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ME314 Final Project

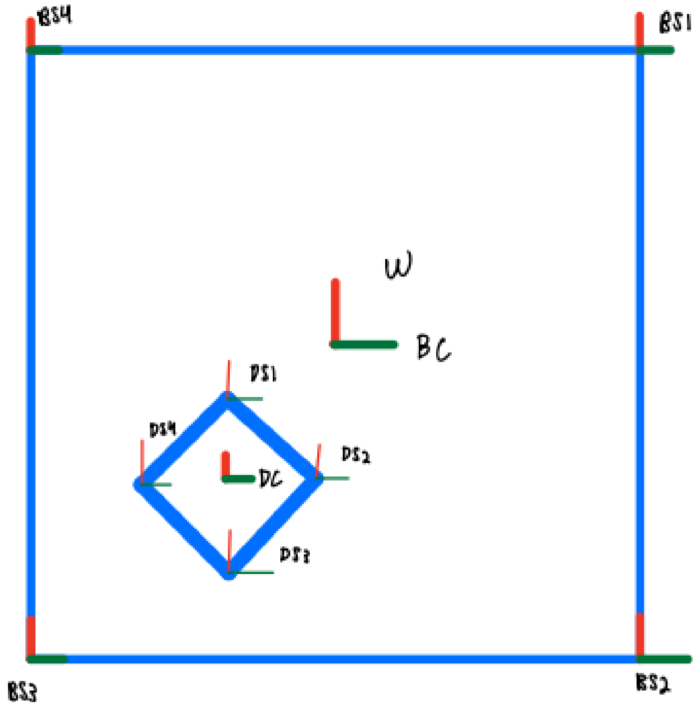

Fig1. Box and dice system with corresponding frames. The world frame {W} and box center frame {BC} are at the same location. BS1 and DS1 correspond to the first corner of the box and die respectively.

Gw_bc :

$$\begin{bmatrix} \cos(\theta_b(t)) & -\sin(\theta_b(t)) & 0.0 & x_b(t) \\ \sin(\theta_b(t)) & \cos(\theta_b(t)) & 0.0 & y_b(t) \\ 0.0 & 0.0 & 1.0 & 0 \\ 0.0 & 0.0 & 0.0 & 1 \end{bmatrix}$$

Gw_bs1:

$$\begin{bmatrix} \cos(\theta_b(t)) & -\sin(\theta_b(t)) & 0 & x_b(t) - 1.5\sin(\theta_b(t)) + 1.5\cos(\theta_b(t)) \\ \sin(\theta_b(t)) & \cos(\theta_b(t)) & 0 & y_b(t) + 1.5\sin(\theta_b(t)) + 1.5\cos(\theta_b(t)) \\ 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Gw_bs2:

$$\begin{bmatrix} \cos(\theta_b(t)) & -\sin(\theta_b(t)) & 0 & x_b(t) + 1.5\sin(\theta_b(t)) + 1.5\cos(\theta_b(t)) \\ \sin(\theta_b(t)) & \cos(\theta_b(t)) & 0 & y_b(t) + 1.5\sin(\theta_b(t)) - 1.5\cos(\theta_b(t)) \\ 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Gw_bs3:

$$\begin{bmatrix} \cos(\theta_b(t)) & -\sin(\theta_b(t)) & 0 & x_b(t) + 1.5\sin(\theta_b(t)) - 1.5\cos(\theta_b(t)) \\ \sin(\theta_b(t)) & \cos(\theta_b(t)) & 0 & y_b(t) - 1.5\sin(\theta_b(t)) - 1.5\cos(\theta_b(t)) \\ 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Gw_bs4:

$$\begin{bmatrix} \cos(\theta_b(t)) & -\sin(\theta_b(t)) & 0 & x_b(t) - 1.5\sin(\theta_b(t)) - 1.5\cos(\theta_b(t)) \\ \sin(\theta_b(t)) & \cos(\theta_b(t)) & 0 & y_b(t) - 1.5\sin(\theta_b(t)) + 1.5\cos(\theta_b(t)) \\ 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Gw_dc:

$$\begin{bmatrix} \cos(\theta_d(t)) & -\sin(\theta_d(t)) & 0.0 & x_d(t) \\ \sin(\theta_d(t)) & \cos(\theta_d(t)) & 0.0 & y_d(t) \\ 0.0 & 0.0 & 1.0 & 0 \\ 0.0 & 0.0 & 0.0 & 1 \end{bmatrix}$$

