Mini Project 1
Mohammed Allama Hossain, Lavanya Gopal
We worked on the questions separately, and then we met up and checked our results with each other and verified it. During the same meeting we collaborated and completed the coding part of the assignment. Lavanya then ran the simulations and I (Allama) compiled the results and completed the documentation.

Question 1.

a) Analytical solution:

Using the given probability density function, we can compute the probability that the lifetime of the satellite exceeds 15 years as:
Let the given probability density function be $f_T(t)$. Then the probability that the lifetime of the satellite exceeds 15 years is given by:

$$P(T > 15) = 1 - P(T <= 15)$$
$$= 1 - F(15)$$
$$= 1 - \int_0^{15} f_T(t)dt$$
$$= 1 - \left\{ \int_0^{15} 0.2e^{-0.1t} - 0.2e^{0.2t} dt \right\}$$
$$= 1 - \left\{ \int_0^{15} 0.2e^{-0.1t} - \int_0^{15} 0.2e^{0.2t} dt \right\}$$
$$= 1 - \left[\left\{ 0.2\left(\frac{e^{-0.1t}}{-0.1} \right) \right\}_0^{15} - \left\{ 0.2\left(\frac{e^{-0.2t}}{-0.2} \right) \right\}_0^{15} \right]$$
$$= 1 - \left[\left\{ -2e^{-0.1t} \right\}_0^{15} + \left\{ e^{-0.2t} \right\}_0^{15} \right]$$
$$= 1 - \left[\left\{ -2e^{-1.5} + 2e^{0} \right\} + \left\{ e^{-3} - e^{0} \right\} \right]$$
$$= 1 - \left[-2e^{-1.5} + e^{-3} + 1 \right]$$
$$= 1 - \left[-0.446260 + 0.049787 + 1 \right]$$
$$= 1 - 0.603527$$
$$= 0.396473$$

b) i)

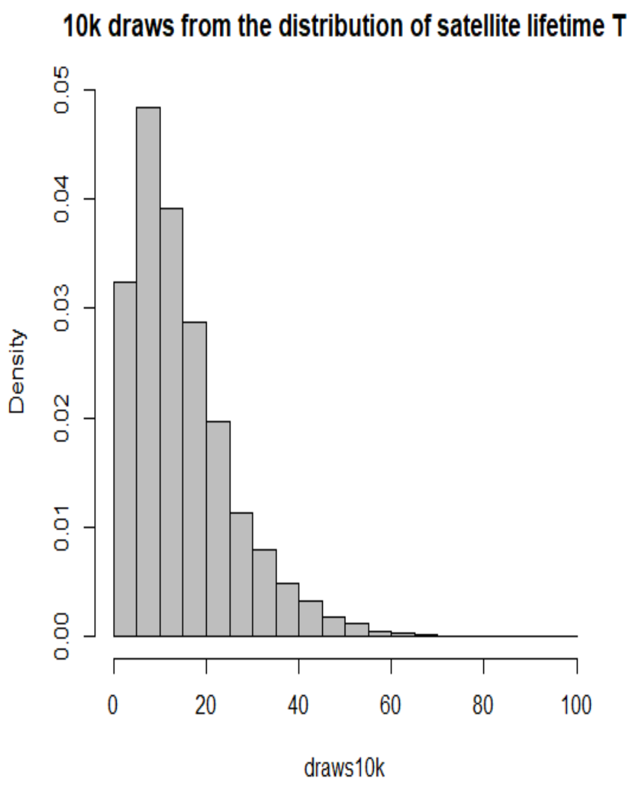
```
> #Question 1)b)i)
> probabDensityFunc <- function(x){return (0.2*exp(-0.1*x)-0.2*exp(-0.2*x))}
```

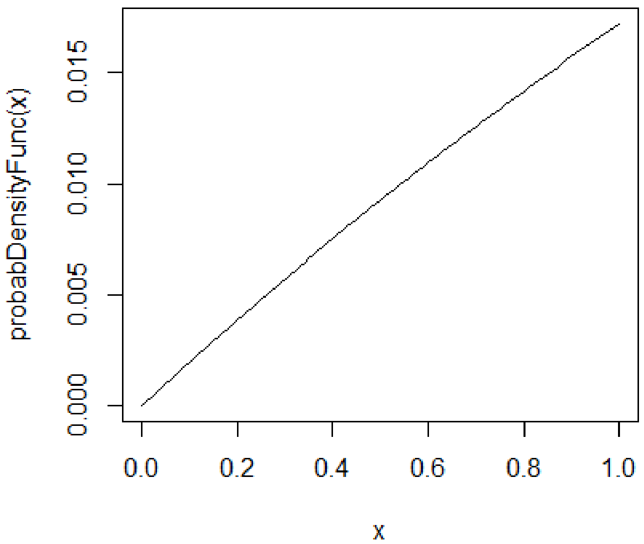
ii)

```
> #ii)
> draws10k <- replicate(10000, max(rexp(n = 1, rate = 1/10), rexp(n = 1, rate
= 1/10)))
```

