Homework 6
Dima Mikhaylov

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LeftFoot      0.3519     0.2961   1.188    0.240
RtFoot        0.1850     0.2816   0.657    0.514
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Residual standard error: 1.796 on 52 degrees of freedom
Multiple R-squared:  0.3688,    Adjusted R-squared:  0.3445
F-statistic: 15.19 on 2 and 52 DF,  p-value: 6.382e-06
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A classmate points out that there appears to be a contradiction in the R output, namely, while the ANOVA F statistic is significant, the t statistics for both predictors are insignificant. Is your classmate's concern warranted? Briefly explain.

4. (No R required) Recall in matrix notation, the least-squares estimators for the regression model can be written as

$$\hat{\boldsymbol{\beta}} = \begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \vdots \\ \hat{\beta}_k \end{bmatrix} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{y}.$$

Fitted values are usually written as

$$\hat{\boldsymbol{y}} = \boldsymbol{X}\hat{\boldsymbol{\beta}} = \boldsymbol{X}(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{y} = \boldsymbol{H}\boldsymbol{y}$$

where $\boldsymbol{H} = \boldsymbol{X}(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'$. $\boldsymbol{H}$ is called the hat matrix. Show that $\boldsymbol{H}$ is idempotent, i.e., $\boldsymbol{H}\boldsymbol{H} = \boldsymbol{H}$.

Proof: $HH = \left(X(X'X)^{-1}X'\right)\left(X(X'X)^{-1}X'\right) \Rightarrow$

Since $E(\varepsilon)=0$ and $(X'X)^{-1}X'X = I \Rightarrow$

$\Rightarrow X(X'X)^{-1}X' = H.$

Therefore $HH = H.$