Gw_ds1 :

$$\begin{bmatrix} \cos(\theta_d(t)) & -\sin(\theta_d(t)) & 0 & x_d(t) - 0.2\sin(\theta_d(t)) + 0.2\cos(\theta_d(t)) \\ \sin(\theta_d(t)) & \cos(\theta_d(t)) & 0 & y_d(t) + 0.2\sin(\theta_d(t)) + 0.2\cos(\theta_d(t)) \\ 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Gw_ds2 :

$$\begin{bmatrix} \cos(\theta_d(t)) & -\sin(\theta_d(t)) & 0 & x_d(t) + 0.2\sin(\theta_d(t)) + 0.2\cos(\theta_d(t)) \\ \sin(\theta_d(t)) & \cos(\theta_d(t)) & 0 & y_d(t) + 0.2\sin(\theta_d(t)) - 0.2\cos(\theta_d(t)) \\ 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Gw_ds3 :

$$\begin{bmatrix} \cos(\theta_d(t)) & -\sin(\theta_d(t)) & 0 & x_d(t) + 0.2\sin(\theta_d(t)) - 0.2\cos(\theta_d(t)) \\ \sin(\theta_d(t)) & \cos(\theta_d(t)) & 0 & y_d(t) - 0.2\sin(\theta_d(t)) - 0.2\cos(\theta_d(t)) \\ 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Gw_ds4 :

$$\begin{bmatrix} \cos(\theta_d(t)) & -\sin(\theta_d(t)) & 0 & x_d(t) - 0.2\sin(\theta_d(t)) - 0.2\cos(\theta_d(t)) \\ \sin(\theta_d(t)) & \cos(\theta_d(t)) & 0 & y_d(t) - 0.2\sin(\theta_d(t)) + 0.2\cos(\theta_d(t)) \\ 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Fig2: The transforms used for {world} to the box corners and dice corners.

To calculate the Euler-Lagrange equations, first the inertia tensor I for both the box and dice were calculated. Then the body angular velocities Vb for each object were calculated using the respective world to box center {gw_bc} and world to dice center {gw_dc} frames. This was used to calculate the rotational kinetic energy KE of the system:

$$KE = \frac{1}{2}(V^b)^T \begin{bmatrix} mI_{n\times n} & 0 \\ 0 & \mathcal{I} \end{bmatrix} V^b$$

The potential energy V was calculated using the y values of the {gw_bc} and {gw_dc}. These were then used to generate the Lagrangian L=KE-V.

The Lagrangian was then used to calculate the Euler-Lagrange (EL):

$$\boxed{\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0}$$

In this system, the system configuration q included an x-coordinate, y-coordinate, and theta rotation for each object, creating the following configuration matrix q which was used for the EL equations:

$$\begin{bmatrix} x_d(t) \\ y_d(t) \\ \theta_d(t) \\ x_b(t) \\ y_b(t) \\ \theta_b(t) \end{bmatrix}$$

The E-L equations were then solved for the accelerations of the system configuration variables.

The external forces added to the system were a vertical force Fyb on the box, and a horizontal force Fthetab on the box. The vertical force Fyb= `9.8*(4*(mb))+100` is meant to counteract the gravitational force on the box to keep it from falling. The extra value added is meant to counteract the downward force that the box will experience as the die bounces on it. The horizontal force Fthetab=`sym.pi/15*sym.sin(t)**2` applies a force on thetab which makes the box spin.

The impacts were calculated using each of the transforms shown in Figure 2. An impact was defined as a corner of the die coming into contact with a side of the box. To do so, each impact corresponds to the distance between the x or y coordinate of the chosen die corner and the x or y coordinate of the chosen box corner. For instance *phi_bs1_ds1=((SE3_inv(g_wbs1)*g_wds1)[0,3])* corresponds to the distance between the x coordinate of the first corner of the die and of the first corner of the box. This distance can be used to check for impact along the entire y-axis of the box corner frame because the x-coordinate will be the same along that axis, and thus can be represented as a line (or wall) for the die to impact with. There are four sides of the box to check, and each of the four die corners are checked for each side resulting in 16 total impacts to check.

Once the impacts are set, the impact condition checks if the resulting x or y coordinates are within a certain tolerance given the current configuration of both the box and die. If any of the impacts are within that tolerance, the impact condition returns true and updates the configuration accelerations to reflect this impact in the impact update.

The impact update solves the impact equations for the new accelerations of the configurations variables after the time of impact τ^+:

$$\frac{\partial L}{\partial \dot{q}}\Big|_{\tau^-}^{\tau^+} = \lambda \frac{\partial \phi}{\partial q}$$

$$\left[\frac{\partial L}{\partial \dot{q}} \cdot \dot{q} - L(q, \dot{q})\right]\Big|_{\tau^-}^{\tau^+} = 0$$

The new accelerations are returned as part of a new configuration that is then integrated to generate the desired trajectory with impact. Once the entire trajectory is generated, it is used to plot and animate the simulation.

In the simulation graphs, the y-coordinate of the die yd shows it bouncing repeatedly up and down while also gradually decreasing in the overall y-value. This seems to correctly be representing the impacts that the die experiences. In the actual animation, the die is properly impacting all the sides of the wall. As the die impacts the box, it does apply force on the box both vertically and horizontally. While this applied force is expected, it seems that the Fyb force on the box does not entirely counteract this force since the box gradually falls down as the die keeps hitting it. The horizontal force on the box does seem to be working since the box spins slightly, yet the force that the die applies to the box seems to skew this intended spin effect on the box. Other than these forces on the box which are slightly altered, the actual dynamics of the die work correctly since the corners of the die that come into contact with the wall react properly by having an acceleration away from the wall after impact.