iii)

```
> #iii
> hist(x = draws10k, main = "10k draws from the distribution of satellite li
fetime T", col = "gray", freq = FALSE)
> curve(probabDensityFunc)
```





iv) The value 15.07093 achieved from the Monte Carlo simulation is very close to the derived value from the probability density function.

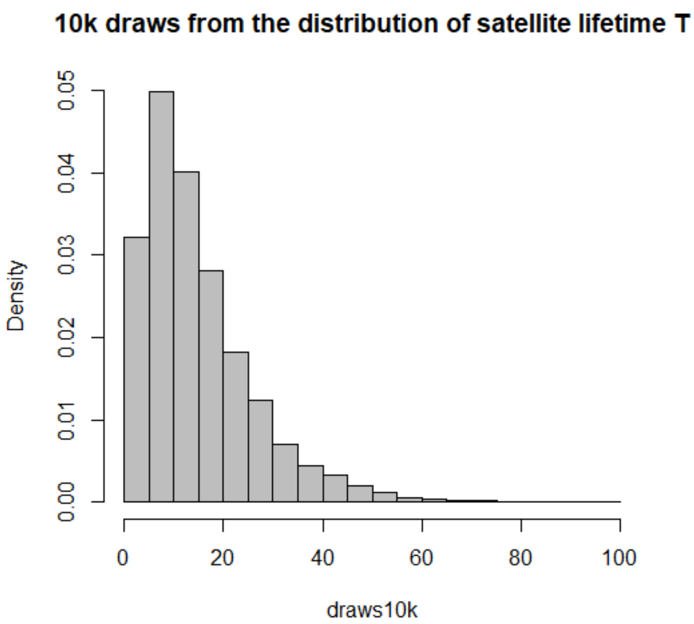
```
> mean(draws10k)
[1] 15.07093
```

v) The probability 0.369615 is almost close to the analytical solution despite the difference in the mean values.

```
> 1 - pexp(15, rate = 1 / mean(draws10k))
[1] 0.369615
```

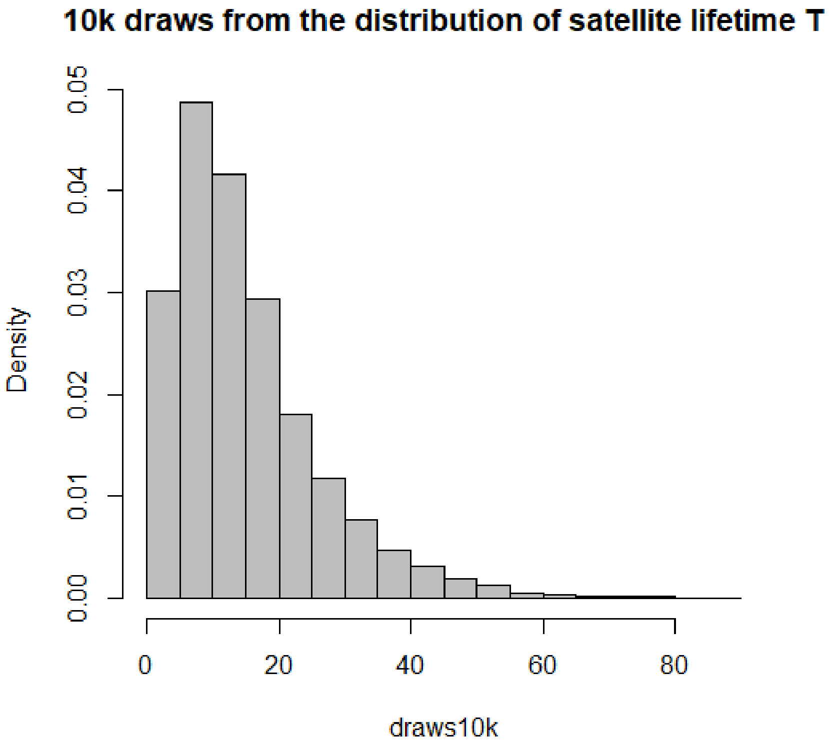
vi) **FIRST ITERATION**

```
> #vi
> #Sample Size = 10000
> #First Iteration
> draws10k <- replicate(10000, max(rexp(n = 1, rate = 1/10), rexp(n = 1, rate
= 1/10)))
> hist(x = draws10k, main = "10k draws from the distribution of satellite life
time T", col = "gray", freq = FALSE)
> mean(draws10k)
[1] 14.93125
> 1 - pexp(15, rate = 1 / mean(draws10k))
[1] 0.3661893
```



