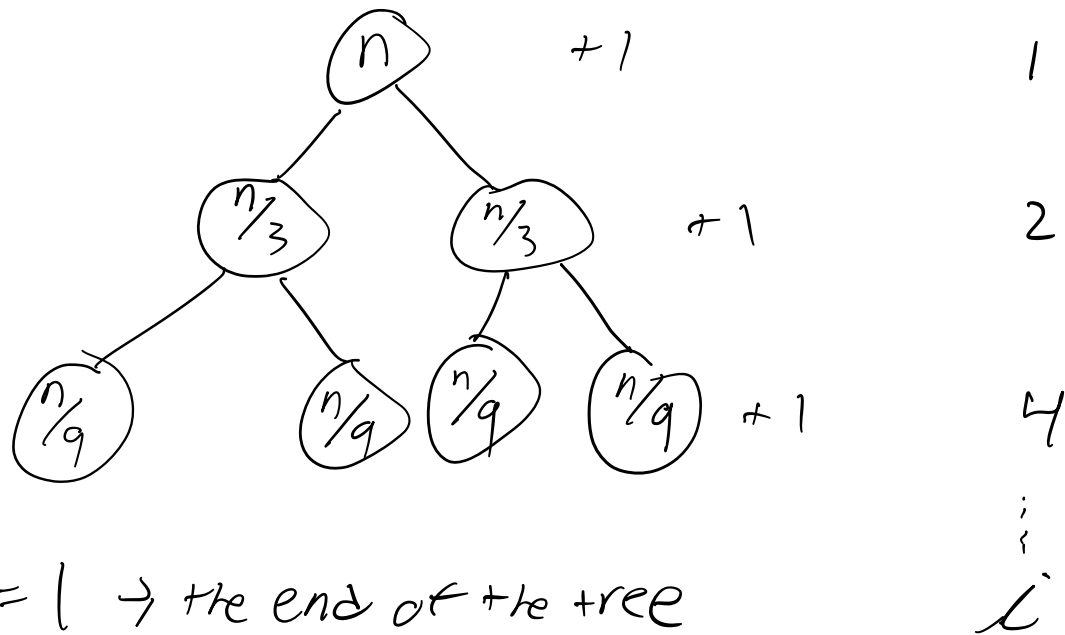


$$1.) W(n) = 2W(n/3) + 1$$

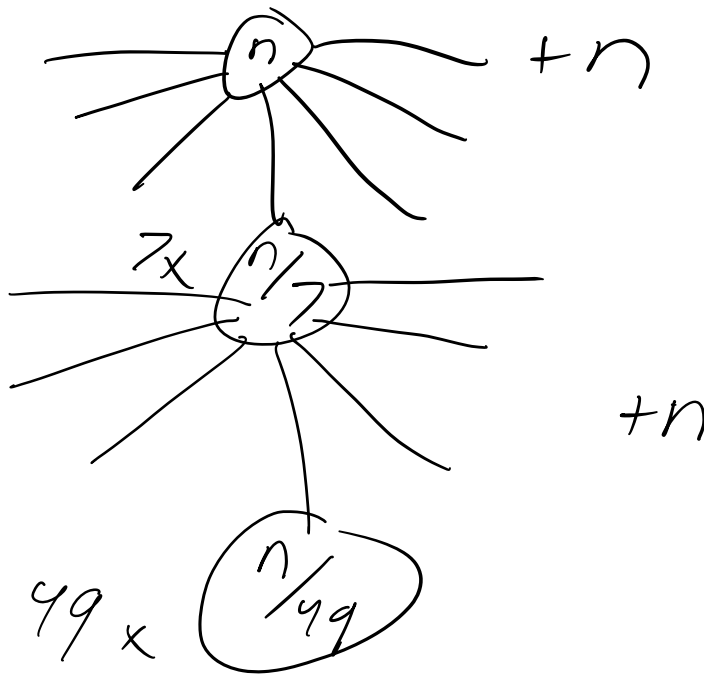


$$w(n) = \sum_{i=0}^{\log_4 n} 5^i$$

$$1 + 5 + 25 + 125 + \dots + 5^{\log_4 n}$$

$$= n^{\log_4 5} \cdot n \rightarrow O(n^{\log_4 5} \cdot n)$$

$$3.) W(n) = 7W(n/7) + n$$



$$1 + 7 + 49 + \dots + 7^{\log_7 n} \cdot n$$

$$7^{\log_7 n} \cdot n$$

$$\frac{n}{7^i} = 1$$

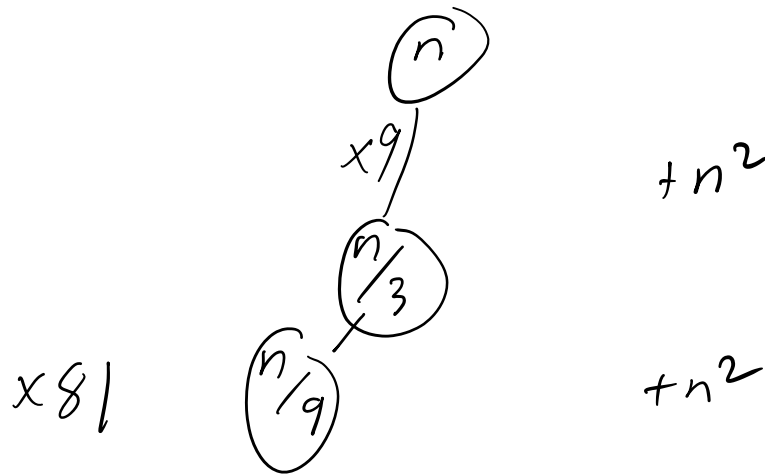
$$\bar{c} = \log_7 n$$

$$\sum_{i=0}^{\log_7 n} 7^i$$

$$\sim \dots$$

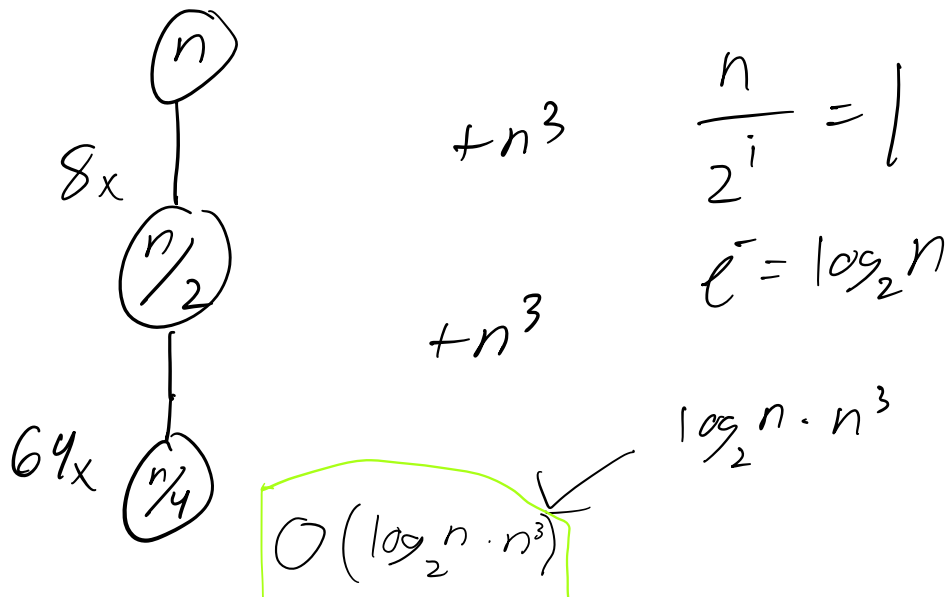
$$= n \cdot n = n^2 \rightarrow \boxed{O(n^2)}$$

$$4.) w(n) = 9w(n/3) + n^2$$



$$\frac{n}{3^i} = 1, \quad i = \log_3 n \cdot n^2 \rightarrow \boxed{O(\log_3 n \cdot n^2)}$$

$$5.) w(n) = 8w(n/2) + n^3$$



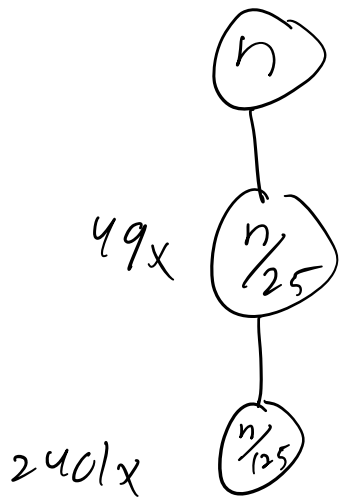
$$\frac{n}{2^i} = 1$$

$$i = \log_2 n$$

$$\log_2 n \cdot n^3$$

$$\boxed{O(\log_2 n \cdot n^3)}$$

6.) $w(n) = 49W(n/25) + n^{3/2} \log n$



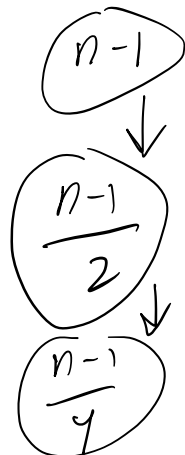
$$+ n^{3/2} \log n \quad \frac{n}{25^i} = 1$$

