Reagan Esteven

Master Theorem:

$T(n) = aT(n/b) + f(n)$

1\. 1. $2W(n/3)+1$

$a=2$ $\quad 2>3^0$

$b=3$ $\quad O(n^{\log_b a})$

$c=1$ $\quad \boxed{O(n^{\log_3 2})}$

$k=0$

2\. $W(n)=5W(\frac{n}{4})+n$

$a=5$ $\quad 5>4^1$

$b=4$ $\quad O(n^{\log_b a})$

$c=1$ $\quad \boxed{O(n^{\log_4 5})}$

$k=1$

3\. $W(n)=7W(\frac{n}{7})+n$

$a=7$

$b=7$ $\quad 7=7^1$

$c=1$ $\quad \boxed{O(n\log n)}$

$k=1$ $\quad O(n^k \log^n n)$

4\. $W(n)=9W(\frac{n}{3})+n^2$

$a=9$

$b=3$ $\quad 9=3^2$

$c=1$ $\quad \boxed{O(n^2\log n)}$

$k=2$

$O(n^k \log n)$

5\. $W(n)=8W(\frac{n}{2})+n^3$

$a=8$

$b=2$ $\quad 8=2^3$

$c=1$ $\quad \boxed{O(n^3\log n)}$

6\. $W(n)=49W(\frac{n}{25})+n^{\frac{3}{2}}\log n$

$a=49$

$b=25$

$f(n)=n^{\frac{3}{2}}\log n$

$k=\frac{3}{2}$

$49<25^{\frac{3}{2}}(125)$

$O(n^k \log^p n)$

$\boxed{O(n^{\frac{3}{2}}\log n)}$

7. $W(n) = W(n-1) + 2$

replace with

$W(n) = [W(n-2)+1] + 2$

$W(n) = W(n-2) + 3$

replace with $n-2$:

$W(n) = [W(n-3)+1] + 3$

$W(n) = W(n-3) + 4$

⋮ until we reach kth level

$W(n) = W(n-k) + k$

If both n & k cancel one another,

then $W(n) = W(n-n) + n$

$W(n) = 1 + n$

$\|$

$\boxed{O(n)}$

8. $W(n) = W(n-1) + n^c$ where $c \geq 1$

$(W(n-1) = W(n-2) + (n-1)^c)$

using this replace $W(n)$:

$W(n) = W(n-2) + (n-1)^c + n^c$

$W(n) = W(n-2) + (n-1)^c + n^c$

replace this with: $W(n-2)$:

$W(n) = W(n-3) + (n-2)^c + (n-1)^c + n^c$

$W(n) = W(n-3) + (n-2)^c + (n-1)^c + n^c$

⋮ continue to kth step

$W(n) = W(n-k) + (n-(k-1))^c + (n-(k-2))^c + \ldots +$
$(n-1)^c + n^2$

consider $n=k$ / then cancel one another out.

then $k=n$

$W(n) = W(n-n) + (n-n+1)^c + (n-n+2)^c + \ldots + (n+1)^2 +$
n^2

$W(n) = 1^c + 2^c + \ldots + n^c = n^{c+1}$

$\boxed{O(n^{c+1})}$

9. $W(n) = W(\sqrt{n}) + 1$

Let $n = 2^k$

thus $W(n) = W(2^{k/2}) + 1$

Using subsitution:

$T(k) = 2^k$, $T(k) = T(\frac{k}{2}) + 1$

using master theorem

$a = 1$ $\quad$ $1 = 2^0$

$b = 2$ $\quad$ $\Theta(n^k \log n)$

$c = 1$

$k = 0$ $\quad$ $T(k) = \Theta(\log n)$

put back in 2 exponent & cancel with ley

$\boxed{\Theta(\log\log n)}$

2. Algorithm a: $5W(\frac{n}{2})+n$

algorithm B: $2(n-1)+1$

algorithm C: $9(n/3)+n^2$

algorithm a: $5W(\frac{n}{2})+n$

$$T(n) = aT(n/b) + f(n)$$

$a=5$ $\quad a > b^k$

$b=2$

$c=1$ $\quad 5 > 2^1$

$k=1$ $\quad \Theta(n^{\log_2 5})$ by master theorem

algorithm B: $2(n-1)+1$

$$W(n) = \sum_{i=0}^{n-1} 2^i = \frac{2^n-1}{2-1} = 2^n - 1 = \Theta(2^n)$$

Since $W(n) \le c2^n + n$

$$W(n) \le 2 \cdot c2^{n-1} + (n-1) + 1$$
$$= c2^n + n$$
$$= \boxed{O(2^n)}$$

algorithm C: $9(n/3)+n^2$

$T(n) = aT(n/b) + f(n)$

$a = 9$ $\qquad a = b^k \; (9 = 3^2)$

$b = 3$ $\qquad \| $

$c = 1$ $\qquad O(n^k \log n)$

$k = 2$ $\qquad \|$

$$\boxed{O(n^2 \log n)}$$

I would choose the algorithm B because its the fastest.