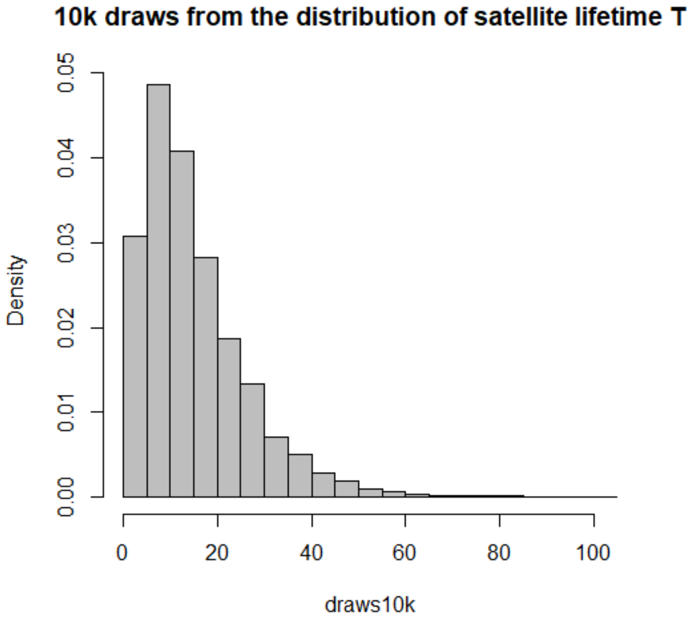
SECOND ITERATION

```
> #Second Iteration
> draws10k <- replicate(10000, max(rexp(n = 1, rate = 1/10), rexp(n = 1, rate
= 1/10)))
> hist(x = draws10k, main = "10k draws from the distribution of satellite life
time T", col = "gray", freq = FALSE)
> mean(draws10k)
[1] 15.08628
> 1 - pexp(15, rate = 1 / mean(draws10k))
[1] 0.3699895
```



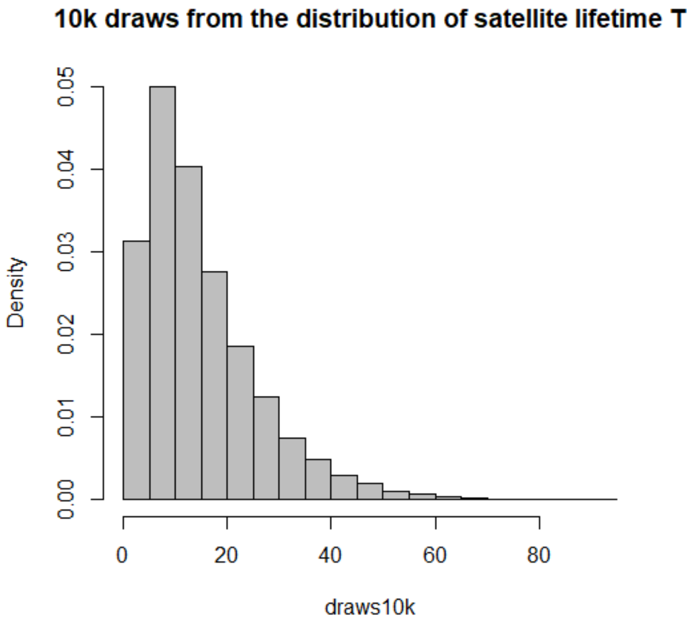
THIRD ITERATION

```
> #Third Iteration
> draws10k <- replicate(10000, max(rexp(n = 1, rate = 1/10), rexp(n = 1, rate = 1
/10)))
> hist(x = draws10k, main = "10k draws from the distribution of satellite lifetim
e T", col = "gray", freq = FALSE)
> mean(draws10k)
[1] 15.16742
> 1 - pexp(15, rate = 1 / mean(draws10k))
[1] 0.3719627
```



