Assignment 2

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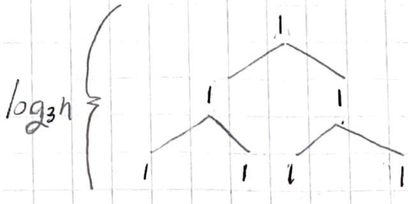
1) a) $W(n) = 2W(n/3) + 1$



$1(c_1)$

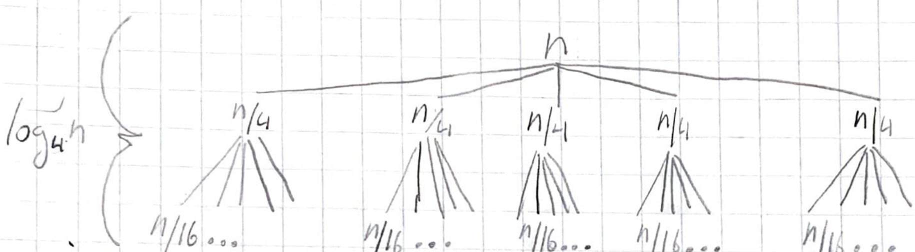
$2(c_1)$

$4(c_1)$

$$W(n) = \sum_{i=0}^{\log n} (2^i c_1) < nc_1 \in O(n)$$

$\boxed{W(n) \in O(n)}$

b) $W(n) = 5W(n/4) + n$



$c_1 n + c_2$

$5(n/4 \, c_1 + c_2)$

$25(n/16 \, c_1 + c_2)$

$$W(n) = \sum_{i=0}^{\log_4 n} \left(5^i \left(\frac{n}{4^i} c_1 + c_2\right)\right) < nc_2 \log_5 n + \frac{5}{4} nc_2$$

$\boxed{W(n) \in O(n \log n)}$

c) $W(n) = 7W(n/7) + n$

level 1: $c_1 n + c_2$

Depth $= \log_7 n$

level 2: $7(n/7 \, c_1 + c_2) = nc_1 + 7c_2$

level 3: $49(n/49 \, c_1 + c_2) = nc_1 + 49c_1$

$$W(n) = \sum_{i=0}^{\log_7 n} (nc_1 + 7^i c_2) < nc_2 \log_7 n + 7c_2 \in O(n \log_7 n)$$

$\boxed{W(n) \in O(n \log n)}$

d) $W(n) = 9W(n/3) + n^2$ level 1: $c_1 n^2 + c_2$

Depth = $\log_3 n$ level 2: $(c_1 (n/3)^2 + c_2) 9 = \frac{n^2}{3} c_1 + 9 c_2$

level 3: $(c_1 (n/9)^2 + c_2) 81 \quad \frac{n^2}{9} c_1 + 81 c_2$

$$W(n) = \sum_{i=0}^{\log_3 n} \frac{n^2}{3^i} + 9^i c_2 < 3n^2 c_1 + 9n c_2 \in O(n^2) \quad \boxed{W(n) \in O(n^2)}$$

e) $W(n) = 8W(n/2) + n^3$ l1: $c_1 n^3 + c_2$

Depth = $\log_2 n$ l2: $8(c_1 (n/2)^3 + c_2) = \frac{n^3}{2} c_1 + 8 c_2$

l3: $64(c_1 (n/4)^3 + c_2) = \frac{n^3}{4} c_1 + 64 c_2$

$$W(n) = \sum_{i=0}^{\log n} \frac{n^3}{2^i} c_1 + 8^i c_2 < 2n^3 c_1 + 8n c_2 \in O(n^3) \quad \boxed{\begin{array}{c} W(n) \in O(n^c) \\ c > 3 \end{array}}$$

f) $W(n) = 49(n/25) + n^{\frac{3}{2}} \log n$

Depth = $\log_{25} n$ l1: $n^{3/2} \log n + c_2$

l2: $49\left(\frac{n^{3/2}}{25} \log(n/25) c_1 + c_2\right) = \frac{49^{3/2}}{25} \log\left(\frac{n}{25}\right) c_1 + 49 c_2$

$$W(n) = \sum_{i=0}^{\log_{25} n} \left(\frac{n^{3/2}}{25^i} \log\left(\frac{n}{25^i}\right) c_1 + 49^i c_2 \right) = \sum \left(\frac{n^{3/2}}{25^i} \log\left(\frac{n}{25^i}\right) c_1 \right) + \sum \left(49^i c_2 \right)$$

$$W(n) < 25 n^{3/2} (\log^2 n + \log n) + 49 n c_2 \Rightarrow 25 n^{3/2} (\log^2 n + \log n)$$ *reduce constant*

$$\boxed{W(n) \in O(\log^2 n)}$$

g) $W(n) = W(n-1) + 2$

$$W(n) = \sum_{i=1}^{n} 2 = n(n-1) \in O(n)$$ $\boxed{W(n) \in O(n)}$

h) $W(n) = W(n-1) + n^c$

$$W(n) = \sum_{i=1}^{n} i^c = \frac{n^c(n^c-1)}{2} \in O(c^n)$$ $\boxed{W(n) \in O(c^n)}$

i) $W(n) = W(\sqrt{n}) + 1 \Rightarrow W(n^{1/2}) + 1$ $\boxed{W(n) \in O(\log n)}$

2)

A: $W(n) = W(n/2) + n$

B: $W(n) = 2W(n-1) + 1$

C: $W(n) = 9W(n/3) + O(n^2)$

- 1b shows that the $w(n)$ for algo A is $\in O(n^2)$
- 1g shows that the $w(n)$ for algo B is $\in O(n)$
- 1e shows that $w(n)$ for algo C is $\in O(n^c)$ such that c is a constant where $c > 2$

I would chose Algo B because $O(n^2)$ and $O(n^c)$ are strictly dominated by $O(n)$.