LOGIC

A <u>Statement/Proposition</u> is a sentence that is true or false, but not both.

<u>Examples of statements/Propositions</u> → can be assessed as true or false

"It is raining outside"

"Today is Tuesday"

"I am taller than my sister"

"Two plus two equals four"

"Two plus two equals five"

<u>Examples of sentences that are NOT logical statements:</u>

"Hello"

"The office"

"Ouch"

"Tuesday"

"California"

Negation

If p is the proposition, then $\sim p$ is the negation.

Common Symbols for negation: $\sim p$ NOT p $\neg p$ $\bar{p}$ $!p$

truth Table

p	$\sim p$
T	F
F	T

The negation of a logical statement must take on the opposite truth value.

Examples

p: "It is raining"

$\sim p$: "It is NOT raining" "It is NOT the case that it is raining"

p: "I am going to the movie"

$\sim p$: "I am not going to the movie"

Conjunction

This is the logical "AND"

Common symbols: p AND q $\quad p \wedge q \quad p \bullet q \quad p \,\&\, q$

$\wedge$ "hat" $\quad \bullet$ "dot" $\quad \&$ ampersand

Truth Table

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

* For an AND statement to evaluate as TRUE, both parts must be true.

* It's analogous to intersection $\cap$ in set theory

Examples

"It is hot AND I am tired"

"$x \geq 2$ AND $x < 5$" $\rightarrow$ true for $2 \leq x < 5$

* Sometimes "but" is understood to be equivalent to "and"

  "It is sunny, but it is not hot" has same meaning as

  "It is sunny and it is not hot"

* The statement "neither p nor q" is equivalent to $\sim p$ and $\sim q$

  "Neither a borrower nor a lender be" is same meaning as

  "Do not be a borrower and do not be a lender"

Disjunction

This is the logical "OR" (Inclusive OR)

Common symbols: p OR q $\qquad$ $\underset{\substack{\uparrow \\ \text{"cup"}}}{p \vee q}$ $\qquad$ $p \,\|\, q$

Truth Table

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

* For an OR statement to evaluate as TRUE, at least one part must be true.
* It is not the exclusive OR
* It's analogous to union $\cup$ in set theory

Examples

"It is Tuesday or it is Wednesday"

"I will pass the class or I will not be happy"

"$x \geq 2$ or $x < 5$" $\rightarrow$ true for all reals

DeMorgan's Laws

Negating Conjunctions and Disjunctions

$$\boxed{\sim(p \wedge q) \equiv \sim p \vee \sim q}$$

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$\sim(p \wedge q)$	
T	T	F	F	F	F	T
T	F	F	T	T	T	F
F	T	T	F	T	T	F
F	F	T	T	T	T	F

same truth values for all cases, so equivalent

$p \wedge q$: "I am hot AND I am tired"

$\sim(p \wedge q)$: "I am NOT hot OR I am NOT tired"

$$\boxed{\sim(p \vee q) \equiv \sim p \wedge \sim q}$$

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$\sim(p \vee q)$	
T	T	F	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	F	T
F	F	T	T	T	T	F

same truth values for all cases, so equivalent

$p \vee q$: "It is Tuesday OR It is Wednesday"

$\sim(p \vee q)$: "It is NOT Tuesdy AND it is NOT Wednesday."

Conditional Statement/Material Implication

"If p, then q" "p implies q"

other common symbols: $p \to q$ $p \Rightarrow q$ $p \supset q$

$\uparrow$ "horseshoe"

truth table

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p is the premise/hypothosis/antecedent

q is the conclusion/consequence

Example: "If you get 100%, then you'll get an A"

consider this my promise → consider all cases and determine if I lied in each case

T → T if it's true that you get 100% and it's true that you get an A, then I didn't lie, ⇒ True

T → F if it's true that you get 100% and it's false that you get an A, then I lied ⇒ False

F → T if you don't get 100%, but you still get an A, then you're happy and my promise was irrelevant. (This is called vacuously true) ⇒ true

F → F If you don't get 100% and you don't get an A, then I did not lie, but my promise was irrelevant anyway. (This is called vacuously true) ⇒ true

* If a premise evaluates false, the implication is always true (called vacuously true, but true nonetheless).

Example: "If I am elected, then I'll work to lower taxes."

consider this as my promise before the election →
consider all cases and determine if I lied in each case.

