Jim Haines Algorithms HW 1

1 a.) $2^{n+1} \in O(2^n)$?

$2^{n+1} \leq C \cdot 2^n$

$2^{n+1} \leq 2 \cdot 2^n$ let $c=2$

$2^{n+1} \leq 2^{n+1}$

$\therefore$ $\boxed{\text{yes}}$

b.) $2^{2n} \in O(2^n)$

$\boxed{\text{No}}$

2^n will always be greater than n, and each exponent shares the same base: 2

$\therefore 2^{2n} \not\leq 2^n \cdot C$

c.) $n^{1.01} \in O(\log^2 n)$

use Lo'hopitals

$\lim_{n\to\infty} \frac{n^{1.01}}{\log^2 n} \quad \lim_{n\to\infty} \frac{1.01n^{.01}}{1/n\ln 2} \ldots \infty$

TOP grows faster than bottom

$\therefore$ $\boxed{\text{NO}}$

D.) $n^{1.01} \in \Omega(\log^2 n)$

$\lim_{n\to\infty} \frac{n^{1.01}}{\log^2 n} \to \lim_{n\to\infty} \frac{1.01n^{.01}}{1/n\ln 2} \to \infty$

So $\boxed{\text{yes}}$ according to limit method

e.) $\sqrt{n} \in O((\log n)^3)$

$\sqrt{n} \leq C \cdot (\log n)^3$

no value of $n \geq n_0$ or C will make the above inequality true

$\therefore$ $\boxed{\text{no}}$, $\sqrt{n}$ grows faster than $(\log n)^3$

f.) $\sqrt{n} \in \Omega((\log n)^3)$

$C \cdot \sqrt{n} \geq \cdot (\log n)^3$

here, $C\sqrt{n}$ will always be greater than $(\log n)^3$ because $\sqrt{n}$ grows faster for large x values

$\therefore$ $\boxed{\text{yes}}$