The use of GPS in gravity surveys

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Global Positioning System (GPS) technology provides a cost-effective surveying method that is replacing traditional methods of precise leveling in exploration. Using satellite-derived GPS coordinates can, however, generate problems, because they will need to be translated to national systems, if this is the preferred local system being used by the explorationists. However, consistent coordinates should be used to compute gravity corrections. This contribution reviews how GPS-derived coordinates are used for different types of gravity surveys and how different coordinate systems generate significant differences in the location, height, and derived gravity values. The use of GPS technology also leads to the term "gravity disturbance" which may be new to many but turns out to be conceptually the more straightforward expression for the anomalous part of the Earth's gravity field. This article also draws attention to recent detailed articles on this subject and on our GPS experience in South America.

GPS was designed to provide an instantaneous absolute positioning using two codes, P and C/A, transmitted by a constellation of satellites. The P code has certain characteristics that allow an accuracy of decimeters in the coordinates but is restricted by the United States to military applications. The C/A code, a free civil code, provides an accuracy of a few tens of meters in the worst case.

An alternative to these codes, now used extensively in geophysical surveys to determine precise 3D position, is the measurement of the phase of the carrier wave which, as such, does not require the need to know or to use the modulations of the signal or the codes transmitted by the satellites. Each satellite transmits two frequencies with the terrestrial receiver designed to receive either one or both frequencies. In the second case, the receiver system can correct for ionospheric refraction, by using the correlation of this effect with frequency. By using the so called "carrier beat frequency" measurements, centimeter accuracy is achieved in (X, Y, Z) or (φ, λ, h) for distances greater than ~25 km from the base station; while for distances less than this receivers using just one frequency are sufficient. In many countries fixed networks of GPS receivers are being established (e.g. the CORE network in South America) and this, where they are available, eliminates the need to establish your own base station. Figure 1 shows the Antuco CORE station in Chile.

It is important to emphasize that the phase measurements are only applied for positioning in the differential (relative) mode. This means that two receivers are needed; one remains fixed in a known position so that differences in coordinates are determined with the roving receiver. The reason for using the differential mode with the phase measurements is that it cancels correlated errors and reduces the number of unknowns. By differentiating with respect to two stations, two satellites, and two epochs—known as single, double, and triple difference respectively (Figure 2)—it is possible to limit the unknowns in a progressive way. The triple difference is always used as a first step because it offers the possibility to provide preliminary coordinates

Figure 1. Gravity observations at the Antuco CORE station in Chile.

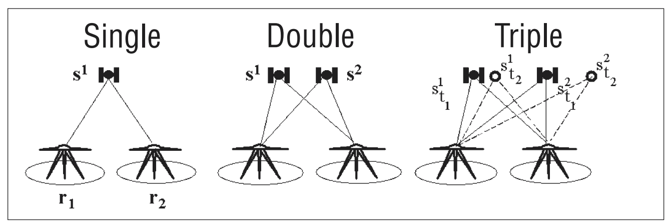

Figure 2. Single, double, and triple differences with satellites in two different positions.

for a point with just four unknowns. However, the coordinates derived in this way are unreliable. The best alternative is to derive the coordinates using the double difference. In this case the complication to be solved for is the "ambiguities"—i.e., the integer number of cycles between the satellite and the station at the first epoch of observations.

Coordinate terminology. The local *horizontal datum*, used for geophysical surveying and mapping, is normally a nationally accepted system that uses a predetermined geodetic datum and ellipsoid. This will normally differ from the WGS84 datum used by the GPS system and the GPS-derived latitudes, longitudes, and ellipsoidal heights (or ellipsoidal coordinates) will need to be transformed into an accepted national or continental datum. This implies that a single point can have more than one set of coordinates by virtue of the effect of the geodetic datum used. It is thus important to ensure that the correct datum and ellipsoid parameters are used. For example, coordinates in some countries can differ from WGS84 by up to 1 km. This can be significant when computing the normal gravity and, thus, we suggest that WGS84 coordinates should be used for gravity data processing especially when using the WGS84 gravity formula (g_{th} in Figure 3).

