Quiz 3

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1a. let $P = 7n-2$ is even
let $q =$ n is even

$p \to q \equiv \sim q \to \sim p$

Assume n is odd

$n = 2k+1$, $k \in \mathbb{Z}$ by def of odd int.

$7n-2 = 7(2k+1)-2$
$= 14k+7-2$
$= 14k+5 = 2(7k+2)+1$
$= 2r+1$, $r = 7k+2$, $r \in \mathbb{Z}$

$\therefore$ $7n-2$ is odd. So since $\sim q \to \sim p$, then $p \to q$ is true by contraposition QED

1b. Assume p & $\sim q$. since $7n-2$ is even, $7n$ is even by addition of 2. Then $7n-n$ must be odd by subtraction of even & odd int. $7n-n \equiv 6n \equiv 2(3n) \equiv 2r$, $r = 3n$
However $7n-n = 2r$ which is even by definition which contradicts our assumptions QED

2. Assume $\sim[(\sim p \wedge w) \to (p \vee \sim s)] \equiv (\sim p \wedge w) \wedge (\sim p \wedge s)$

$\sim p = F$ $\quad w = T$
$p = T$ $\quad s = T$

$Q \to p$ ✓ $\quad Q = F$

$(R \vee S) \vee W$ ✓ $\quad R = T$

$U \Leftrightarrow \sim Q$ ✓ $\quad U = T$

$(R \wedge U) \Rightarrow P$ ✓

$w \to (R \vee P)$ ✓

Invalid

3. Assume $\sim[\sim S \to \sim U] \equiv \sim S \wedge U$

$S = T \quad U = T$

$U \to R$ ✓ $R = T$

P ✓ $P = T$

$Q \to (R \to S)$ ✓ $Q = F$

$P \vee Q$ ✓

$\boxed{\text{Invalid}}$

4.
1 $(T \vee C) \to (V \wedge P)$

2 $P \to O$

3 $\sim O$

$\therefore$

4 $\sim T$

$\sim P$ by Modes Tollens (2,3)

$\sim(T \vee C)$ Mode Tollens (1,2)

$\sim T \wedge \sim C$ Negation of a conjunction

5.
a. $\forall x \exists y (x^2 = y)$ True. You can pick a y such that $y = x^2$

b. $\forall x \exists y (x = y^2)$ False. $x = -1$, $y = i$ which isn't in domain

c. $\exists x \forall y (xy = 0)$ True. if $x = 0$ $xy = 0$

d. $\exists x \exists y (x + y \neq y + x)$ False. Contradicts commucative

$x = 1$, $y = 1$

e. $\forall x (x \neq 0 \to \exists y (xy = 1))$ True. For any $n \neq 0$ $n\left(\frac{1}{n}\right) = 1$

f. $\exists x \forall y (y \neq 0 \to xy = 1)$ False. if $y = 2$ then $x = \frac{1}{2}$. If

$y = 3 \quad x = 1/3 \quad \& \quad \frac{1}{2} \neq \frac{1}{3}$

g. $\forall x \exists y (x + y = 1)$ True. For any n, $n + (-(n-1)) = 1$

$n - n + 1 = 1 \Rightarrow 1 = 1$ ✓

h. $\exists x \exists y (x + 2y = 2 \wedge 2x + 4y = 5)$ False. Don't know how a counter example works here.

$$2x + 4y = 5 \qquad 2x + 4y = 5$$
$$-2(x + 2y = 2) \Rightarrow -2x - 4y - 4$$
$$0 \neq 1$$

i. $\forall x \exists y (x + y = 2 \wedge 2x + 4y = 5)$ False. theres a unique solution,

$$2x + 4y = 5 \qquad 2x + 4y = 5$$
$$-2(x + y = 2) \Rightarrow -2x - 2y = -4$$
$$2y = 1$$
$$y = \frac{1}{2}$$

$x + \frac{1}{2} = 2$

$x = \frac{3}{2}$

$x = 1 \rightarrow y = 1 \wedge y = \frac{3}{4}$

$1 \neq \frac{3}{4}$

j. $\forall x \forall y \exists z (z = (x + y)/2)$ True. You can choose a z such that $z = \frac{x+y}{2}$

6. Assume a, b, c are even if their sum is even then $\sim q \rightarrow \sim p$ & therefore $p \rightarrow q$

$a = 2k$, $b = 2l$, $c = 2m$

$2k + 2l + 2m = 2(k + l + m)$. $k+l+m$ is an int, therefore if a, b, c are even, $a+b+c$ is even by def of an even int.

Proof by contraposition QED

7. Assume (rational)/(irrational) -> rational

$\frac{a}{b} x = \frac{m}{n}$

$x = \frac{mb}{na}$

x must be rational but we assumed x was irrational.

QED proof by contradiction

8. $\frac{a}{b}$ = rational, r = irrational, $\frac{a/b + r}{2}$ = irrational bc $\frac{a}{b} + r$ is irrational

assume $a/b < r$

$r + \frac{a}{b} < r + r \Rightarrow r + \frac{a}{b} < 2r \Rightarrow \underline{\frac{r + \frac{a}{b}}{2} < r}$

$\frac{a}{b} < r \Rightarrow \frac{a}{b} + \frac{a}{b} < \frac{a}{b} + r \Rightarrow 2\frac{a}{b} < \frac{a}{b} + r$

$\Rightarrow \underline{\frac{a}{b} < \frac{\frac{a}{b} + r}{2}}$

$$\frac{a}{b} < \frac{\frac{a}{b} + r}{2} < r$$

QED

9. $\lfloor 8 \rfloor \quad \lfloor 5 \rfloor \quad \lfloor 3 \rfloor$

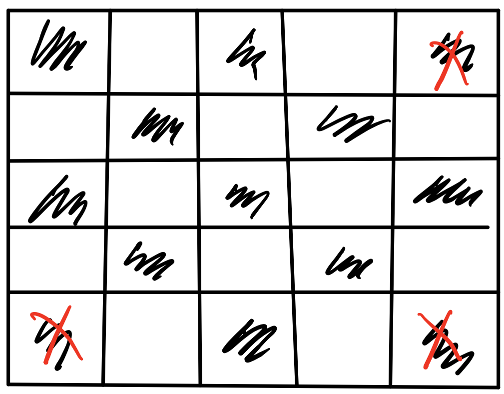
8 0 0

3 5 0

3	2	3
6	2	0
6	0	2
1	5	2
1	4	3 ✓

Not sure how to formerly prove this but following these steps makes it possible.

10.



False even though number of squares is even, because a domino covers 1 black & 1 white if # of black $\neq$ # of white impossible. Since all 4 corners are black removing any 3 corners means # black = 10 & # white = 12. $10 \neq 12$