Trig Integrals

- $\int sin(x) = -cos(x); \int cos(x) = sin(x); \int tan(x) = \ln|sec(x)|$
- $\int sec(x) = \ln|sec(x) + tan(x)|; \int sec^2(x) = tan(x); \int cot = -\ln|csc(x)|$
- $\int csc^2(x) = -cot(x); \int sec(x)tan(x) = sec(x); \int csc(x)cot(x) = -csc(x)$

Properties of Exponents

- $a^{m+n} = a^m \cdot a^n$
- $a^{m-n} = a^m/a^n$

Properties of logs

- $\ln|m^n| = n\ln|m|$
- $\ln|mn| = \ln|m| + \ln|n|$
- $\ln|\frac{m}{n}| = \ln|m| - \ln|n|$

Methods of Solving Diff eqs

Separable

- of the form: $g(y)y\ \ = f(x)$

Linear

- of the form: $f_n(x)y^{(n)} + f_{n-1}(x)y^{(n-1)} + ... + f_1(x)y' + f_0(x)y = F(x)$
- examples:
  - $y' + y = 2x$
  - $xy' - 2y = x^5$
- solved using this substitution: $e^{\int f_0(x)dx}$

Change of Variable

- of the form: $y' = F(x, y)$
- substitute: $V = \frac{y}{x}$

Bernoulli

- of the form $y' + p(x)y = q(x)y^n$
- substitute: $u = y^{1-n}$

Exact

- of the form: $M(x, y) + N(x, y)y' = 0$
- if $M_y = N_x$ simply integrate $\int Mdx; \int Ny$
  - remember the int of M is missing $g(y)$ and the int of N is missing $f(x)$
- by looking at the two see hte missing piece, plug it in, and finally set it equal to C

Exact w/ Int Fact

- if $\frac{M_y - N_x}{N} = f(x)$ use this integrating factor: $e^{\int f(x)}$
- alternatively if $\frac{M_y - N_x}{M} = g(y)$ use this integrating factor: $e^{\int g(y)}$

Theoroms

General Sol

- n unknown constants where N is the order
- let $y = f(x)$ be a sol. NTS $y = f(x)$ can be written as the sol by considering an IVP (make sure to write matching initial conds) $y(0) = f(x); y'(0) = f'(0)$
- solve the system of equation $f(0), f'(0)$ for the constants in terms of $f(0), f'(0)$
- plug this in. You should see that this is the proposed solutiion given some choice of constants

EU for Linear DE

- given a linear DE in standard form, if all the functions of x are continuous on some interval I, then any IVP of the form $y(x_0) = z_0, y'(x_0) = z_1, ..., y^{(n-1)}(x_0) = z_0 n - 1; x_0 \in I$ will have a unique sol

EU from 1st ODE

- given an IVP: $y' = F(x, y); y(x_0) = y_0$ if $F(x, y)$ and $\frac{\partial F}{\partial y}$ are continuous around (x_0, y_0) the IVP will have a unique sol

Methods of solving 2nd order diff eq

Case 0 (missing y and y')

- of the form: $y'' = f(x)$
- simply take a double integral

Case 1 (missing y)

- of the form: $y'' = f(x, y')$
- substitute $u = y'; u' = y''$

Case 2 (missing x)

- $y'' = f(y, y')$
- substitute $V = y'; V'V = y''$
  - **NOTE** y is treated as the independent variable

Gotchas

- When getting to a sol to a diff eq you can treat the constants as just that, but to solve for the constants, differentiate the sol so that the relation holds $y = cx^3; y = 3cx^2$
- `Piece Wise`: solve each part but at the end you need to solve for the constants in terms of one of them by passing in the values at the intersection and solving

Misc

- do LHS RHS