Table 1 shows an example of a simple horizontal translation for three widely spaced South America stations from

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Table 1. *Coordinates in two different reference systems*

Country	Latitude	Longitude	Altitude	Reference system
Brazil	-19 45 43.3459	-48 06 05.6732	754.1502	WGS-84
Brazil	-19 45 41.6527	-48 06 04.0639	*763.2821*	SAD-69
Argentina	-35 10 29.0200	-59 15 46.6400	45.9100	WGS-84
Argentina	-35 10 27.3038	-59 15 44.4610	*31.9277*	SAD-69
Venezuela	5 16 42.0800	-61 08 04.8600	1 254.5400	WGS-84
Venezuela	5 16 43.2237	-61 08 03.0228	*1 271.0418*	SAD-69

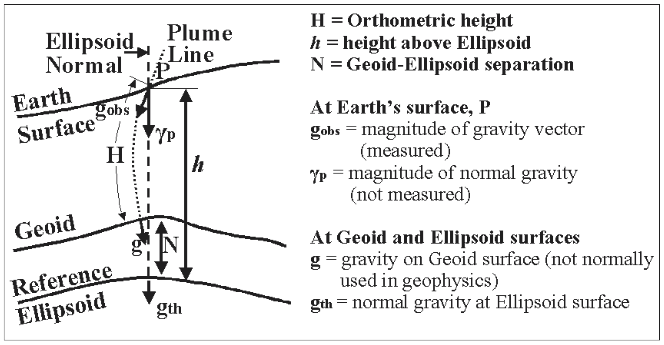

Figure 3. *Definitions of terms used at the Earth, geoid, and ellipsoid surfaces.*

WGS-84 to the South American Datum 1969, SAD-69. The following translation parameters were used:

$$\Delta X\ +66.87\ \text{m};\ \Delta Y\ -04.37\ \text{m};\ \Delta Z\ +38.52\ \text{m} \qquad (1)$$

Often seven transformation parameters are used, thus allowing incorporation of axis-rotation and scale factor. These translations are applied to the geocentric Cartesian coordinates before the transformation to geodetic coordinates using the appropriate ellipsoid parameters.

The ellipsoidal height resulting from the transformation (italics in Table 1) should not be used since this is a horizontal transformation. The vertical transformation is achieved using the ellipsoidal height and geoid model.

National height systems, used to determine heights of benchmarks and topographic maps, are traditionally based on a reference datum of H=0 representing mean sea level at a given location. For mainland United Kingdom, for example, the height reference system used is based on the mean sea level at Newlyn in Cornwall.

For inland areas of continents, using sea level height references is not without its problems due to the propagation of precise spirit leveling errors, reference system biases, and other temporal effects such as glacial rebound, etc. In central Eastern Europe, the Baltic (Kronshtadt) height reference system gives heights that differ by up to 2 m from the Adriatic (Trieste) height reference system.

For a country such as Brazil, having a single reference tide gauge is impractical and the introduction of GPS geometric leveling has highlighted the problems with the older *orthometric height* system. If orthometric or precise leveling is used to link two tide gauges, then for various reasons the difference is not necessarily zero. First, due to hydrodynamic effects (currents, temperature) the mean sea level at the two tide gauges will not necessarily be on the same equipotential surface resulting in spatial differences of the mean sea level (sea surface topography). This is not necessarily a function of distance. Second, errors in orthometric leveling tend to increase with distance. Third, and conceptually more complicated, is the fact that the equipotential surfaces are not parallel in a geometrical sense; i.e. what two different equipotential surfaces have in common is their difference in potential and not the difference in the distance or separation between the two surfaces. So, the results of orthometric leveling depend on the paths taken, and as such orthometric corrections seek to make the leveling path-independent.