FOURTH ITERATION

```
> #Fourth Iteration
> draws10k <- replicate(10000, max(rexp(n = 1, rate = 1/10), rexp(n = 1, rate =
1/10)))
> hist(x = draws10k, main = "10k draws from the distribution of satellite lifet
ime T", col = "gray", freq = FALSE)
> mean(draws10k)
[1] 14.88513
> 1 - pexp(15, rate = 1 / mean(draws10k))
[1] 0.3650515
```



Comparison Table:

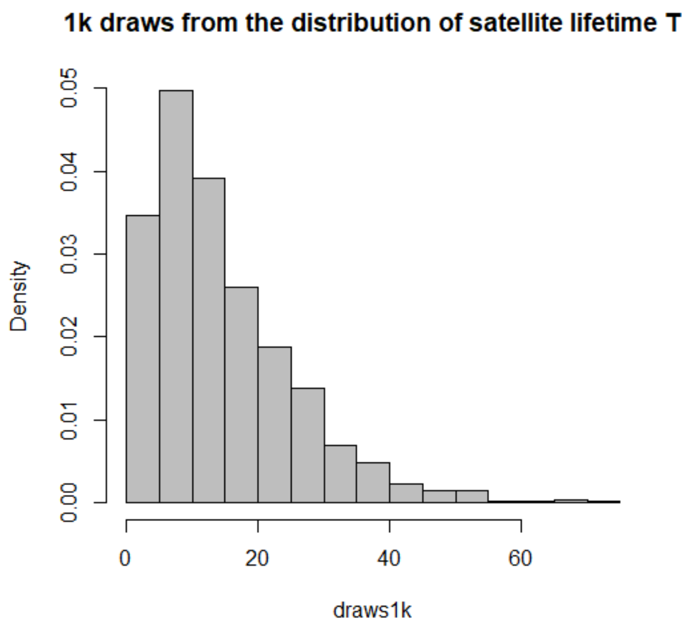
Test for Sample Size 10000	First Iteration	Second Iteration	Third Iteration	Fourth Iteration	Fifth Iteration
E(T)	15.07093	14.93125	15.08628	15.16742	14.88513
P(T>15)	0.369615	0.3661893	0.3699895	0.3719627	0.3650515

The expected value E(T) is very close to the actual value as seen from the data compiled in the above table and the probability distribution P(T>15) is also close to the analytically derived probability value with slight variations. Therefore we can conclude that Central Limit Theorem holds true.

c) Sample Size: 1000

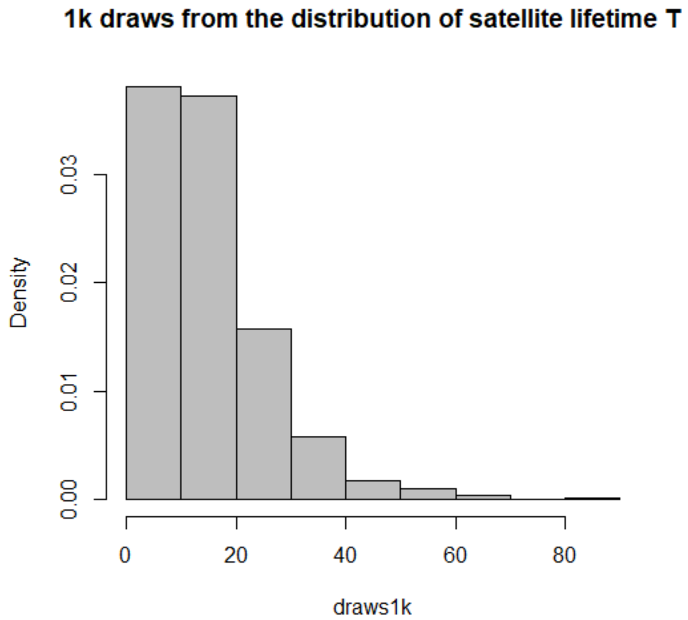
FIRST ITERATION

```
> #c
> #Sample Size = 1000
> #First Iteration, SampleSize = 1000
> draws1k <- replicate(1000, max(rexp(n = 1, rate = 1/10), rexp(n = 1, rate = 1/1
0)))
> hist(x = draws1k, main = "1k draws from the distribution of satellite lifetime
T", col = "gray", freq = FALSE)
> mean(draws1k)
[1] 14.56739
> 1 - pexp(15, rate = 1 / mean(draws1k))
[1] 0.357115
```



