Hypothesis testing for the mean

→ It is a statistical procedure used to determine whether a sample mean provides enough evidence to support or reject a claim about a population mean.

→ Two competing statements.

null Hypothesis (H_0): It representing what has been assumed about the value of the parameter

alternative Hypothesis (H_1) OR (H_a) : It represents an alternative claim about the value of the parameter.

→ There are two possible conclusions.

(I) Reject Ho

(II) do not reject Ho.

Ex A criminal trial in the Form of Hypothesis test.

Ho: Defendant is innocent

H_a: " is guilty.

		Reality: Ho true: Defendant did not commit crime	Ho false: Defendant did commit crime
Jury's Decision	Reject Ho: Find defendant guilty	Type 1 error	correct decision
	Do not Reject Ho: find defendant not guilty	correct decision	Type-II error

Type 1 error: Reject Ho when Ho is true. The jury convicts an innocent person.

Type II error: Do not reject Ho when Ho is false. Jury acquits a guilty person.

correct decisions :

Reject H_0 when H_0 is false : The jury convicts a guilty person.

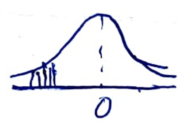
Do not reject H_0 when H_0 is true : The jury acquits an innocent Person.

The probability of a type-I error is denoted α.

" type-II " " β.

* p-value is measures the strength of evidence against H_0.

  if the p-value is less than α, reject H_0. otherwise fail to reject H_0.

Form of Hypothesis Test	p-value
Left-tailed test $H_0: \mu \geq \mu_0$ versus $H_a: \mu < \mu_0$ 	$P(t < t_{data})$
Right-tailed test $H_0: \mu \leq \mu_0$ versus $H_a: \mu > \mu_0$	$P(t > t_{data})$
Two tailed test $H_0: \mu = \mu_0$ versus $H_a: \mu \neq \mu_0$	If $t_{data} < 0$, then p-value $= 2 \cdot P(t < t_{data})$ If $t_{data} > 0$, then p-value $= 2 \cdot P(t > t_{data})$

where μ_0 represents a hypothesized value of μ.

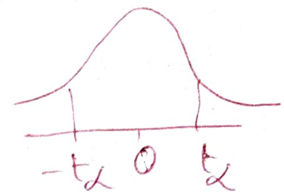
→ population standard deviation is known, use z-test

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

If σ is unknown, use t-test

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$ where $\bar{x}$ = sample mean
s = sample standard deviation
n = sample size.

Q two tail hypothesis test :



S-1 set hypothesis
$H_0 : \mu = 2.4$ Vs $H_a : \mu \neq 2.4$

The null hypothesis will be rejected if the p-value is less than 0.05.

Here $\mu_0 = 2.4$, $\bar{x} = 1.607$, $s = 1.892$, $n = 28$.

S-2 Choose statistics (t_{data})
Thus.
$$t_{data} = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{1.607 - 2.4}{1.892/\sqrt{28}} = -2.2178$$

This value will be check from t-table

As $t_{data} < 0$,

S-3 Rejection Rule
$$\text{P-value} = 2 \cdot P(t < t_{data}) = 2 \cdot P(t < (-2.2178))$$
$$= 2 \cdot P(t > 2.2178)$$
$$\cancel{= 2(P(t < -2.2178))} = 2 \cdot (0.01758)$$ 0.015
$$= 0.035$$

As per t-table

$df = n - 1 = 28 - 1 = 27$, & value is (2.21) So
It will lies in between 0.02 & 0.01 column of table.

So $\frac{0.02 + 0.01}{2} = 0.015$

Thus $0.015 \times 2 = 0.030$

S-4 conclusion
~~As~~ P value is 0.035 which is less than $\alpha = 0.05$.

we reject H_0.

Q/ A sample of 100 donors to a charity has a mean donation amount of \$55 with a sample standard deviation of \$25. Test using $\alpha = 0.05$ whether the population mean donation amount exceeds \$50.

Given

Sample size $= n = 100$

" mean $(\bar{x}) = \$55$ $\qquad \alpha = 0.05, \ \mu_0 = 50$

" SD $(s) = \$25$

(I) we define Hypothesis

$H_0 : \mu < 50 \quad vs \quad H_1 : \mu \geq 50$

(II) Choose the statistics (t_{data})

$$t_{data} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{55 - 50}{25\sqrt{100}} = \frac{5}{25/10} = 2.0$$

(III) Rejection Rule

IF p-value $< \alpha = 0.05$, then reject the $H_0 : \mu < 50$

In this case, $0.02 < \text{p-value} < 0.025$ $\quad = t = 2 \quad 0.025 + 0.02$

$$\text{p-value} = P(T > t_{data}) = P(T > 2) = 0.02412$$

(IV) conclusion :

since p-value $< \alpha = 0.05$ we must reject H_0.

Thus population mean ≥ 50