Satellite positioning systems (GPS) are increasingly being used to determine the vertical coordinate. The accuracy using GPS can range from a few meters to a few centimeters and are in general 2-5 times worse than the horizontal accuracy. In the first case the C/A code is used with the DGPS correction and gives an accuracy of 1-5 m on baselines shorter than ~25 km. Accuracies of between a few centimeters and 1 m can be achieved using single frequency receivers with short periods of observation, of about 20 minutes, using the triple difference solution. This means that the ambiguities will not be resolved and the base line distance is restricted to within 25 km. If the "carrier beat phase" methodology is used with periods of observation increased to 1-2 hours with single frequency receivers, (or double frequency for base lines longer than 25 km), an accuracy of a few centimeters is achieved. This improved accuracy uses the "double difference" method to solve for the ambiguities and is made possible by the longer observational period and error modeling. The above set of timing requirements is likely to decrease significantly when the current GPS (U.S. system) is upgraded and supplemented by Galileo (European system) in ~2007.

GPS directly provides the *geometric* or *ellipsoidal height* h (height above the ellipsoid defined by WGS84).

To convert the ellipsoid height h to an orthometric height H requires the height of the geoid N to be known, where N is the separation between the geoid and the ellipsoid (Figure 3), so H = h - N. This has problems because the geoid surface is not precisely known everywhere and its calculation is continually being updated.

Gravity terminology. Gravity corrections are often, and incorrectly, thought of as the result of correcting the surface gravity observation, g_{obs}, down to a datum. The correct way of viewing such corrections is correcting normal gravity to the observational point at the Earth's surface. The magnitude of the gravity acceleration, g_{obs}, measured at the earth's surface, is a scalar quantity (Figure 3). If g_{obs} is corrected using ellipsoid- and/or geoid-based corrections, then different gravity terminology needs to be used and the resulting gravity values will be numerically different.

Using H, the height in meters relative to the geoid surface (traditional processing), the free air and Bouguer gravity anomalies, expressed in mGal, can be simply expressed (ignoring the second-order free air correction effect, the curvature (Bullard B) correction and terrain corrections) as:

$$\text{Free air anomaly} = \text{Faa} = g_{obs} - (g_{th} - 0.3086H)$$
$$\text{Bouguer anomaly} = \text{Faa} - 0.04191\rho H$$

where g_{obs} is the observed gravity, g_{th} is normal gravity based on the WGS84 ellipsoid formula, the free air correction is 0.3086 H, the Bouguer correction for an infinite slab is $0.04191\rho H$, where H is in meters and ρ is density in g/cc.

Using h, the height in meters relative to the ellipsoid surface (new GPS processing), then the above expressions become:

$$\text{Free air disturbance} = \text{Fad} = g_{obs} - (g_{th} - 0.3086h)$$
$$\text{Bouguer disturbance} = \text{Fad} - 0.04191\rho h$$

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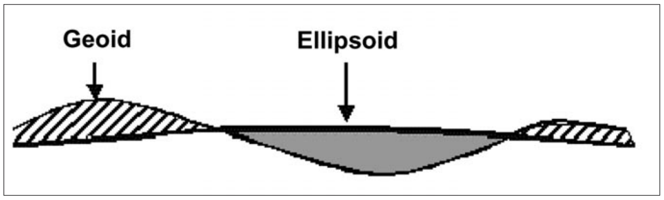

Figure 4. Geoid and ellipsoid surfaces.

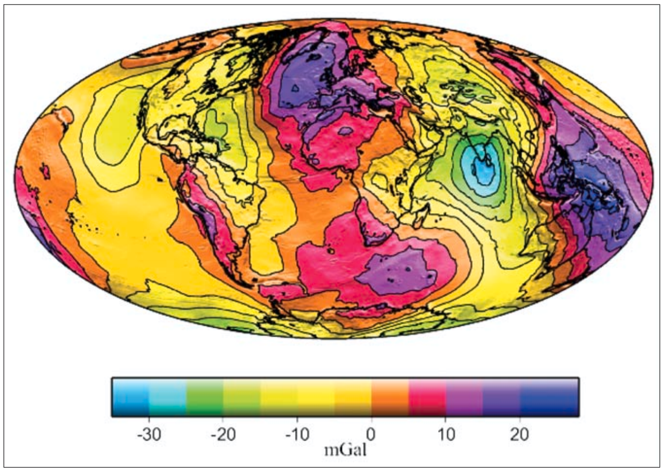

Figure 5. Indirect effect for the free air values generated from the height difference (EGM96 geoid-ellipsoid).