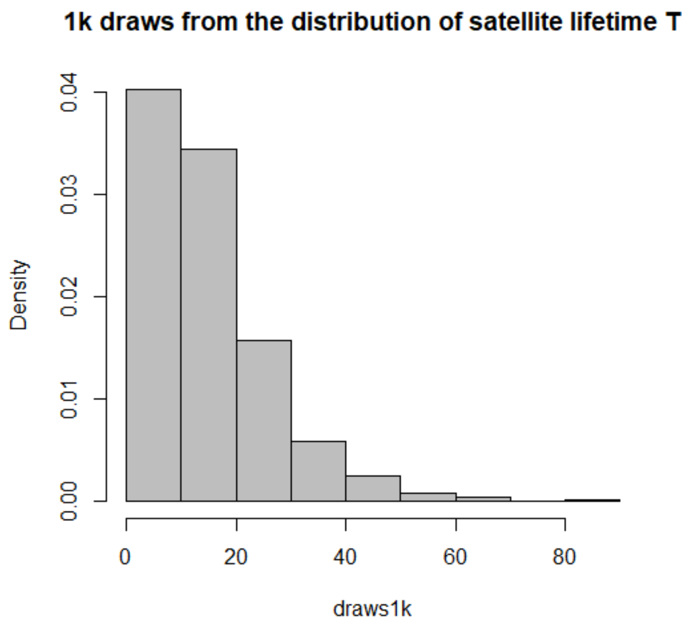
SECOND ITERATION

```
> #Second Iteration, SampleSize = 1000
> draws1k <- replicate(1000, max(rexp(n = 1, rate = 1/10), rexp(n = 1, rate = 1/10
)))
> hist(x = draws1k, main = "1k draws from the distribution of satellite lifetime T
", col = "gray", freq = FALSE)
> mean(draws1k)
[1] 15.00954
> 1 - pexp(15, rate = 1 / mean(draws1k))
[1] 0.3681134
```



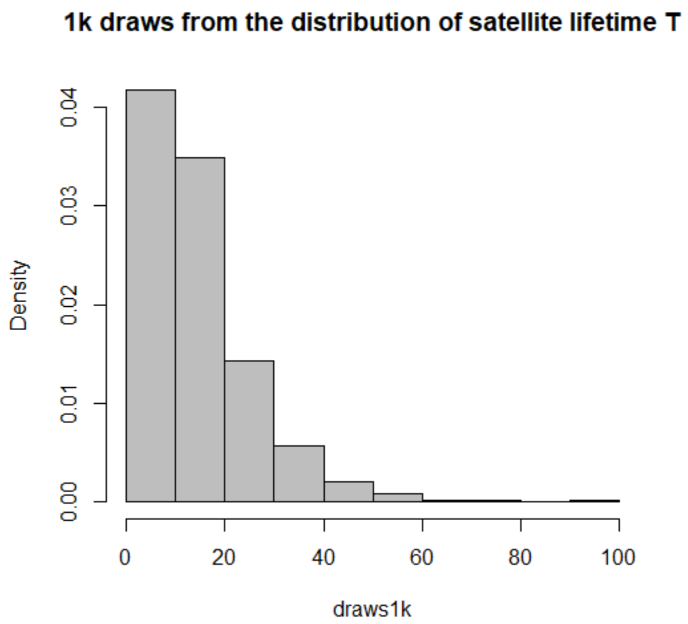
THIRD ITERATION

```
> #Third Iteration, SampleSize = 1000
> draws1k <- replicate(1000, max(rexp(n = 1, rate = 1/10), rexp(n = 1, rate = 1/10
)))
> hist(x = draws1k, main = "1k draws from the distribution of satellite lifetime T
", col = "gray", freq = FALSE)
> mean(draws1k)
[1] 14.97528
> 1 - pexp(15, rate = 1 / mean(draws1k))
[1] 0.3672726
```



