1a) $W(n) = 2W(n/3) + 1$ $T(n) = aT(n/b) + f(n)$ where $f(n) = cn^k$

$a = 2$ $b = 3$ $f(n) = 1$

$a > b^k$

$2 > 3^0$ $O(n^{\log_b a})$

$2 > 1$ $\boxed{O(n^{\log_3 2})}$ based on master Method

1b) $W(n) = 5W(n/4) + n$

$T(n) = aT(n/b) + f(n)$ where $f(n) = cn^k$

$a = 5$ $a > b^k$

$b = 4$ $5 > 4^1$

$c = 1$ $O(n^{\log_b a})$ Based on Master Method

$k = 1$ $\boxed{O(n^{\log_4 5})}$

1c) $W(n) = 7W(n/7) + n$

$a = 7$ $b = 7$ $c = 1$ $k = 1$

$7 = 7^1$

$O(n^k \log n)$ Based on Master Method

$\boxed{\Theta(n \log n)}$

1d) $W(n) = 9W(n/3) + n^2$

$a = 9$ $b = 3$ $c = 1$ $k = 2$

$9 = 3^2$

$9 = 9$ Based on Master Method

$O(n^k \log n)$

$\boxed{O(n^2 \log n)}$

1e) $W(n) = 8W(n/2) + n^3$

$a = 8$ $b = 2$ $c = 1$ $k = 3$

$8 = 2^3$, $8 = 8$

$O(n^k \log n)$

$\boxed{O(n^3 \log n)}$

1f) $W(n) = 49W(n/25) + n^{3/2}\log n$

$a = 49$ $b = 25$ $f(n) = n^{3/2}\log n$

$n^{3/2}\log n = n^k \log^p n$ $k = 3/2$ $p = 1$

$a > b^k$

$a = b^k$

$a < b^k$

$49 < 25^{3/2}$

$49 < 125$

$O(n^k \log^p n)$

$\boxed{O(n^{3/2} \log n)}$ Based on master method

$$w(n) = \begin{cases} 1 & n=0 \\ w(n-1)+2 & n>0 \end{cases}$$

19) $w(n) = w(n-1) + 2$

substitute $w(n-1)$

$w(n) = w(n-1) + 1$

$w(n-1) = w(n-2) + 1$

$w(n-2) = w(n-3) + 1$

$w(n) = [w(n-2) + 1] + 2$ ← substitute

$w(n) = w(n-2) + 3$

$w(n) = [w(n-3) + 1] + 3$ ← substitute

$w(n) = w(n-3) + 4$

⋮ cont'd for k times

$w(n) = w(n-k) + k$

assume $n-k=0$, $\therefore n=k$ then $w(n) = w(n-n) + n$

$w(n) = w(0) + n$

$w(n) = 1 + n$

$\boxed{O(n)}$

(h) $w(n) = w(n-1) + n^c$ where $c \geq 1$

$$w(n) = \begin{cases} 1 & n=0 \\ w(n-1) + n^c & n>0 \text{ and } c \geq 1 \end{cases}$$

$w(n) = w(n-1) + n^c$

$\therefore w(n) = w(n-1) + n^c$

$\therefore w(n-1) = w(n-2) + (n-1)^c$ → substitute

$w(n) = [w(n-2) + (n-1)^c] + n^c$

$w(n) = w(n-2) + (n-1)^c + n^c$

$\therefore w(n-2) = w(n-3) + (n-2)^c$ → substitute

$w(n) = [w(n-3) + (n-2)^c] + (n-1)^c + n^c$

$w(n) = w(n-3) + (n-2)^c + (n-1)^c + n^c$

$w(n) = w(n-k) + (n-(k-1))^c + (n-(k-2))^c + \dots + (n-1)^c + n^c$

Assume $n-k=0$, $\therefore n=k$

$w(n) = w(n-n) + (n-n+1)^c + (n-n+2)^c + \dots (n-1)^c + n^c$

$w(n) = w(0) + 1^c + 2^c + \dots + (n-1)^c + n^c$

$\therefore 1^c + 2^c + \dots n^c \leq \underbrace{n^c + n^c + \dots + n^c}_{n} = n^{c+1}$

$\boxed{O(n^{c+1})}$

1 i) $w(n) = w(\sqrt{n}) + 1$

consider $n = 2^K$, then

$w(n^k) = w(\frac{2^K}{k}) + 1$

consider $w(2^K) = T(K)$, then

$T(K) = T(\frac{K}{2}) + 1$

Now, using master method for $T(K)$

$T(n) = aT(n/b) + f(n)$ $\quad f(n) = cn^K$

$a = 1$ $\quad K = 0$ $\quad a = b^K$

$b = 2$ $\quad c = 1$ $\quad 1 = 2^0$

$1 = 1$

$O(n^K \log n)$

$O(\log n)$

$\boxed{\text{now, } w(n) = O\log(\log n)}$

2) algorithm a: $5W(n/2) + n$

$T(n) = a\,T(n/b) + f(n)$ $\qquad$ $f(n) = cn^k$

$a = 5$ $\quad$ $c = 1$ $\qquad$ $a > b^k$

$b = 2$ $\quad$ $k = 1$ $\qquad$ $5 > 2^1$

$\boxed{O(n^{\log_2 5}) \text{ by Master Method}}$

algorithm b: $2W(n-1) + 1$

$$W(n) = \sum_{i=0}^{n-1} 2^i = \frac{2n-1}{2-1} = 2n - 1 = O(2n)$$

since $W(n) \leq c \cdot 2^n + n$

$W(n) \leq 2 \cdot c2^{n-1} + (n-1) + 1$

$= c2^n + n$

$= \boxed{O(2^n)}$

algorithm c: $9W(n/3) + n^2$

$T(n) = a\,T(n/b) + f(n)$ $\qquad$ $f(n) = cn^k$ $\qquad$ $3^k = n$ $k = \log n$

$a = 9$ $\quad$ $c = 1$ $\qquad$ $a = b^k$

$b = 3$ $\quad$ $k = 2$ $\qquad$ $9 = 3^2$

$9 = 9$

$O(n^k \log n) \Rightarrow \boxed{O(n^2 \log n)}$

by Master Method

$\boxed{\text{Algorithm B is fastest so I choose B.}}$