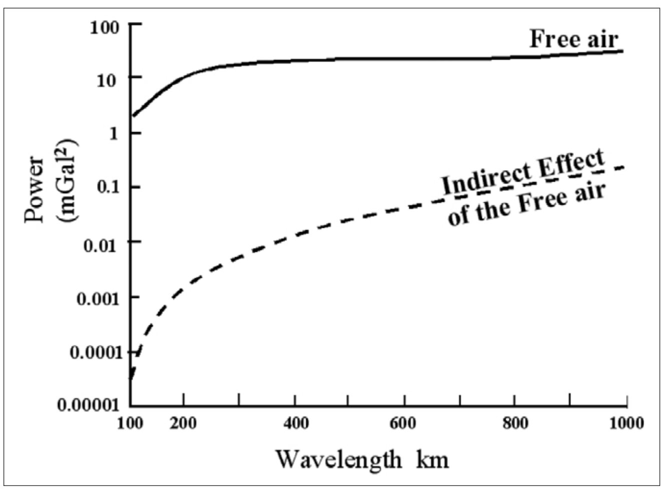

Figure 6. Power spectrum of free air and indirect effect free air for a 10-minute grid of South America covering the area 85W to 34W and 56S to 15N.

The magnitudes of these anomaly and disturbance values will be different because H and h have different values. Thus, merging old surveys using H and modern gravity surveys using h will give a further level of complexity to be resolved.

Which formula is the more correct? From the above equations and Figure 4, the gravity disturbance is the difference between g_{obs} at the observational surface and g_{th} at the ellipsoid surface and its upward continuation to the measurement surface by the use of the free air correction, 0.3086h and the Bouguer correction 0.04191ρh.

Traditional geophysical processing has determined gravity anomalies using H, because only H (not h) was available in the past. Using H undercorrects for the hatched areas in Figure 4, where the geoid surface is above the ellipsoid surface, and overcorrects for the part of the geoid surface that is below the ellipsoid surface.

The gravity effect for these two different reference surfaces (Figure 5) is known as the indirect effect. The difference between the geoid and ellipsoid surfaces can be as large as ~100 m and generates a maximum indirect effect of ~30 mGal in the free air value or ~20 mGal in the Bouguer value. So should geophysicists change their terminology and procedures and introduce the more correct term "gravity disturbance"?

Implications for gravity exploration. For solid earth geophysical studies, studying large regions of the Earth, the traditional method of calculating gravity anomalies has ignored the indirect effect and, thus, subsurface mass distribution may have been under- or overestimated. The spectral plot of the indirect effect for South America is shown in Figure 6. It indicates that for small-scale surveys, less than a few hundred km in dimension, the variation within the survey area is likely to be small. This is quantified by using an azimuthally averaged power spectrum for South America in Figure 6. The ratio between the indirect effect free air and free air amplitudes decreases from ~10^{-2} at ~1000-km wavelength to ~10^{-5} at ~100-km wavelength. Local large gravity anomalies can still produce significant indirect effects. The global free air representation of the indirect effect is produced from EGM96 and hence represents the regional or long wavelength component of this correction.

Satellite altimeter-derived gravity: The satellite altimeter method to derive marine gravity uses the principle that the mean sea surface is an equipotential surface of the gravity field. Free air gravity is derived from either converting along track sea surface gradients or directly from the mean sea surface shape. The gravity values relate to the sea surface (i.e., the geoid) and hence are clearly gravity anomalies rather than gravity disturbances. To derive the gravity disturbance requires the application of the indirect effect.