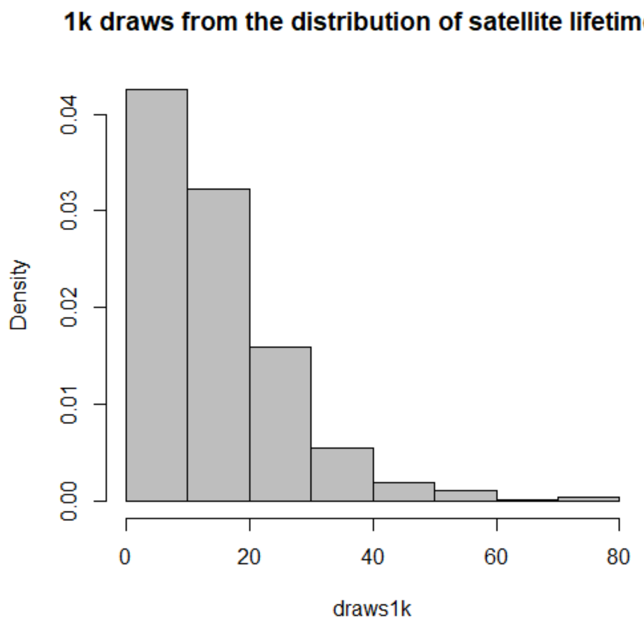
FOURTH ITERATION

```
> #Fourth Iteration, SampleSize = 1000
> draws1k <- replicate(1000, max(rexp(n = 1, rate = 1/10), rexp(n = 1, rate = 1/10)
))
> hist(x = draws1k, main = "1k draws from the distribution of satellite lifetime T"
, col = "gray", freq = FALSE)
> mean(draws1k)
[1] 14.61472
> 1 - pexp(15, rate = 1 / mean(draws1k))
[1] 0.358308
```



FIFTH ITERATION

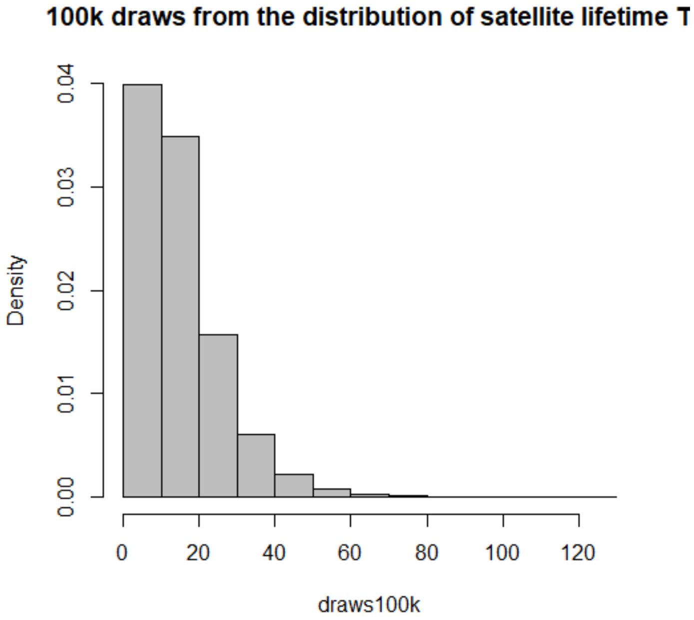
```
> #Fifth Iteration, SampleSize = 1000
> draws1k <- replicate(1000, max(rexp(n = 1, rate = 1/10), rexp(n = 1, rate = 1/1
0)))
> hist(x = draws1k, main = "1k draws from the distribution of satellite lifetime
T", col = "gray", freq = FALSE)
> mean(draws1k)
[1] 14.71134
> 1 - pexp(15, rate = 1 / mean(draws1k))
[1] 0.3607313
```



Sample Size 100000

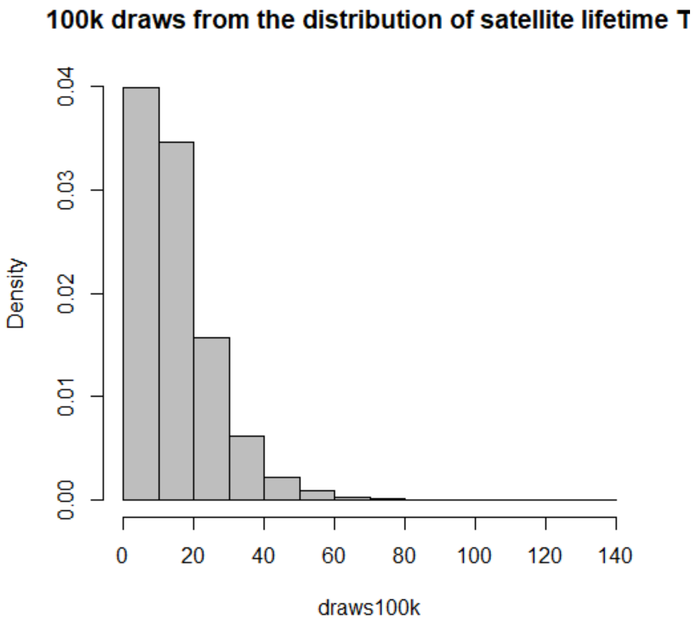
FIRST ITERATION

```
> #Sample Size = 100000
> #First Iteration, SampleSize = 100000
> draws100k <- replicate(100000, max(rexp(n = 1, rate = 1/10), rexp(n = 1, rate =
1/10)))
> hist(x = draws100k, main = "100k draws from the distribution of satellite lifet
ime T", col = "gray", freq = FALSE)
> mean(draws100k)
[1] 14.98801
> 1 - pexp(15, rate = 1 / mean(draws100k))
[1] 0.3675852
```