T→T if I get elected and I work to lower taxes, then I certainly didn't lie ⇒ True

T→F if I get elected, but I don't work to lower taxes, then I lied. ⇒ False

F→T if I don't get elected, I could still work to lower taxes by lobbying, petitioning, etc. However, my promise really doesn't matter ⇒ True

F→F If I don't get elected and I don't work to lower taxes, then I didn't break my promise. I'm not bound to do anything ⇒ True

⇒ The last two cases, where the premise is false, are vacuously true

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Other Examples

"If you drink beer, then you are at least 21 years old"

"If you are an LA Dodger, then you are an athlete"

"If the opposite angles of a quadrilateral are congruent, then it is a parallelogram"

Converse/Contrapositive/Inverse

Conditional: $p \to q$

Converse: $q \to p$

Contrapositive: $\sim q \to \sim p$

Inverse: $\sim p \to \sim q$

The conditional and its contrapositive are logically equivalent

p	q	$\sim p$	$\sim q$	$\sim q \to \sim p$			$p \to q$
T	T	F	F	F	T	F	T
T	F	F	T	T	F	F	F
F	T	T	F	F	T	T	T
F	F	T	T	T	T	T	T

same values for all cases,
so they are logically equivalent

$$(p \to q) \equiv (\sim q \to \sim p)$$

Also, the converse and the inverse are logically equivalent
$\Rightarrow$ they are contrapositives of each other.

$$(q \to p) \equiv (\sim p \to \sim q)$$

Examples

Conditional: "If it's a square, then it's a rectangle" $\square \to ▭$

converse: "If it's a rectangle, then it's a square" $▭ \to \square$

contrapositive: "If it's NOT a rectangle, then it's NOT a square"

$\sim ▭ \to \sim \square$

inverse: "If it's NOT a square, then it's NOT a rectangle"

$\sim \square \to \sim ▭$

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conditional: "If you get 100%, then you'll get an A"

converse: "If you get an A, then you got 100%"

contrapositive: "If you didn't get an A, then you didn't get 100%"

inverse: "If you didn't get 100%, then you won't get an A"

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conditional: "If you're an LA Dodger, then you're an athlete"

converse: "If you're an athlete, then you're an LA Dodger"

contrapositive: "If you're not an athlete, then you're not an LA Dodger"

inverse: "If you're not an LA Dodger, then you're not an athlete"

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conditional: "If I'm elected, then I'll work to lower taxes"

converse: "If I work to lower taxes, then I got elected"

contrapositive: "If I don't work to lower taxes, then I didn't get elected"

inverse: "If I'm not elected, then I won't work to lower taxes"

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conditional: "If you drink beer, then you're at least 21 years old"

converse: "If you're at least 21 years old, then you drink beer."

contrapositive: "If you're not at least 21, then you don't drink beer"

inverse: "If you don't drink beer, then you are not at least 21"

Biconditional

"if and only if" "iff" $\leftrightarrow$ $\Leftrightarrow$

sometimes called "material equivalence"

$$(p \leftrightarrow q) \equiv \Big[\underbrace{(p \to q)}_{\text{conditional}} \wedge \underbrace{(q \to p)}_{\text{converse}} \Big]$$

p	q	$p \leftrightarrow q$	$p \to q$	$q \to p$	$(p \to q)$	$\wedge$	$(q \to p)$
T	T	T	T	T	T	T	T
T	F	F	F	T	F	F	T
F	T	F	T	F	T	F	F
F	F	T	T	T	T	T	T

same values for all cases, so logically equivalent

Examples

"I won't be tired if and only if I get enough sleep"

has same meaning as

"If I'm not tired, then I got enough sleep AND If I got enough sleep, then I'm not tired."

Also,

"Not being tired is a necessary and sufficient condition for getting enough sleep"

Order of Precedence

1. $\sim$
2. $\wedge$
3. $\vee$
4. $\rightarrow$ $\leftrightarrow$ (Associate with symbol on right first)

* Associate with operator of higher precedence

Examples

$$\sim p \wedge q \equiv ((\sim p) \wedge q)$$

$$p \wedge \sim q \equiv (p \wedge (\sim q))$$

$$p \wedge q \vee r \equiv ((p \wedge q) \vee r)$$

$$p \vee q \wedge r \equiv (p \vee (q \wedge r))$$

$$p \rightarrow q \rightarrow r \equiv (p \rightarrow (q \rightarrow r))$$

$$p \rightarrow q \leftrightarrow r \equiv (p \rightarrow (q \leftrightarrow r))$$

$$p \rightarrow q \vee r \wedge s \equiv (p \rightarrow (q \vee (r \wedge s)))$$

$$p \wedge \sim q \leftrightarrow r \rightarrow s \wedge t \equiv$$
$$((p \wedge (\sim q)) \leftrightarrow (r \rightarrow (s \wedge t)))$$

Exclusive Disjunction

This is the "exclusive OR"

Common symbols: $p \oplus q$ $\quad$ p XOR q $\quad$ $p \veebar q$

Meaning: "p or q, but not both"

Truth Table

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

* For an XOR statement to evaluate as TRUE, exactly one part must be true.
* It's analogous to symmetric difference Δ in set theory.

