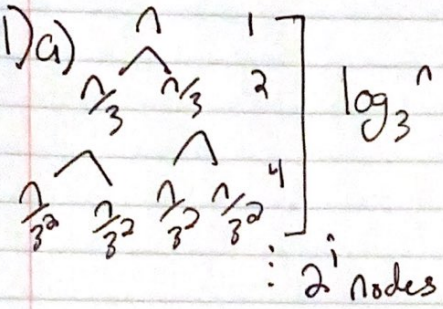
Assignment 2

1)a) 

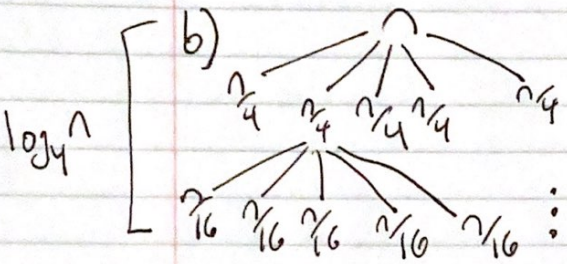
$W(n) = 2W(n/3) + 1$

cost of each node = 1

$2^i \cdot 1 = 2^i$ Cost per level

$\sum_{i=0}^{\log_3 n} 2^i = \frac{2}{1} \cdot 2^{\log_3 n} = 2 \cdot n^{\log_3 2}$

$= O(n^{\log_3 2})$

b) 

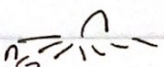
$W(n) = 5W(n/4) + n$ cost of each node = $n/4^i$

$5^i \cdot n/4^i = (\frac{5}{4})^i n$ cost per level

$\sum_{i=0}^{\log_4 n} (\frac{5}{4})^i n =$

$= O(n^{\log_4 5})$

c) $W(n) = 7W(n/7) + n$

height = $\log_7 n$ 

7^i nodes cost of each node = $n/7^i$

$7^i \cdot \frac{n}{7^i} = n$ cost per level

$\sum_{i=0}^{\log_7 n} n =$

$= O(n \cdot \log n)$

d) $W(n) = 9W(n/3) + n^2$

9^i nodes height = $\log_3 n$

$9^i \cdot \frac{n^2}{9^i} = n^2$ cost per level

cost of each node = $n^2/9^i$

Cost of tree $\sum_{i=0}^{\log_3 n} n^2 =$

$O(n^2 \cdot \log n)$

e) $W(n) = 8W(n/2) + n^3$

8^i nodes height $= \log_2 n$

cost per level $= 8^i \cdot \frac{n^3}{8^i} = n^3$

cost of each node $= \frac{n^3}{8^i}$

cost of tree $= \sum_{i=0}^{\log_2 n} n^3 = O(n^3 \cdot \log n)$

f) $W(n) = 49W(n/25) + n^{3/2}\log n$

49^i nodes height $= \log_{25} n^2$

cost per node $= (\frac{n}{25})^{3/2} \log \frac{n}{25}$

cost per level $= 49^i \cdot$

$C(0) = 49 \cdot (\frac{n}{25})^{3/2} \log \frac{n}{25}$

$= \frac{49 n^{3/2}}{125} \log \frac{n}{25}$

root dominated

so total cost is $O(C(r))$

, r being $n^{3/2}\log n$

so $O(n^{3/2}\log n)$

g) $W(n) = W(n-1) + 2$

$\sum_{i=0}^{n} 2 = O(n)$ ← constant

$W(n-1)+2 = W(n-2)+4 = \ldots$

h) $W(n) = W(n-1) + n^c$, $c \geq 1$

$W(n-1)+n^c = W(n-2) + (n-1)^c + n^c$

$= W(n-3) + (n-2)^c + (n-1)^c + n^c$

$\Rightarrow n^1 n^c = n^{c+1}$

$O(n^{c+1})$

i) $W(n) = W(\sqrt{n}) + 1$

height $= \log_2 \log_2 n$

cost per node $= 1$

1 node per level

$\Rightarrow$ cost per level $= 1$

$\sum_{i=0}^{\log_2 \log_2 n} 1 = O(\log_2 \log_2 n)$