Option Pricing Using Jump Diffusion Models

1. Title Page

- **Project Title**: Option Pricing Using Jump Diffusion Models
- **Your Name**
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- **Date of Submission**

2. Abstract

This project explores the application of Jump Diffusion models in option pricing, specifically the Merton Jump Diffusion Model. Traditional Black-Scholes models assume continuous price changes in underlying assets, but real markets often experience sudden jumps. The Merton Jump Diffusion Model extends the Black-Scholes framework by incorporating these jumps, resulting in more accurate option pricing in markets prone to large, unexpected movements. The project compares the pricing results from the Merton model with the Black-Scholes model, highlighting the impact of jumps on option prices.

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4. Introduction

Background

The Black-Scholes model is a cornerstone of option pricing, relying on the assumption of continuous and normally distributed returns. However, empirical evidence shows that asset prices can experience sudden jumps due to market events, which the Black-Scholes model fails to capture. The Jump Diffusion model, proposed by Robert C. Merton, addresses this limitation by adding a jump component to the asset price dynamics, allowing for more realistic modeling of asset returns.

Objective

The primary objective of this project is to implement the Merton Jump Diffusion Model for option pricing and compare its performance with the traditional Black-Scholes model. The project aims to demonstrate how incorporating jumps can lead to more accurate option pricing in markets with significant discontinuities.

Scope

This project focuses on pricing European call options using the Merton Jump Diffusion Model and comparing the results with the Black-Scholes model. The analysis includes sensitivity analysis to understand the impact of different parameters on option prices.

5. Literature Review

Option Pricing Models

The Black-Scholes model, introduced in 1973, revolutionized the field of finance by providing a closed-form solution for pricing European options. However, its assumptions of constant volatility and continuous price movements are often violated in practice.

Jump Diffusion Models

The Jump Diffusion model, introduced by Merton in 1976, extends the Black-Scholes model by incorporating a jump process into the asset price dynamics. This model captures both the continuous price changes and the occasional large jumps, offering a more comprehensive framework for option pricing.

6. Methodology

Data Collection and Preprocessing

- **Data Sources**: Historical price data for the underlying asset and option prices are obtained from financial databases such as Bloomberg or Yahoo Finance.
- **Data Cleaning**: The data is cleaned to remove outliers and handle missing values, ensuring accuracy in model implementation.

Jump Diffusion Model Implementation

Merton Jump Diffusion Model: The Merton Jump Diffusion Model extends the Black-Scholes model by adding a Poisson jump process to the asset price dynamics. The asset price $S(t)$ under the Merton model is given by:

$$dS(t) = (r - \lambda \kappa) S(t) dt + \sigma S(t) dW(t) + S(t) dJ(t)$$

Where:

- r is the risk-free rate.
- σ is the volatility of the asset.
- $W(t)$ is a Wiener process representing the continuous part of the returns.
- $J(t)$ is a Poisson jump process with intensity λ.
- κ represents the average jump size.

Option Pricing with Merton Model

The Merton Jump Diffusion Model is used to price European call options. The code provided calculates the option price by summing the contributions from different possible numbers of jumps.

Comparison with Black-Scholes Model

The option prices obtained from the Merton model are compared with those from the Black-Scholes model to assess the impact of jumps on pricing.

7. Results

Option Pricing Comparison

- **Merton Jump Diffusion Model Option Price**: The calculated price reflects the impact of potential jumps, resulting in a different (typically higher) option price compared to the Black-Scholes model.
- **Black-Scholes Model Option Price**: This price is based on the assumption of continuous price changes without jumps.

Sensitivity Analysis

A sensitivity analysis is performed to study the impact of varying key parameters like volatility (σ), jump intensity (λ), and jump size (μ_j) on the option price.

8. Discussion

Insights

- **Impact of Jumps**: The Merton model accounts for sudden large movements in the underlying asset price, which are not captured by the Black-Scholes model. This results in higher option prices, especially for out-of-the-money options.
- **Parameter Sensitivity**: The option price is particularly sensitive to the jump intensity (λ\lambdaλ) and the average jump size (μj\mu_jμj). As these parameters increase, the option price tends to rise, reflecting the higher risk of sudden large price changes.

Limitations

- **Model Complexity**: The Merton Jump Diffusion Model is more complex than the Black-Scholes model, requiring additional parameters that need to be estimated accurately.
- **Assumptions**: The model assumes jumps follow a Poisson distribution and that the jump size distribution is log-normal, which may not always reflect real market conditions.

9. Conclusion

The Merton Jump Diffusion Model provides a more realistic framework for option pricing in markets where asset prices experience sudden jumps. While more complex than the Black-Scholes model, it offers greater accuracy in pricing options, particularly in volatile markets. The sensitivity analysis highlights the importance of accurately estimating jump-related parameters to obtain reliable option prices.

10. References

- Merton, R. C. (1976). "Option Pricing When Underlying Stock Returns Are Discontinuous". Journal of Financial Economics, 3(1-2), 125-144.
- Black, F., & Scholes, M. (1973). "The Pricing of Options and Corporate Liabilities". Journal of Political Economy, 81(3), 637-654.

11. Appendices

- **Appendix A**: Full Python Code Implementation
- **Appendix B**: Additional Sensitivity Analysis Results