Example: "I am sleeping or I am awake, but not both"

$$p \oplus q \equiv (p \vee q) \wedge \sim(p \wedge q)$$

p	q	$p \vee q$	$\sim(p \wedge q)$	$(p \vee q) \wedge \sim(p \wedge q)$
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	T	F

↑ same truth table as $p \oplus q$ so it's equivalent

$\oplus$ is non-standard, because it can be written with standard symbols: $\sim \wedge \vee$

Negating the Conditional

$$\boxed{\sim(p \to q) \equiv (p \wedge \sim q)}$$

it's NOT another conditional!

p	q	$p \to q$	$\sim(p \to q)$	$\sim q$	$p \wedge \sim q$		
T	T	T	F	F	T	F	F
T	F	F	T	T	T	T	T
F	T	T	F	F	F	F	F
F	F	T	F	T	F	F	T

same truth values for all cases → logically equivalent

NOTE: the negation ALWAYS has the opposite truth value

Examples

$p \to q$: "If the Super Bowl is live on TV, then it is Sunday"

$\sim(p \to q)$: "The Super Bowl is live on TV and it is NOT Sunday"

$p \to q$: "If the shape is a square, then it is a rectangle"

$\sim(p \to q)$: "The shape is a square and it is NOT a rectangle"

$p \to q$: "If you get 100%, then you'll get an A"

$\sim(p \to q)$: "You get 100% and you do NOT get an A"

$p \to q$: "If you're an LA Dodger, then you're an athlete"

$\sim(p \to q)$: "You're an LA Dodger and you are NOT an athlete"

$p \to q$: "If I go home for Christmas, then I'll see my family and my mom will not be sad"

$\sim(p \to q)$: "I go home for Christmas, but I do not see my family OR my mom will be sad"

Negating the Biconditional

$$\boxed{\sim(p \leftrightarrow q) \equiv p \leftrightarrow \sim q}$$

p	q	$p \leftrightarrow q$	$\sim(p \leftrightarrow q)$	$\sim q$	$p \leftrightarrow \sim q$
T	T	T	F	F	T F F
T	F	F	T	T	T T T
F	T	F	T	F	F T F
F	F	T	F	T	F F T

same values for all cases, so logically equivalent

Example

$p \leftrightarrow q$: "I will not be tired if and only if I get enough sleep"

$\sim(p \leftrightarrow q)$: "I will not be tired if and only if I do NOT get enough sleep"

NOTE: $(p \leftrightarrow q) \equiv (q \leftrightarrow p)$

so $\sim(p \leftrightarrow q) \equiv \sim(q \leftrightarrow p) \equiv \sim q \leftrightarrow p$

$\sim(p \leftrightarrow q)$: "I will be tired if and only if I get enough sleep"

Note that

$$\sim(p \leftrightarrow q) \equiv p \oplus q$$

exclusive OR

Calculus Example

$\boxed{\text{Differentiability Implies Continuity}}$

"If a function f is differentiable (smooth) at a, then f is continuous at a." $p \to q$

<u>Contrapositive</u>: "If f is NOT continuous at point a, then f is NOT differentiable (smooth) at point a."

$\sim q \to \sim p$

<u>Converse</u>: "If f is continuous at point a, then f is differentiable (smooth) at a."

$q \to p$

<u>False</u>, counterexamples

$f(x) = |x|$ is continuous at $x=0$, but corner there

$f(x) = x^{2/3}$ is continuous at $x=0$, but cusp there

<u>inverse</u>: "If f is NOT differentiable (smooth) at a, then f is NOT continuous at a."

$\sim p \to \sim q$

<u>False</u>, counterexamples

$f(x) = |x|$ has corner at $x=0$, so not differentiable, but it is continuous at $x=0$

$f(x) = x^{2/3}$ has cusp at $x=0$, so not differentiable, but it is continuous at $x=0$

<u>Negation</u> $\sim (p \to q) \equiv p \wedge \sim q$

"A function f is differentiable (smooth) at a, but f is NOT continuous at a."