Airborne gravity: Scalar and vector gravity measurements are now routinely observed in exploration. This has been possible by the use of GPS, which tracks the geometric 3D motion and position of the gravity sensor/aircraft. All 3D positional measurements onboard the aircraft are made by GPS, thus determining gravity disturbances would be less error prone than trying to correct to orthometric heights. Airborne gravity gradiometer measurements (gradients and tensors) also rely on GPS but are insensitive to long wavelength gravity variations and since such data undergo different forms of processing compared to conventional gravity, there are no similar problems with the indirect effect.

Marine gravity: Shipborne gravity measurements are collected at the sea surface and tidal effects have either been corrected for or minimized at cross-over correction and microleveling stages of processing. To our knowledge, no attempt has been made to output accurate ship-based GPS heights due possibly to the ship motion noise. In light of such problems, the free air gravity anomaly can at present only be converted to the free air disturbance by applying the indirect effect. For many marine surveys, imprecise bathymetry still remains a major source of error. Depth errors up to 10% are common (i.e., 300 m in 3 km of water!). This results from using the wrong velocities to convert the two-way transit times to depth, which significantly affects the calculation of Bouguer and isostatic anomalies and can be a greater problem than any geoid/ellipsoid differences.

Land gravity: For stand-alone gravity surveys, the speed and efficiency of GPS-based surveying are replacing conventional surveying methods such that surveys can be reduced to free air disturbance without any assumptions being made. Converting the data to Bouguer and isostatic disturbances is straightforward using traditional methods by using h rather

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Figure 7. Gravity fieldwork in Paraguay and Rio Negro, Brazil using GPS methods.

than H. If the gravity survey is regional in character (Figure 7) and involves the integration of existing surveys, then working with orthometric heights, H, is recommended and requires that the appropriate geoid be used.

Conclusions. The term *gravity disturbance*, which is familiar in geodesy, is unlikely to be readily accepted by geophysicists until possibly a body such as SEG provides clear guidelines to the acquisition, processing, and documentation of gravity surveys using GPS measurements. The terms "anomaly" and "disturbance" will have to coexist because not all surveys use ellipsoidal heights. The amplitude variation of the indirect effect at the scale that exploration surveys normally operate (<100 km) will in general have an insignificant effect on any resulting interpretation because long wavelength or DC terms are often removed from Bouguer data during modeling to remove unwanted long wavelength regional gravity effects. Figure 5 indicates that spatial variation of the free air indirect effect could be as large as 30 mGal or 20 mGal for the Bouguer indirect effect. Such shifts between Bouguer anomaly-disturbance between adjacent surveys could generate problems when integrating old and new surveys. The gravity differences between overlapping surveys and the global variation of the indirect effect could however provide a "fingerprint" to the type of processing that has occurred if full processing details are not readily available.

Good survey practice should always dictate that a full set of meta data for both data acquisition and processing be included with the listing of the principal facts of a survey and reproduced on all map legends. This will only be achieved if clients insist on such conditions/specification as part of their standard survey contract.

Clearly ship, satellite, and older land data naturally produce gravity anomalies, while newer land and airborne data will most readily be gravity disturbances. Possible problems envisaged by having these two gravity systems working side by side are in the calculation of terrain corrections using GPS station heights with digital terrain maps based on a national coordinate system, and in data compilations where there is potential for mixing gravity anomalies and disturbances as well as surveys with different coordinate systems.

Suggested reading. General text: *GPS—Theory and Practice* by Hofmann-Wellenhof et al. (Springer, 1997). Articles: "Geodetic versus geophysical perspectives of the 'gravity anomaly'" by Hackney and Featherstone (*Geophysical Journal International*, 2003) and "Ellipsoid, geoid, gravity, geodesy, and geophysics" by Li and Götze (GEOPHYSICS, 2001). TLE

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