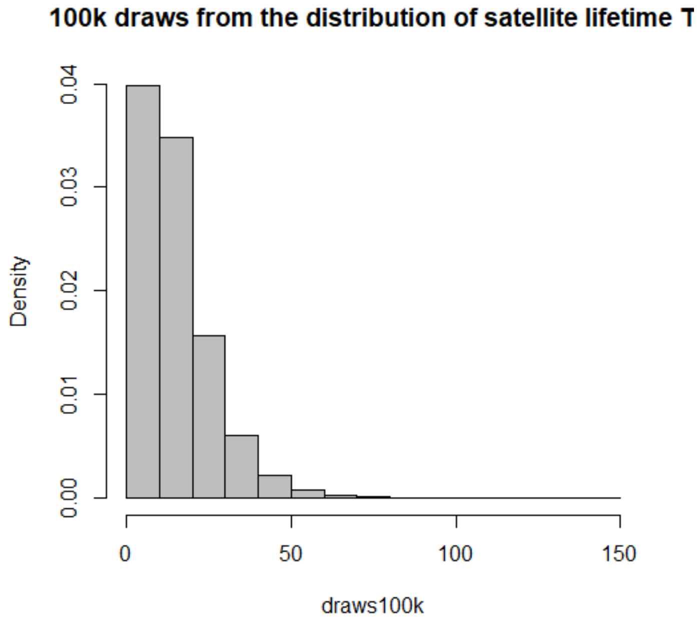
SECOND ITERATION

```
> #Second Iteration, SampleSize = 100000
> draws100k <- replicate(100000, max(rexp(n = 1, rate = 1/10), rexp(n = 1, rate =
1/10)))
> hist(x = draws100k, main = "100k draws from the distribution of satellite lifet
ime T", col = "gray", freq = FALSE)
> mean(draws100k)
[1] 15.01277
> 1 - pexp(15, rate = 1 / mean(draws100k))
[1] 0.3681925
```



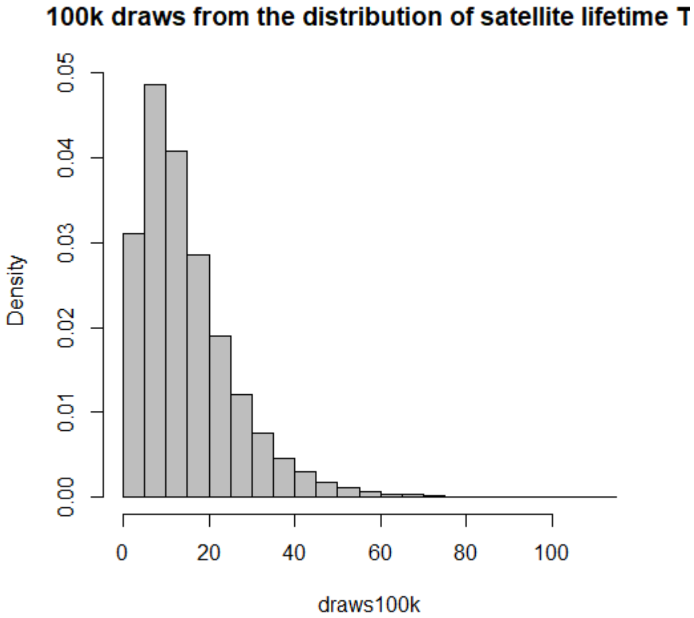
THIRD ITERATION

```
> #Third Iteration, SampleSize = 100000
> draws100k <- replicate(100000, max(rexp(n = 1, rate = 1/10), rexp(n = 1, rate =
1/10)))
> hist(x = draws100k, main = "100k draws from the distribution of satellite lifeti
me T", col = "gray", freq = FALSE)
> mean(draws100k)
[1] 15.00557
> 1 - pexp(15, rate = 1 / mean(draws100k))
[1] 0.368016
```



