CMPS 2200 Assignment 2

Name:

In this assignment we'll work on applying the methods we've learned to analyze recurrences, and also see their behavior in practice. As with previous assignments, some of of your answers will go in main.py.. You should feel free to edit this file with your answers; for handwritten work please scan your work and submit a PDF titled assignment-02.pdf and push to your github repository.

- 1. Derive asymptotic upper bounds of work for each recurrence below.
- W(n) = 2W(n/3) + 1.

 $(W(n)) = aW(\frac{n}{h}) + f(n)$

· Since [ogo A = [ogo A] < [ogo A].
· W(n) = 5W(n/4) + n.

logus >1, Wan) E O (nlogus)

• W(n) = 7W(n/7) + n.

log_7=1 W(n) E O (n logn)

• $W(n) = 9W(n/3) + n^2$.

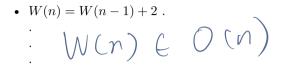
log 3 ? = 3 >2 W(n) & O(13)

• $W(n) = 8W(n/2) + n^3$.

: 109 28 = 4 > 3 W(n) E

• $W(n) = 49W(n/25) + n^{3/2} \log n$.

: log 25 47 = 1.209 < 3



• $W(n) = W(n-1) + n^c$, with $c \ge 1$.

 $(n) \in O(n^c)$

Win) E D (logh) • $W(n) = W(\sqrt{n}) + 1$ 2. Suppose that for a given task you are choosing between the following three

algorithms:

Algorithm \mathcal{A} solves problems by dividing them into five subproblems of half the size, recursively solving each subproblem, and then combining the solutions in linear time.

Algorithm \mathcal{B} solves problems of size n by recursively solving two subproblems of size n-1 and then combining the solutions in constant time.

• Algorithm \mathcal{C} solves problems of size n by dividing them into nine subproblems of size n by dividing them into nine

What are the asymptotic running times of each of these algorithms? Which algorithm would you choose? algorithm would you choose? Choose 3. Now that you have some practice solving recurrences, let's work on

implementing some algorithms. In lecture we discussed a divide and conquer algorithm for integer multiplication. This algorithm takes as input two *n*-bit strings $x = \langle x_L, x_R \rangle$ and $y = \langle y_L, y_R \rangle$ and computes the product xy by using the fact that $xy = 2^{n/2}x_Ly_L + 2^{n/2}(x_Ly_R + x_Ry_L) + x_Ry_R$. Use the stub functions in main.py to implement Karatsaba-Ofman algorithm algorithm for integer multiplication: a divide and conquer algorithm that runs in subquadratic time. Then test the empirical running times across a variety of inputs to test whether your code scales in the manner described by the asymptotic runtime. Please refer to Recitation 3 for some basic implementations, and Eqs (7) and (8) in the slides https://github.com/allan-tulane/cmps2200-slides/blob/main/module-02recurrences/recurrences-integer-multiplication.ipynb