]')
ylabel('Error [%]')
```

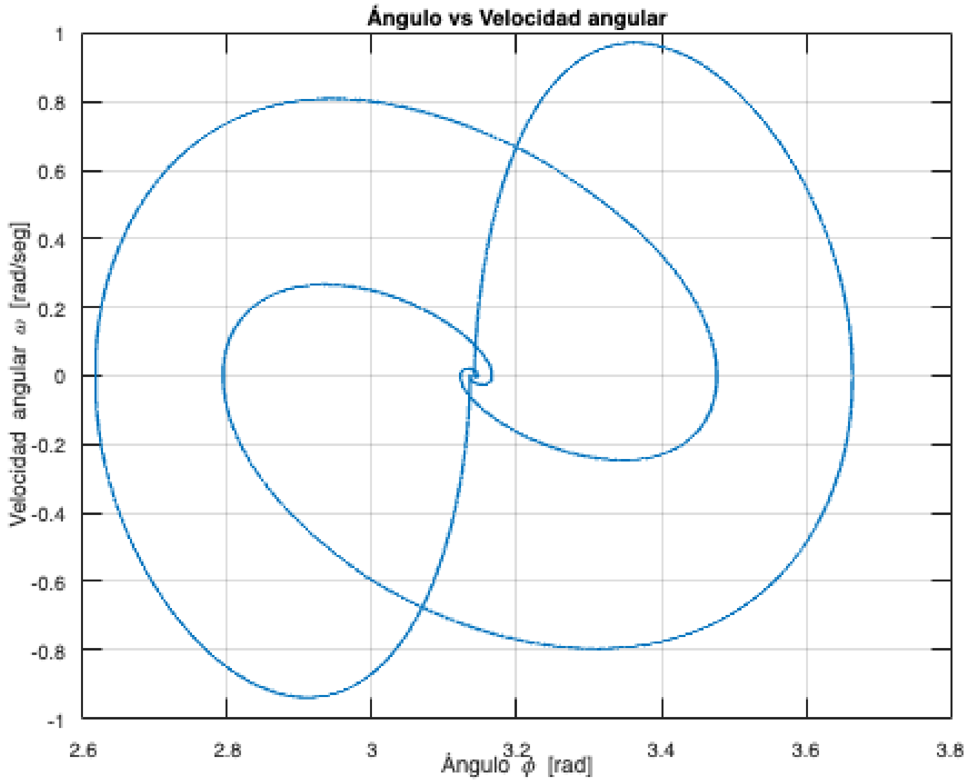



```
plot(t,omega,'LineWidth',1.5);
grid;
title('Velocidad angular')
xlabel('Tiempo [seg]')
ylabel('Velocidad angular [rad/s]')
```

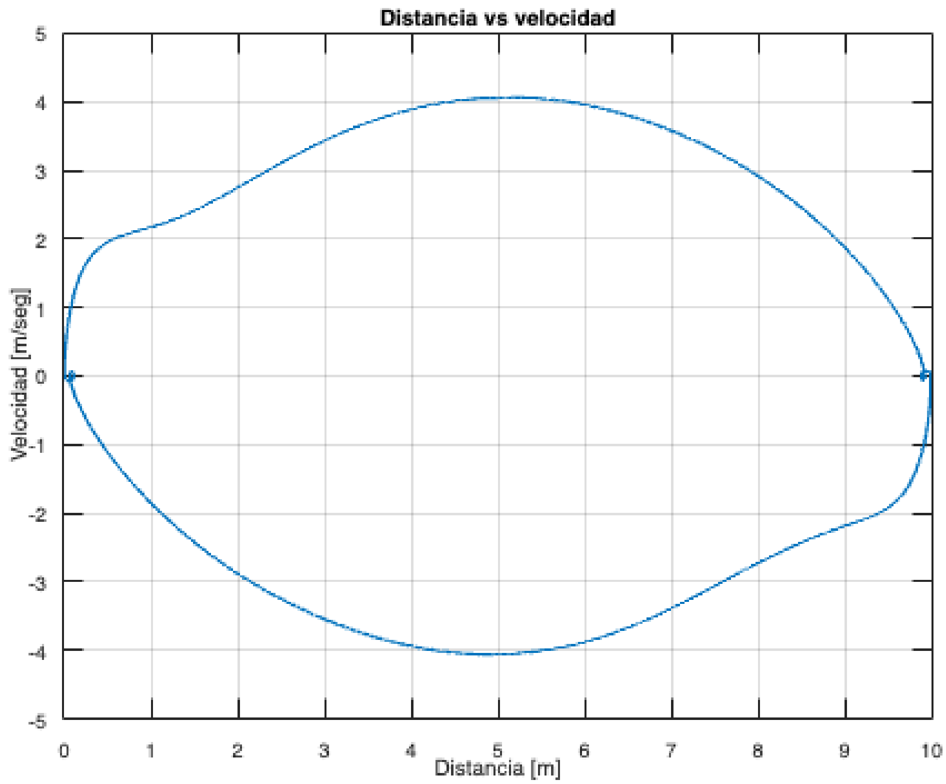


Los planos de fase son los siguientes:

```
plot(phi,omega,'LineWidth',1.5);
title('Ángulo vs Velocidad angular');
xlabel('Ángulo \phi [rad]');
ylabel('Velocidad angular \omega [rad/seg]');
grid;
```




```
plot(delta,deltaP,'LineWidth',1.5);
title('Distancia vs velocidad');
xlabel('Distancia [m]');
ylabel('Velocidad [m/seg]');
grid;
```


Distancia vs velocidad
Velocidad [m/seg]
Distancia [m]
5
4
3
2
1
0
-1
-2
-3
-4
-5
0
1
2
3
4
5
6
7
8
9
10