$$+ n^{3/2} \log n \quad i = \log_{25} n$$

$$49^i \rightarrow 49^{\log_{25} n}$$

$$O(n^{\log_{25} 49} + n^{3/2} \log n)$$

7.) $w(n) = w(n-1) + 2$



$$+ 2$$

$$\frac{n-1}{n^i} = 1$$

$$+ 2$$

$$i = \log_n (n-1)$$

$$~~i = \log(n)~~$$

$$O(n)$$

8.) $w(h) = w(h-1) + n^c$, with $c \geq 1$
constant at each level

$$\sum_{i=1}^{n-1} n^c$$

$$O(n^2)$$

$$9.) w(n) = w(\sqrt{n}) + 1$$

$$\begin{array}{c} \sqrt{n} \\ \searrow \\ \sqrt{\sqrt{n}} \\ \searrow \\ \sqrt[4]{n} \end{array} + 1$$

$$\frac{\sqrt{n}}{n^i} = 1 \quad O(\log(\sqrt{n}))$$

2.)

A: $w(n) = 5w(n/2) + n$ Lowest work

B: $w(n) = 2w(n-1) + 1$

C: $w(n) = 9w(n/3) + n^2$

A: $\begin{matrix} (n) \\ | \\ 5 \times \frac{n}{2} \\ \vdots \end{matrix}$ $\frac{n}{2^i} = 1 \quad \log n$ $n^{\log 2} = O(n^2)$

B: $\begin{matrix} n-1 \\ \vdots \\ 2^i + 1 \end{matrix}$ irrelevant $\rightarrow O(2^n)$

C: $\begin{matrix} n \\ \downarrow \\ 5 \times \frac{n}{3} \\ \vdots \\ 25 \times \frac{n}{9} \end{matrix}$ $+n^2$ $\frac{n}{3^i} \quad i = \log n$ $+n^2$ $O(\log(n) \cdot n^2)$