Basic Operations

- `Matrix add/sub` & `Scalar add/sub`: element wise
- `Matrix Multiplication`: let R_i be a row of the left matrix, C_j be a column of the right matrix, and e_{ij} be element of resultant matrix. Then $e_{ij} = R_i \cdot C_j$
  - ex: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}; B = \begin{bmatrix} w & x \\ y & z \end{bmatrix}; AB = \begin{bmatrix} aw+by & ax+bz \\ cw+dy & cx+cz \end{bmatrix}$
  - **note**: requires nxm and mxp matrix and results in nxp
  - def incase I need it for proof: $\sum_{n=1}^{n} A_{ij}B_{ji}$
- `Transpose`: rows become columns
  - ex: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}; A^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$
- `Trace`: sum of diagonal $\sum A_{ii}$
- `UT`: 0's in top right; `LT`: 0's in bottom left; `Diagonal`: both UT and LT
- `Symmetric`: $A^T = A$; `Skew Symmetric`: $A^T = -A$

ERO's

- if you augment the indentity and take that matrix to rref the augmented portion is the inverse of the orignal matrix
- if you take a matrix to ref (technically can be UT, LT, or diag) the product of the diagonal is the determinant except
  - for every swap of rows multiply the det by -1^n
  - for every scale multiply the `determinant` by $\frac{1}{scale}$
    - ex: $R_1 \to \frac{1}{2}R_1$ then $2det$

Rank

- Consider a system $Ax = b$ w/ A mxn, x nx1, b mx1 then
  - if $rank(A^\#) = rank(A) = n$ there is a `unique sol`
  - $rank(A^\#) \neq rank(A)$ `no sol` (more specifically $rank(A^\#) = rank(A) + 1$)
  - $rank(A^\#) = rank(A) < n$ `inf sol` with $n - rank(A)$ params
  - `big theorom`: if $rank(A^\#) = rank(A)$ `consistent`. if $rank(A^\#) \neq rank(A)$ `inconsistent`
  - `homogenous version` (constants = 0):
    - always have sol $x = 0$
    - unique sol of $x = 0$ if $rank(A) = n$
    - inf sol (includes $x = 0$) if $rank(A) < n$

Property of Transpose

- $(A+B)^T = A^T + B^T$
- $(AB)^T = B^T A^T$

Property of Inverse

- $(AB)^{-1} = B^{-1}A^{-1}$
- $(A^T)^{-1} = (A^{-1})^T$

Determinant

- Def for Proof:
  - `determinant`: let A be an nxn matrix. $det(A) = \sum_{n!} \sigma(p_1, p_2 ... p_n) a_{1p_1} a_{2p_2} ... a_{np_n}$
- `Thm`: A is invertible $\leftrightarrow det(A) \neq 0$

Properties

- $det(A^T) = det(A)$
- $det(A^{-1}) = \frac{1}{det(A)}$
- $det(kA) = k^n det(A)$

Vector Spaces

Let $u, v, w \in V$. Let k_1, k_2 be scalars - Closures under $\oplus$ - $u \oplus v \in V$ $(\forall u, v \in V)$ - if you add 2 vectors inside a given vector space, the sum should also exist in that space - Closure under $\odot$ - $k_1 \odot u \in V$ $(\forall u \in V, \forall k_1$ in field$)$ - $\oplus$ is Commutatuve - $u \oplus v \equiv v \oplus u$ - $\oplus$ is associative - $(u \oplus v) \oplus q \equiv u \oplus (v \oplus w)$ - $\exists$ a **zero vector**, denoted $\vec{0}$ such that - $v \oplus \vec{0} = v$ $\forall v \in V$ - Additive inverse - $\forall v \in V \exists -v \in V$ such that - $v \oplus (-v) = \vec{0}$ - Unit property - $1 \odot v = v$ $\forall v \in V$ - Associativity of scalar multiplication - $k_1 \odot (k_2 \odot v) \equiv (k_1 k_2) \odot V$ - Scalar multiplication over vector addition - $k_1 \odot (u \oplus v) \equiv (k_1 \odot u) \oplus (k_1 \odot v)$ - Scalar addition over scalar multiplication - $(k_1 + k_2) \odot u \equiv (k_1 \odot) \oplus (k_2 \odot u)$