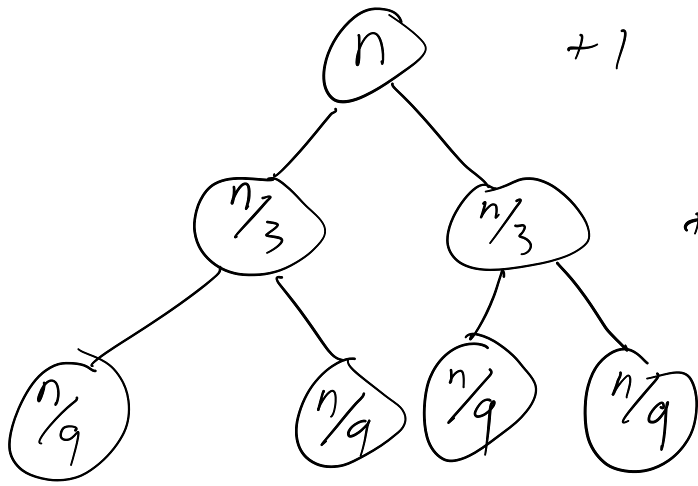
1.) $W(n) = 2W(n/3) + 1$



$+1$ 1

$+1$ 2

$+1$ 4

$\vdots$

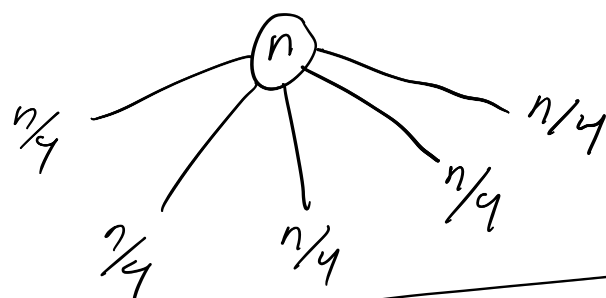
i

$\frac{n}{3^i} = 1 \rightarrow$ the end of the tree

so, $i = \log_3 n$ $w(n) = \sum_{i=0}^{\log_3 n} 2^i$

$1 + 2 + 4 + 8 + \ldots + 2^{\log_3 n} = n^{\log_3 2} \rightarrow \boxed{O(n^{\log_3 2})}$

2.) $W(n) = 5W(n/4) + n$



$+ n$ 1

$+ n$

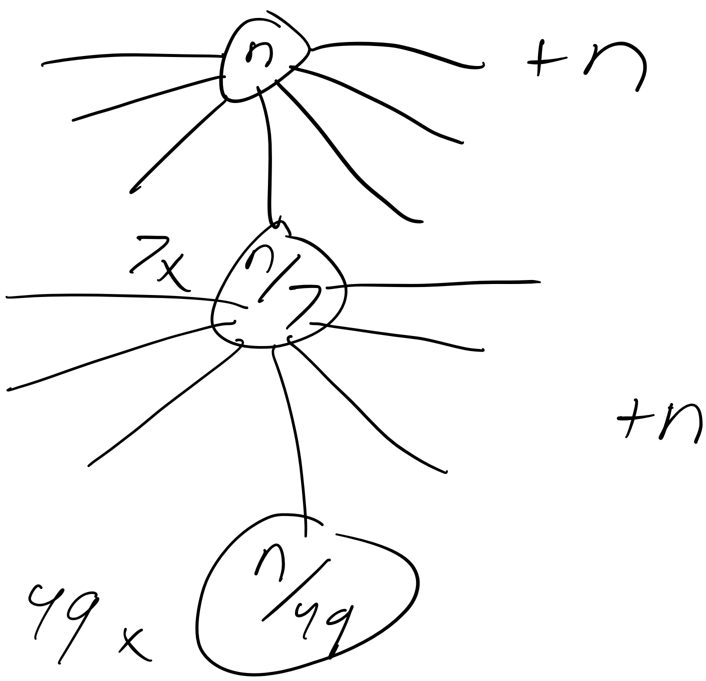
$5 \times n/16$, 25 total in level 2 $\frac{n}{4^i} = 1$, $i = \log_4 n$

$$w(n) = \sum_{i=0}^{\log_4 n} 5^i$$

$$1 + 5 + 25 + 125 + \dots + 5^{\log_4 n}$$

$$= n^{\log_4 5} \cdot n \rightarrow \boxed{O\left(n^{\log_4 5} \cdot n\right)}$$

