

Ch-6 Multivariate Statistics

Two sample T-test for Difference in means

$$t_{data} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{(s_1^2/n_1) + (s_2^2/n_2)}}$$

df = $n_1 - 1 + n_2 - 1 = n_1 + n_2 - 2$
will be minimum ~~2539-1~~

Q Page-189 ie $n_1 - 1 = 804 - 1$
 $n_2 - 1 = 804 - 1$

S-1 State hypotheses

$H_0 = \mu_1 = \mu_2$ (no-difference)
 $H_1: \mu_1 \neq \mu_2$ (means are different)

This is two-tail test.

S-2 compute test statistics

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{(s_1^2/n_1) + (s_2^2/n_2)}} = \frac{1.5714 - 1.5361}{\sqrt{\frac{(1.3186)^2}{2539} + \frac{(1.3851)^2}{804}}} = 0.6595$$

S-3

P-value (use z-table because sample is too large)

P-value = $2 \cdot P(t > t_{data}) = P(t > 0.6595)$

$1 - 0.7421 = 0.2579$
 $2 \times 0.2579 = 0.5158$

from z-table row 0.6 & column 0.06 = $0.6 + 0.06 = 0.66$
value is 0.7454, from right tail = $1 - 0.7454 = 0.2546$

S-4 conclusion

$P = 0.508 > 0.05$
do not reject the null hypothesis.

Two Sample Z-Test for Difference in Proportions

$$Z_{data} = \frac{p_1 - p_2}{\sqrt{P_{pooled} \cdot (1 - P_{pooled}) \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

where $P_{pooled} = \frac{x_1 + x_2}{n_1 + n_2}$ ~~the~~

Ex $x_1 = 707$ of $n_1 = 2529$ customers on the training set belonging to the voice mail plan while $x_2 = 215$ of $n_2 = 804$ customers on the test set.

$$\text{So } p_1 = \frac{x_1}{n_1} = \frac{707}{2529} = 0.2796$$

$$p_2 = \frac{x_2}{n_2} = \frac{215}{804} = 0.2674$$

$$P_{pooled} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{707 + 215}{2529 + 804} = 0.2766$$

S-1 Hypothesis

$$H_0: \pi_1 = \pi_2 \quad \text{Vs} \quad H_1: \pi_1 \neq \pi_2$$

S-2

$$\begin{aligned} Z_{data} &= \frac{p_1 - p_2}{\sqrt{P_{pooled} \cdot (1 - P_{pooled}) \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \\ &= \frac{0.2796 - 0.2674}{\sqrt{0.2766 \cdot (0.7234) \left(\frac{1}{2529} + \frac{1}{804} \right)}} \\ &= 0.6736 \end{aligned}$$

$$p\text{-value} = 2 \cdot P(Z > 0.6736)$$

$$= 0.5006$$

from Z-table
 0.6 (row) 40.07 (column)
 0.7485

s-y conclusion
 $p\text{-value} = 0.5006 > 0.05$
 so do not reject H_0 .

TEST FOR HOMOGENEITY OF PROPORTIONS (chi-square distribution)
observed frequency (Marital status)

Dataset	Married	Single	other	Total
Training set	410	340	250	1000
Test set	95	85	70	250
Total	505	425	320	1250

s-1 Hypothesis set

$H_0: P_{\text{married, training}} = P_{\text{married, test}}$

$P_{\text{single, training}} = P_{\text{single, test}}$

$P_{\text{other, training}} = P_{\text{other, test}}$

H_a : At least one of the claims in H_0 is wrong.

3-2 Expected frequency

$$\text{married, training} = \frac{(1000) \cdot 505}{1250} = 404$$

$$\text{Expected frequency} = \frac{(\text{row total}) (\text{column total})}{\text{grand total}}$$

Expected frequencies

Dataset	Married	Single	Other	Total
Training set	404	340	256	1000
Test set	101	85	64	250
Total	505	425	320	1250

3 calculate test statistic χ^2 data

cell	observed freq.	Expected freq.	$\frac{(\text{Obs} - \text{exp})^2}{\text{exp}}$
Married, training	410	404	$\frac{(410 - 404)^2}{404} = 0.09$
Married, test	95	101	0.36
Single, training	340	340	0
Single, test	85	85	0
Other, training	250	256	0.14
Other, test	70	64	0.56

$$\text{sum} = \chi^2_{\text{data}} = 1.15$$

$$p\text{-value} = P(\chi^2 > \chi^2_{\text{data}}) = P(\chi^2 > 1.15) = 0.5627$$

$$\textcircled{1} \text{ test of independence: } df = (r-1)(c-1) = 2$$

$$\text{i.e. } df = (\text{no. of rows} - 1)(\text{no. of columns} - 1)$$

$$= (2-1)(3-1) = 1 \times 2 = 2$$

$$\textcircled{2} \text{ goodness of fit}$$

$$df = k-1 \quad (k = \text{no. of categories})$$

$$= 6-1$$

$$\text{no. of } p\text{-value} = 2 \times 0.5627$$

Conclusion

$p\text{-value} < \alpha$ then reject H_0

but here $0.5627 \not< 0.05$ then observed frequencies represent proportions that are significantly different for the frequencies a test-deciding.

