

Multivariate Statistics

Two sample T-test for difference in means

$$t_{\text{data}} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{(s_1^2/n_1) + (s_2^2/n_2)}}$$

$$\text{df} = n_1 - 1 + n_2 - 1 = n_1 + n_2 - 2$$

$$\text{minimum } 25.29 - 1 \\ \text{ie } n_1 - 1 = 25.29 - 1 \\ n_2 - 1 = 80.4 - 1$$

$$\begin{aligned} & \text{S-1} \\ & \text{state hypotheses} \\ & H_0: \mu_1 = \mu_2 \quad (\text{no-difference}) \\ & H_1: \mu_1 \neq \mu_2 \quad (\text{means are different}) \end{aligned}$$

This is two-tail test.

S-2
complete test statistics

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{(s_1^2/n_1) + (s_2^2/n_2)}} = \frac{1.5714 - 1.5361}{\sqrt{\frac{(1.3126)^2}{25.29} + \frac{(1.3251)^2}{80.4}}} = 0.6595$$

S-3

P-value Use 2-table because sample is ~~not~~ ^{t-test even} t-test even

$$P\text{-value} = 2 \cdot P(t > t_{\text{data}}) = P(t > 0.6595)$$

$$\begin{aligned} & 1 - 0.7421 \\ & = 0.257 = 0.257 = 0.514 \\ & \text{from z-table now } 0.6 \text{ & column } 0.06 = 0.6 + 0.06 = 0.66 \\ & \text{data is } 0.7454, \text{ from right tail } = 1 - 0.7454 = 0.2546 \end{aligned}$$

Two Sample Z-Test for Difference in Proportions

$$Z_{\text{data}} = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_{\text{pooled}} \cdot (1 - \hat{p}_{\text{pooled}}) \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

where $\hat{p}_{\text{pooled}} = \frac{x_1 + x_2}{n_1 + n_2}$ & ~~reject~~

Ex
 $x_1 = 707$ of $n_1 = 2529$ customers on the training set belonging to the voice-mail plan while $x_2 = 215$ of $n_2 = 804$ customers in the test set.

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{707}{2529} = 0.2796$$

$$\hat{p}_2 = \frac{x_2}{n_2} = \frac{215}{804} = 0.2674$$

$$\hat{p}_{\text{pooled}} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{707 + 215}{2529 + 804} = 0.2766$$

S-1 Hypothesis

$$H_0 : \pi_1 = \pi_2 \text{ vs } H_1 : \pi_1 \neq \pi_2$$

S-2

$$Z_{\text{data}} = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_{\text{pooled}} \cdot (1 - \hat{p}_{\text{pooled}}) \left(\left(\frac{1}{n_1} + \frac{1}{n_2} \right) \right)}}$$

$$= \frac{0.2796 - 0.2674}{\sqrt{0.2766 \cdot (0.7234) \left(\left(1/2529 \right) + \left(1/804 \right) \right)}}$$

$$= 0.6736$$

P-value is

$$P\text{-value} = 2 \cdot P(Z > 0.6736) \\ = 0.5006$$

from Z-table

0.6 (row) & 0.07 (column)
0.7485

Sy conclusion $= 0.25 / 2 = 1 - 0.7485 = 0.2515$
 $P\text{-value} = 0.5006 > 0.05$ do not reject H_0 .

TEST FOR HOMOGENEITY OF PROPORTIONS (chi-square distribution)

observed frequency (Marital status)

Dataset	Married	Single	Other	Total
Training set	410	340	250	1000
Test set	95	85	70	250
Total	505	425	320	1250

Hypothesis set

$$H_0 : P_{\text{married, training}} = P_{\text{married, test}}$$

$$P_{\text{single, training}} = P_{\text{single, test}}$$

$$P_{\text{other, training}} = P_{\text{other, test}}$$

H_a : At least one of the claims in H_0 is wrong.

$$\text{Expected frequency} = \frac{\text{row total} \times \text{column total}}{\text{grand total}}$$

Expected frequencies

Dataset	Married	Single	Other	Total
Training set	404	340	256	1000
Test set	101	85	64	250
Total	505	425	320	1250

calculate test statistic χ^2 data

cell	Observed freq.	Expected freq.	$\frac{(Obs - Exp)^2}{Exp}$
Married, training	410	404	$\frac{(410 - 404)^2}{404} = 0.09$
Married, test	95	101	0.36
Single, training	340	340	0
Single, test	85	85	0
Other, training	250	256	0.14
Other, test	70	64	0.56
sum = $\chi^2_{\text{data}} = 1.15$			

P-value = $P(\chi^2 > \chi^2_{\text{data}}) = P(\chi^2 > 1.15) = 0.5627$

① test of independence : df = $(r-1)(c-1) = 2$

i.e. df = $(\text{no. of rows} - 1)(\text{no. of columns} - 1)$

$$= (2-1)(3-1) = 1 \times 2 = 2$$

② goodness of fit

No choice \Rightarrow p-value $\rightarrow 0.5627$

$$\begin{aligned} df &= k-1 \quad (k = \text{no. of categories}) \\ &= 6-1 \end{aligned}$$

Conclusion

p-value $< \alpha$ then reject H_0

but here $0.5627 \not< 0.05$ then observed frequencies
represent proportions that are significantly different
for the fractions in test data.

Lenovo