<u>False</u>, many counterexamples

$f(x) = x^2$ is diff at $x=0$, and it is countinuous there.

Calculus Example

$\boxed{\text{Fermat's Theorem}}$ "If f has a local max or min at c, and $f'(c)$ exists, then $f'(c)=0$."

simplified version for example: $p \to q$

"If f has local extrema at c, then c is a critical number."

<u>Contrapositive</u>: $\sim q \to \sim p$

"If c is NOT a critical number of f, then there is not a local extrema at c." true

<u>Converse</u>: $q \to p$

"If c is a critical number, then there's a local extrema at c."

False → counterexample

$f(x)=x^3$ $f'(x)=3x^2=0 \to CN$ $c=0$

But NO local extrema at 0

<u>inverse</u>: $\sim p \to \sim q$

"If there is NOT a local extrema at c, then c is NOT a critical number"

False → counterexample → $f(x)=x^3$

there is no local extrema at $x=0$, then $x=0$ is a CN,

<u>Negation</u> $\sim(p \to q) \equiv p \wedge \sim q$

"f has a local extrema at c, but c is NOT a critical number."

False → counterexample → $f(x)=x^2$

it has a local min at $x=0$, but $x=0$ is a CN.

Calculus Example

Divergence Test

"If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$." True

Contrapositive: "If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges." True

$\sim q \to \sim p$

converse: "If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges."

$q \to p$ False, counterexample $a_n = \frac{1}{n}$

$\lim_{n\to\infty} \frac{1}{n} = 0$, but $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

inverse: "If $\sum_{n=1}^{\infty} a_n$ diverges, then $\lim_{n\to\infty} a_n \neq 0$"

$\sim p \to \sim q$ False, counterexample $a_n = \frac{1}{n}$

$\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, but $\lim_{n\to\infty} \frac{1}{n} = 0$

Negation $\sim(p \to q) \equiv p \wedge \sim q$

"$\sum_{n=1}^{\infty} a_n$ converges and $\lim_{n\to\infty} a_n \neq 0$."

False, as it should be, since it negates something that is true.

However, some counterexamples can show it, too:

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges but $\lim_{n\to\infty} \frac{1}{n^2} = 0$

$\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$ converges but $\lim_{n\to\infty} \left(\frac{1}{2}\right)^n = 0$

Tautology vs. Contradiction

Tautology → A statement/proposition that is always <u>true</u> regardless of the truth values of the individual parts of the statement.

Contradiction → A statement/proposition that is always <u>false</u> regardless of the truth values of the individual parts of the statement.

Contingency → A statement/proposition that isn't a tautology or contradiction.

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p	p	$\vee$	$\sim p$
T	T	T	F
F	F	T	T

always true

$p \vee \sim p$ is a tautology

p	p	$\wedge$	$\sim p$
T	T	F	F
F	F	F	T

always false

$p \wedge \sim p$ is a contradiction

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p	q	$(\sim p$	$\wedge$	$q)$	$\rightarrow$	$(\sim$	$(q \rightarrow p))$
T	T	F	F	T	T	F	T
T	F	F	F	F	T	F	T
F	T	T	T	T	T	T	F
F	F	T	F	F	T	F	T

always true

$(\sim p \wedge q) \rightarrow (\sim (q \rightarrow p))$ is a tautology

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p	q	$(\sim p$	$\wedge$	$(p \rightarrow q))$	$\rightarrow$	$\sim q$
T	T	F	F	T	T	F
T	F	F	F	F	T	T
F	T	T	T	T	F	F
F	F	T	T	T	T	T

Neither

$(\sim p \wedge (p \rightarrow q)) \rightarrow \sim q$ is a contingency

Basic Logic Identities

1) Commutative $p \vee q \equiv q \vee p$ $\quad p \wedge q \equiv q \wedge p$

2) Associative $(p \vee q) \vee r \equiv p \vee (q \vee r)$ $\quad (p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

3) Distributive $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

4) Double Negation $\sim(\sim p) \equiv p$

5) DeMorgan $\sim(p \wedge q) \equiv \sim p \vee \sim q$ $\quad \sim(p \vee q) \equiv \sim p \wedge \sim q$

