Assignment 2

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Derive asymptotic upper bounds of work:

Part 1. $W(n) = 2W(n/3) + 1$

$T(n) = aT(n/b) + f(n)$

$a = 2$

$b = 3$ $\quad 2 > 3^0$

$f(n) = 1$

$1 < \log_3(2)$ $\quad n^{\log_b(a)} \leftarrow$ upper bound $\rightarrow n^{\log_3(2)}$

2. $W(n) = 5W(n/4) + n$

$a = 5$

$b = 4$ $\quad n < n^{\log_4(5)}$

$f(n) = n$

$\Theta(n^{\log_4(5)})$

3. $7W(n/7) + n$

$a = 7$

$b = 7$ $\quad \log_7 7 = 1$

$f(n) = n$ $\quad n = n^1$

$W(n) = \Theta(n \log n)$

4. $W(n) = 9W(n/3) + n^2$

$a = 9$ $\quad \log_3(9) = 2$

$b = 3$

$f(n) = n^2$ $\quad n^2 = n^{\log_3(9)}$

$W(n) = \Theta(n^2 \log n)$

5. $W(n) = 8W(n/2) + n^3$

$a = 8$ $\quad \log_2(8) = 3$

$b = 2$ $\quad n^3 = n^{\log_2(8)}$

$f(n) = n^3$ $\quad W(n) = \Theta(n^3 \log n)$

6. $W(n) = 49W(n/25) + n^{3/2}\log n$

$a=49$

$b=25$

$f(n) = n^{3/2}\log n$

$\log_{25}(49) = 1$

$n^{3/2}\log n > n^{\log_{25}(49)}$

$W(n) = \Theta(n^{3/2}\log n)$

7. $W(n) = W(n-1)+2$

Every step you add 2 so it's linear

$W(n) = \Theta(n)$

8. $W(n) = W(n-1)+n^c$

$W(\overset{+1}{n}-2)+(\overset{+1}{n}-1)^c+n^c$

$W(n) = \Theta(n^{c+1})$

9. $W(n) = W(\sqrt{n})+1$

$\sqrt{n}$ each step

$\sqrt{n} \rightarrow \log_2 n$

$W(n) = \Theta(\log_2(n))$

Part 2. Algorithm A: $W(A) = 5W(n/2)+n$

$a=5$

$b=2$

$f(n)=n$

$n < n^{\log_2 5}$

$W(\text{Algo } A) = \Theta(n^{\log_2(5)})$

Algorithm B: $W(n) = 2W(n-1) + 1$

$2 \cdot 2W(n-2) + 1 + 1$

$2^2 \cdot 1 + 2 \cdot 1 + 1$ $\qquad W(n) = \Theta(2^n)$

$2^n \cdot 1$

Algorithm C: $W(n) = 9W(n/3) + O(n^2)$

$a = 9$

$b = 3$

$f(n) = O(n^2)$

9	$n/3$
$9 \cdot 9$	$n/3^2$
$9 \cdot 9 \cdot 9$	$n/3^3$
9^k	$n/3^k$

$W(n) = O(n^2 \cdot \log_3 n)$

I would choose algorithm C because for larger input it has to potential to be faster than Algorithm A. Algorithm B is the worst.