Stat 6021: Confidence Intervals and Hypothesis Testing in Logistic Regression

Read this after Section 4 from Guided Notes

1 Interpreting Confidence Intervals for Logistic Regression Coefficients

In Section 13.2.3 of your textbook, we have the following interpretation of the coefficient β_1 in a logistic regression model: For a one-unit increase in the predictor x_1, the odds ratio is $\exp(\beta_1)$. In other words, the odds are multiplied by a factor of $\exp(\beta_1)$ when the predictor x_1 increases by a unit.

Suppose we find a confidence interval for β_1 using

$$\hat{\beta}_1 - z_{\alpha/2} se(\hat{\beta}_1) \leq \beta_1 \leq \hat{\beta}_1 + z_{\alpha/2} se(\hat{\beta}_1) \tag{1}$$

Let L and U denote the lower and upper bounds of the interval in (1). Exponentiating (1), we obtain

$$\exp\left[\hat{\beta}_1 - z_{\alpha/2} se(\hat{\beta}_1)\right] \leq \text{ odds ratio } \leq \exp\left[\hat{\beta}_1 + z_{\alpha/2} se(\hat{\beta}_1)\right]$$

Thus, we can interpret the confidence as: For a one-unit increase in x_1, the odds gets multiplied by a factor between $\exp(L)$ and $\exp(U)$. We typically check to see if 0 lies within the interval in (1), since $\exp(0) = 1$, i.e., multiplying by a factor of 1 means the value has not changed.

2 Overview of Likelihood Ratio Tests in Logistic Regression

The various likelihood ratio tests in logistic regression are analagous to the partial F, ANOVA F, and t tests in linear regression using ordinary least squares. The likelihood ratio tests involve comparing the model fit using deviance between two models, and then evaluate if the presence of the additional predictors improve the model fit enough to justify making the model more complex. Consider a first-order logistic regression with k predictors.

2.1 Testing a Subset of Coefficients

Suppose we are testing whether we can drop the first q predictors from the model, where $q \leq k$. We have the following:

$$\begin{aligned} H_0 &: \beta_1 = \beta_2 = \cdots = \beta_q = 0 \\ H_a &: \text{not all of } \beta_j \text{ in } H_0 \text{ equal zero.} \end{aligned}$$

The test statistic is the difference in the model deviances between both models, $D(\beta_1, \cdots, \beta_q) - D(\beta_1, \cdots, \beta_k)$, which is then compared to a χ_2 distribution with q degrees of freedom. In R, the model deviances are called the residual deviance.

2.2 Testing All Coefficients

To test if all the coefficients are equal to zero, the test statistic is the difference in the deviances between our model and the intercept-only model. In R, this test statistic can be found by computing the difference between the null deviance and residual deviance, and compare the test statistic to a χ^2 distribution with k degrees of freedom. This test is just a special case of testing a subset of coefficients with $q = k$.

2.3 Testing a Single Coefficient

The Wald test for a single coefficient is just a special case of testing a subset of coefficients with $q = 1$. Assuming we are looking to drop the first predictor, the hypotheses become

$$\begin{aligned} H_0 &: \beta_1 = 0 \\ H_a &: \beta_1 \neq 0 \end{aligned}$$

The Wald test statistic will give the same result as using the difference in the model deviances with and without x_1.