6) Identity $p \wedge T \equiv p$ $\quad p \vee F \equiv p$

7) Domination $p \vee T \equiv T$ $\quad p \wedge F \equiv F$

8) Idempotent $p \vee p \equiv p$ $\quad p \wedge p \equiv p$

9) Negation $p \vee \sim p \equiv T$ (tautology) $\quad p \wedge \sim p \equiv F$ (contradiction)

10) Absorption $p \vee (p \wedge q) \equiv p$ $\quad p \wedge (p \vee q) \equiv p$

Prove Basic Identities → Truth Table

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r) \quad \text{Distributive}$$

p	q	r	$p \wedge (q \vee r)$		$(p \wedge q)$	$\vee$	$(p \wedge r)$
T	T	T	T	T T T	T	T	T
T	T	F	T	T T F	T	T	F
T	F	T	T	F T T	F	T	T
T	F	F	F	F F F	F	F	F
F	T	T	F	T T T	F	F	F
F	T	F	F	T T F	F	F	F
F	F	T	F	F T T	F	F	F
F	F	F	F	F F F	F	F	F

same value for all cases, so logically equivalent

More Basic Logic Identities

$p \to q \equiv \sim p \vee q$

$p \to q \equiv \sim q \to \sim p$ (contrapositive)

$p \vee q \equiv \sim p \to q$

$p \wedge q \equiv \sim (p \to \sim q)$

$\sim (p \to q) \equiv p \wedge \sim q$

$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$

$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

$(p \leftrightarrow q) \equiv [(p \to q) \wedge (q \to p)]$

$p \leftrightarrow q \equiv \sim p \leftrightarrow \sim q$

$p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

$\sim (p \leftrightarrow q) \equiv p \leftrightarrow \sim q$

* All can be proved with truth table

Prove: $p \to q \equiv \sim p \vee q$

p	q	$p \to q$	$\sim p$	$\vee$	q
T	T	T	F	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	T	T	F

same value for all cases

Prove: $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

p	q	r	$(p \to r)$	$\wedge$	$(q \to r)$	$(p \vee q)$	$\to$	r
T	T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F	F
T	F	T	T	T	T	T	T	T
T	F	F	F	F	T	T	F	F
F	T	T	T	T	T	T	T	T
F	T	F	T	F	F	T	F	F
F	F	T	T	T	T	F	T	T
F	F	F	T	T	T	F	T	F

same value for all cases

Two Ways To Prove An Identity

$$\sim(p \vee (\sim p \wedge q)) \equiv \sim p \wedge \sim q$$

① Truth Table

p	q	$\sim($	p	$\vee$	$(\sim p$	$\wedge$	$q))$	$\sim p$	$\wedge$	$\sim q$
T	T	F	T	T	F	F	T	F	F	F
T	F	F	T	T	F	F	F	F	F	T
F	T	F	F	T	T	T	T	T	F	F
F	F	T	F	F	T	F	F	T	T	T

negate this

same values for all cases

② Direct Proof using Identities → Start on one side and use chain of equivalences (like trig proofs)

$$\begin{aligned}
\text{Left} &\equiv \sim(p \vee (\sim p \wedge q)) \\
&\equiv \sim p \wedge \sim(\sim p \wedge q) && \text{DeMorgan's Law} \\
&\equiv \sim p \wedge [\sim(\sim p) \vee \sim q] && \text{DeMorgan's Law} \\
&\equiv \sim p \wedge (p \vee \sim q) && \text{Double Negation} \\
&\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q) && \text{Distribution} \\
&\equiv F \vee (\sim p \wedge \sim q) && \text{Negation Law/contradiction} \\
&\equiv (\sim p \wedge \sim q) \vee F && \text{Commutative} \\
&\equiv \sim p \wedge \sim q && \text{Identity} \\
&\equiv \text{Right}
\end{aligned}$$

Implication - Alternate Phrasing

These are all equivalent:

"if p, then q" "p implies q" "q follows from p"

"q if p" "p only if q" "p is sufficient for q"

"q is necessary for p"

Example

A: "I get wet if it rains"

This has meaning "If it rains, then I get wet"

rain → wet

B: "I get wet only if it rains"

If this is true, then it means that if it rains, I could get wet, but there is no guarantee. However, if I get wet, it must have rained.

This has meaning "If I get wet, then it rained"

wet → rain

Also, consider that if it does not rain, then I do not get wet.

But if I do not get wet, it doesn't imply that it did not rain.

