CMPS 2200 Assignment 1

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In this assignment, you will learn more about asymptotic notation, parallelism, functional languages, and algorithmic cost models. As in the recitation, some of your answer will go here and some will go in `main.py`. You are welcome to edit this `assignment-01.md` file directly, or print and fill in by hand. If you do the latter, please scan to a file `assignment-01.pdf` and push to your github repository.

1. (2 pts ea) **Asymptotic notation**

- 1a. Is $2^{n+1} \in O(2^n)$? Why or why not? . Yes
  - $f(n) = 2^{n+1}$
  - $g(n) = 2^n$
  - $\lim_{n \to \infty} \frac{f(n)}{g(n)} = \frac{2^{n+1}}{2^n} = 2$

  Since the limit is constant, $2^{n+1} \in O(2^n)$

- 1b. Is $2^{2^n} \in O(2^n)$? Why or why not? No
  - $f(n) = 2^{2n}$
  - $g(n) = 2^n$
  - $\lim_{n \to \infty} \frac{f(n)}{g(n)} = \frac{2^{2n}}{2^n} = 2^n_{n \to \infty} = \infty$

  since the limit is not a constant, $2^{2^n} \notin O(2^n)$

- 1c. Is $n^{1.01} \in O(\log^2 n)$? No
  - $f(n) = n^{1.01}$
  - $g(n) = \log^2 n$
  - $\lim_{n \to \infty} \frac{f(n)}{g(n)} = \frac{n^{1.01}}{\log^2 n}$

  $\therefore$ $n^{1.01}$ grows polynomially with n, $\log^2 n$ grows logarithmically with n, $n^{1.01}$ grows much faster than $\log^2 n$

  $\therefore n^{1.01} \notin O(\log^2 n)$ No

- 1d. Is $n^{1.01} \in \Omega(\log^2 n)$? Yes
  - $f(n) = n^{1.01}$
  - $g(n) = \log^2 n$
  - we want to find c, n_0:
  - $f(n) \geq c \cdot g(n)$ for all $n \geq n_0$

    $n^{1.01} \geq c \log^2 n$

  $\log(n^{1.01}) \geq \log(c \cdot \log^2 n)$

  $1.01 \log(n) \geq \log(c) + 2\log(\log n)$

  $1.01 \geq \frac{\log(c)}{\log(n)} + \frac{2\log(\log n)}{\log(n)}$

  As n approach infinity:

  $1.01 \geq 0 + 2 \cdot 0$ which is alway true

  $\therefore$ Yes

  $6 \times 5 \times 4 \times 3 \times 2$

- 1e. Is $\sqrt{n} \in O((\log n)^3)$? Yes
  - We want to find c, n
  - $\sqrt{n} \leq c \times (\log n)^3$
  - $c^2 \geq \frac{n}{(\log n)^6}$

  $\lim_{n \to \infty} \frac{n}{(\log n)^6} = \frac{1}{6(\log n)^5 \cdot \frac{1}{n}} = \frac{n}{6(\log n)^5} = \frac{n}{720 \log n} \approx 0$

  $\because$ the limit is 0

  $\therefore c^2$ alway larger than $\frac{n}{(\log n)^6} \to$ Yes

- 1f. Is $\sqrt{n} \in \Omega((\log n)^3)$?

  $f(n) = \sqrt{n}$

  $g(n) = (\log n)^3$

  we want to find,

  $\sqrt{n} \geq c \cdot (\log n)^3$

  $n \geq c^2 (\log n)^6$

  $c^2 \leq \frac{n}{(\log n)^6}$

  $\lim_{n \to \infty} \frac{n}{(\log n)^6} = 0$

  $c^2 \leq 0$,

  it is not alway true

  $\therefore \sqrt{n} \notin \Omega((\log n)^3)$

2. **SPARC to Python**

Consider the following SPARC code of the Fibonacci sequence, which is the series of numbers where each number is the sum of the two preceding numbers. For example, 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610 . . .

```
foo x =
  if  x ≤ 1  then
    x
  else
    let  (ra, rb) = (foo (x − 1))  ,  (foo (x − 2))  in
      ra + rb
    end.
```

- 2a. (6 pts) Translate this to Python code – fill in the `def foo` method in `main.py`
- 2b. (6 pts) What does this function do, in your own words?

This function uses recursion to compute Fibonacci numbers by breaking down the problem into smaller subproblems and combine the result to get the Fibonacci number for the given index X.

3. **Parallelism and recursion**

Consider the following function:

```python
def longest_run(myarray, key)
    """
     Input:
       `myarray`: a list of ints
       `key`: an int
     Return:
       the longest continuous sequence of `key` in `myarray`
    """
```

E.g., `longest_run([2,12,12,8,12,12,12,0,12,1], 12) == 3`

- 3a. (7 pts) First, implement an iterative, sequential version of `longest_run` in `main.py`.
- 3b. (4 pts) What is the Work and Span of this implementation?

$O(n)$

$O(n)$

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- 3c. (7 pts) Next, implement a `longest_run_recursive`, a recursive, divide and conquer implementation. This is analogous to our implementation of `sum_list_recursive`. To do so, you will need to think about how to combine partial solutions from each recursive call. Make use of the provided class `Result`.
- 3d. (4 pts) What is the Work and Span of this sequential algorithm?

$O(\log n)$

$O(\log n)$

- 3e. (4 pts) Assume that we parallelize in a similar way we did with `sum_list_recursive`. That is, each recursive call spawns a new thread. What is the Work and Span of this algorithm?

$O(1)$

$O(\log n)$