

# Stat 6021: Standard Error for the Mean Response

Read this after Section 3 in Guided Notes.

In Module 1, we have shown that

$$Var(\hat{\beta}_1) = \frac{\sigma^2}{\sum (x_i - \bar{x})^2} \quad (1)$$

and that

$$Cov(\bar{y}, \hat{\beta}_1) = 0 \quad (2)$$

We will use these properties to derive the standard error of the mean response.

## 1 Properties of the Estimated Mean Response for Given Predictor

Let  $x_0$  denote the value of the predictor for which we want to estimate the mean response for, which is in turn denoted by  $\mu_{y|x_0}$ . Then we have the following for the expected value and variance for the estimate as:

$$\begin{aligned} E(\hat{\mu}_{y|x_0}) &= E(\hat{\beta}_0 + \hat{\beta}_1 x_0) \\ &= \beta_0 + \beta_1 x_0. \end{aligned}$$

$$\begin{aligned} Var(\hat{\mu}_{y|x_0}) &= Var(\hat{\beta}_0 + \hat{\beta}_1 x_0) \\ &= Var(\bar{y} + \hat{\beta}_1(x_0 - \bar{x})) \\ &= Var(\bar{y}) + (x_0 - \bar{x})^2 Var(\hat{\beta}_1) + 2Cov(\bar{y}, \hat{\beta}_1) \\ &= \frac{\sigma^2}{n} + (x_0 - \bar{x})^2 \frac{\sigma^2}{\sum (x_i - \bar{x})^2} \text{ using (1) and (2)} \\ &= \sigma^2 \left[ \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{\sum (x_i - \bar{x})^2} \right] \end{aligned}$$