$\left.\begin{array}{l}\text{"I only get wet if it rains"} \\ \text{"I get wet only when it rains"} \\ \text{"The only time I get wet is when it rains"}\end{array}\right\}$ All have same meaning as B

Necessary vs. Sufficient

"p is a sufficient condition for q" means "if p, then q"

→ the occurrence of p is sufficient to guarantee the occurence of q

"p is a necessary condition for q" means "if not p, then not q"
OR "if q, then p"

→ if p does not occur, then q can't occur either

<u>Example</u>: A number being divisible by 10 is sufficient for it to be even, but NOT necessary (there exist even numbers that are not divisible by 10).

"If a number is divisible by 10, then it is even"

<u>Example</u>: An integer greater than two being odd is necessary for it to be prime, but it is NOT sufficient (there exist composite odd numbers like 33)

"If an integer greater than 2 is prime, then it is odd"

<u>Example</u>: Being a square is not a necessary condition for being a rectangle, since there are rectangles that are not squares.

Being a square is a sufficient condition for being a rectangle.

"If it's a square, then it's a rectangle."

"only if" vs "if"

A statement that uses "only if" is the converse of a statement with "if" in its place.

A statement that uses "necessary" is the converse of a statement with "sufficient" in its place.

Conditional	Converse
If p, then q	If q, then p
$p \to q$	$q \to p$
p only if q	p if q
p is sufficient for q	p is necessary for q

Example

Conditional	Converse
"If it snows, then I get cold"	"If I get cold, then it snows"
"It snows only if I get cold"	"It snows if I get cold"
"Snowing is sufficient for me to get cold"	"Snowing is necessary for me to get cold"

$\Uparrow$ Note that there could be other ways for me to get cold → Snowing is sufficient, but not only way.

$\Uparrow$ this implies that the only way I get cold is if it snows → snowing is necessary

Logic Practice

For each conditional statement, find each of these:

(a) converse
(b) contrapositive
(c) inverse
(d) negation

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1. If it rained, then flowers bloomed.
2. If fish are plentiful, then bears are happy.
3. If you don't eat your breakfast, then you'll be hungry.
4. If a shape is not a parallelogram, then it's not a rhombus.
5. If Joey is a bungee jumper, then he likes dangerous things.
6. If Mary went to the movies, then she ate popcorn.
7. If Mary went to the movies, then she ate popcorn and drank soda.
8. If I go hiking or biking, then I'll be happy and I'll sleep well.

Logic Solutions

1) (a) If flowers bloomed, then it rained.
(b) If flowers didn't bloom, then it didn't rain.
(c) If it didn't rain, then flowers didn't bloom.
(d) It rained and flowers didn't bloom.

2) (a) If bears are happy, then fish are plentiful.
(b) If bears are not happy, then fish are not plentiful
(c) If fish are not plentiful, then bears are not happy.
(d) Fish are plentiful and bears are not happy.

3) (a) If you're hungry, then you didn't eat your breakfast.
(b) If you're not hungry, then you ate your breakfast.
(c) If you eat your breakfast, then you will not be hungry.
(d) You don't eat your breakfast, but you are not hungry.

4) (a) If it's not a rhombus, then it's not a parallelogram. F
(b) If it is a rhombus, then it is a parallelogram. T
(c) If it is a parallelogram, then it is a rhombus. F
(d) The shape is not a parallelogram, but it is a rhombus. F

5) (a) If he likes dangerous things, then he's a bungee jumper.
(b) If he doesn't like dangerous things, then he's not a bungee jumper.
(c) If he is not a bungee jumper, then he doesn't like dangerous things.
(d) Joey is a bungee jumper, but he doesn't like dangerous things

6) (a) If she ate popcorn, then she went to the movies.
(b) If she didn't eat popcorn, then she didn't go to the movies.
(c) If she didn't go to the movies, then she didn't eat popcorn.
(d) She went to the movies, but she didn't eat popcorn.

7) (a) If she ate popcorn and drank soda, then she went to the movies.
(b) If she didn't eat popcorn OR didn't drink soda, then she didn't go to movies.
(c) If she didn't go to the movies, then she didn't eat popcorn OR she didn't drink soda.
(d) She went to the movies and she didn't eat popcorn OR she didn't drink soda.

8) (a) If I'm happy and I sleep well, then I go hiking or biking
(b) If I'm not happy or I don't sleep well, then I didn't hike and I didn't bike.
(c) If I don't hike and I don't bike, then I won't be happy OR I won't sleep well.
(d) I go hiking or biking, but I'm not happy OR I didn't sleep well.