3.) $W(n) = 7W(n/7) + n$



$$\frac{n}{7^i} = 1$$

$$i = \log_7 n$$

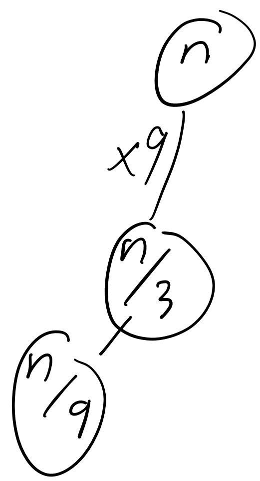
$$1 + 7 + 49 + \dots + 7^{\log_7 n} \cdot n$$

$$\sum_{i=0}^{\log_7 n} 7^i$$

$$\cancel{7}^{\log_7 n} \cdot n$$

$= n \cdot n = n^2 \rightarrow \boxed{O(n^2)}$

4.) $w(n) = 9w(n/3) + n^2$



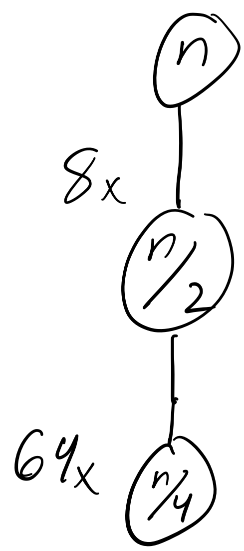
$+n^2$

x81

$+n^2$

$\frac{n}{3^i} = 1$, $i = \log_3 n \cdot n^2 \rightarrow \boxed{O(\log_3 n \cdot n^2)}$

5.) $w(n) = 8w(n/2) + n^3$



$+n^3$

$\frac{n}{2^i} = 1$

$i = \log_2 n$

$+n^3$

$\log_2 n \cdot n^3$

$\boxed{O(\log_2 n \cdot n^3)}$

6.) $w(n) = 49\,W(n/25) + n^{3/2} \log n$

n

$+n^{3/2} \log n$

$49\times \quad n/25$

$+n^{3/2} \log n$

$2401\times \quad n/125$

$$\frac{n}{25^i} = 1$$

$$i = \log_{25} n$$

$$49^i \to 49^{\log_{25} n}$$

$$\boxed{O\left(n^{\log_{25} 49} + n^{3/2} \log n\right)}$$

7.) $w(n) = w(n-1) + 2$

$n-1$

$+2$

$\frac{n-1}{2}$

$+2$

$\frac{n-1}{4}$

$$\frac{n-1}{n^i} = 1$$

$$i = \log_n (n-1)$$

$$i = \log(n)$$

$$\boxed{O(\log(n))}$$

8.) $W(n) = w(n-1) + n^c$, with $c \geq 1$

constant at each level

$\frac{n-1}{n^c} + n^c$ $\boxed{O(n^2)}$

9.) $W(n) = W(\sqrt{n}) + 1$

$\sqrt{n}$

$\downarrow$

$\frac{\sqrt{n}}{2}$

$\downarrow$

$\frac{\sqrt{n}}{4} + 1$

$\frac{\sqrt{n}}{n^i} = 1$ $\boxed{O(\log(\sqrt{n}))}$

2.)

$\boxed{A: \; w(n) = 5w(n/2) + n}$ Lowest work

B: $w(n) = 2w(n-1) + 1$

C: $wn = 9w(n/3) + n^2$

A: n

$5x \quad \frac{n}{2}$

$\vdots$

$\frac{n}{2^i} = 1$

$\log n$

$n^{\log n} = \boxed{O(n^2)}$

B: $n-1$

$\vdots$

$2x \quad 2^i + 1$

irrelevant

$\rightarrow \boxed{O(2^n)}$

C: n

$\downarrow$

$5x \quad n/3 \quad + n^2$

$\vdots$

$25x \quad n/9 \quad + n^2$

$\vdots$

$\frac{n}{3^i} \quad i = \log n$

$\boxed{O(\log(n) \cdot n^2)}$