CMPS 2200 Assignment 1

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In this assignment, you will learn more about asymptotic notation, parallelism, functional languages, and algorithmic cost models. As in the recitation, some of your answer will go here and some will go in `main.py`. You are welcome to edit this `assignment-01.md` file directly, or print and fill in by hand. If you do the latter, please scan to a file `assignment-01.pdf` and push to your github repository.

1. (2 pts ea) **Asymptotic notation**

- 1a. Is $2^{n+1} \in O(2^n)$? Why or why not? .
  - $\exists c, n_0$ s.t. $0 \le f(n) \le c g(n)\ \forall n \ge n_0$
  - $2^{n+1} \le c \cdot 2^n$
  - $\ln 2^{n+1} \le \ln(c \cdot 2^n)$
  - $n+1 \cdot \ln 2 \le \ln c + n \cdot \ln 2$
  - $n+1 \le \ln c + n$ $\Rightarrow 1 \le \ln c$

  Let $c \in \mathbb{R}$ s.t. $c > e$, then the inequality is always true, and the Big-O definition is true, so $2^{n+1} \in O(2^n)$ ■ QED
- 1b. Is $2^{2^n} \in O(2^n)$? Why or why not?
  - $\exists c, n_0$ s.t. $0 \le 2^{2^n} \le c \cdot 2^n\ \forall n \ge n_0$
  - $2^{2^n} \le c \cdot 2^n$
  - $2^{2 \cdot n} \le c \cdot 2^n$
  - $2^n \cdot 2^n \le c \cdot 2^n$
  - $2^n \le c$ $\Rightarrow$ Contradiction, as 2^n is greater than a constant. Thus, $2^{2^n} \notin O(2^n)$ ■ QED
- 1c. Is $n^{1.01} \in O(\log^2 n)$?
  - $\exists c, n_0$ s.t. $0 \le n^{1.01} \le c \cdot \log^2 n\ \forall n \ge n_0$
  - $\lim_{n\to\infty} \frac{n^{1.01}}{\log^2 n} = \frac{\infty}{\infty}$
  - L'hopitals $\rightarrow$ $\lim_{n\to\infty} \frac{1.01 n^{.01}}{\frac{2\log x}{x}} = \frac{2.02 n^{.01}}{2 \log n} = \frac{\infty}{\infty}$ $\Rightarrow \lim_{n\to\infty} \frac{.0202 n^{-.99}}{\frac{2}{n}} = \frac{.0202 n}{2 n^{.99}} = \frac{\infty}{\infty}$
  - $\lim_{n\to\infty} \frac{.0202}{1.98 n^{-.01}} = \frac{.0202 n^{.01}}{1.98} = \infty$

  Since the $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$, $n^{1.01} \notin O(\log^2 n)$ ■ QED
- 1d. Is $n^{1.01} \in \Omega(\log^2 n)$?
  - $\exists c, n_0$ s.t. $n^{1.01} \ge c \cdot \log^2 n\ \forall n \ge n_0$
  - $\lim_{n\to\infty} \frac{n^{1.01}}{\log^2 n} = \frac{\infty}{\infty}$
  - $= \lim_{n\to\infty} \frac{1.01 n^{.01}}{\frac{2\log x}{x}} = \frac{2.02 n^{.01}}{2 \log n} = \frac{\infty}{\infty}$ $\Rightarrow \lim_{n\to\infty} \frac{.0202 n^{-.99}}{\frac{2}{n}} = \frac{.0202 n}{2 n^{.99}} = \frac{\infty}{\infty}$
  - $= \lim_{n\to\infty} \frac{.0202}{1.98 n^{-.01}} = \frac{.0202 n^{.01}}{1.98} = \infty$

  Because the $\lim \frac{f(n)}{g(n)} = \infty$, $n^{1.01} \in \Omega(\log^2 n)$ ■ QED

  $$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \begin{cases} 0 & \Rightarrow f(n) \in O(g(n)) \\ c > 0 & \Rightarrow f(n) \in \Theta(g(n)) \\ \infty & \Rightarrow f(n) \in \Omega(g(n)) \end{cases}$$
- 1e. Is $\sqrt{n} \in O((\log n)^3)$?
  - $\exists c, n_0$ s.t. $0 \le \sqrt{n} \le c \cdot (\log n)^3\ \forall n \ge n_0$
  - $\lim_{n\to\infty} \frac{\sqrt{n}}{(\log n)^3} = \frac{\infty}{\infty}$
  - $= \lim_{n\to\infty} \frac{\frac{1}{2} n^{-1/2}}{3(\log n)^2 \frac{1}{n}} = \frac{\frac{1}{2}\sqrt{n}}{3\log^2 n} = \frac{\infty}{\infty}$ $\Rightarrow = \lim_{n\to\infty} \frac{\frac{1}{4} n^{-1/2}}{6 \log n \frac{1}{n}} = \frac{\frac{1}{4}\sqrt{n}}{6 \log n} = \frac{\infty}{\infty}$
  - $= \lim_{n\to\infty} \frac{\frac{1}{8} n^{-1/2}}{6 \cdot \frac{1}{n}} = \frac{\frac{1}{8} n^{1/2}}{6} = \frac{\sqrt{n}}{48} = \infty$

  The $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$, so thus $\sqrt{n} \notin O((\log n)^3)$ ■ QED
- 1f. Is $\sqrt{n} \in \Omega((\log n)^3)$?
  - $\exists c, n_0$ s.t. $\sqrt{n} \ge c \cdot (\log n)^3$

  $\lim_{n\to\infty} \frac{\sqrt{n}}{(\log n)^3} = \frac{\infty}{\infty}$

  $= \lim_{n\to\infty} \frac{\frac{1}{2} n^{-1/2}}{3(\log n)^2 \frac{1}{n}} = \frac{\frac{1}{2}\sqrt{n}}{3\log^2 n} = \frac{\infty}{\infty}$ $\Rightarrow = \lim_{n\to\infty} \frac{\frac{1}{4} n^{-1/2}}{6 \log n \frac{1}{n}} = \frac{\frac{1}{4}\sqrt{n}}{6 \log n} = \frac{\infty}{\infty}$

  $= \lim_{n\to\infty} \frac{\frac{1}{8} n^{-1/2}}{6 \cdot \frac{1}{n}} = \frac{\frac{1}{8} n^{1/2}}{6} = \frac{\sqrt{n}}{48} = \infty$

  Since the $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$, $\sqrt{n} \in \Omega((\log n)^3)$ ■ QED