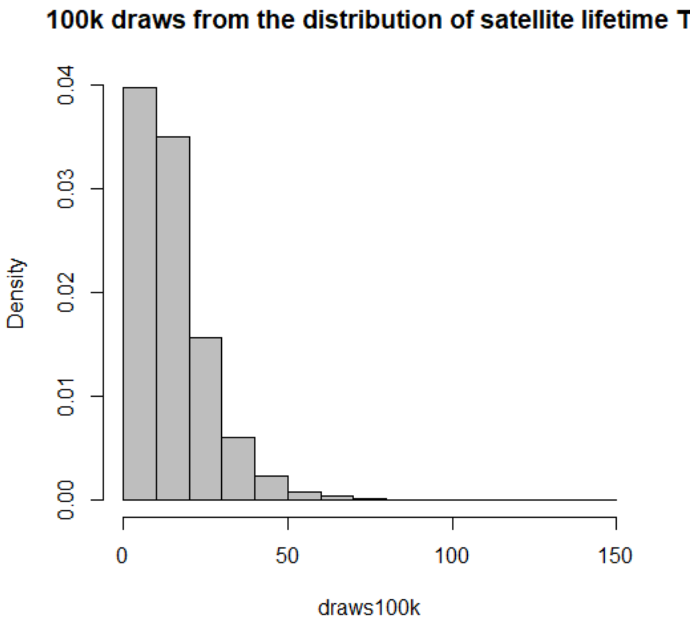
FOURTH ITERATION

```
> #Fourth Iteration, SampleSize = 100000
> draws100k <- replicate(100000, max(rexp(n = 1, rate = 1/10), rexp(n = 1, rate =
1/10)))
> hist(x = draws100k, main = "100k draws from the distribution of satellite lifeti
me T", col = "gray", freq = FALSE)
> mean(draws100k)
[1] 15.07215
> 1 - pexp(15, rate = 1 / mean(draws100k))
[1] 0.3696448
```



FIFTH ITERATION

```
> #Fifth Iteration, SampleSize = 100000
> draws100k <- replicate(100000, max(rexp(n = 1, rate = 1/10), rexp(n = 1, rate =
1/10)))
> hist(x = draws100k, main = "100k draws from the distribution of satellite lifeti
me T", col = "gray", freq = FALSE)
> mean(draws100k)
[1] 15.02857
> 1 - pexp(15, rate = 1 / mean(draws100k))
[1] 0.3685795
```



Test for Sample Size 1000					
	First Iteration	Second Iteration	Third Iteration	Fourth Iteration	Fifth Iteration
E(T)	14.56739	15.00954	14.97528	14.61472	14.71134
P(T>15)	0.357115	0.3681134	0.3672726	0.358308	0.3607313
Test for Sample Size 10000					
	First Iteration	Second Iteration	Third Iteration	Fourth Iteration	Fifth Iteration
E(T)	15.07093	14.93125	15.08628	15.16742	14.88513
P(T>15)	0.369615	0.3661893	0.3699895	0.3719627	0.3650515
Test for Sample Size 100000					
	First Iteration	Second Iteration	Third Iteration	Fourth Iteration	Fifth Iteration
E(T)	14.98801	15.01277	15.00557	15.07215	15.02857
P(T>15)	0.3675852	0.3681925	0.368016	0.3696448	0.3685795

It can be seen from the compiled data above that for higher sample sizes, the variations amongst the derived value reduces and this is in coherence with the Central Limit Theorem. The test for sample size 100000 has the least variation amongst the simulations of expected value E(T) and probability P(T>15).

Question 2:

The probability that a point lies in a circle with centre (0.5, 0.5) inscribed in a square with coordinates (0, 0), (1, 0), (0, 1), (1, 1) is given by

Area of the circle / Area of the square

= pi/4

Now we will run a simulation with 10000 iterations and we will generate random deviates of a uniform distribution for the variables x and y and we will check if the number falls within the range of the circle or not.

```
> #Question 2
> iter <- 10000
> x <- runif(iter, min = 0, max = 1)
> y <- runif(iter, min = 0, max = 1)
> circle <- (x - 0.5)^2 + (y - 0.5)^2 <= 0.5^2
> MonteCarloPi <- (sum(circle)/iter)*4
> MonteCarloPi
[1] 3.1496
```

It can be seen from the above result that the simulated value of pi achieved was 3.1496 which is close to the actual value of pi 3.14159.