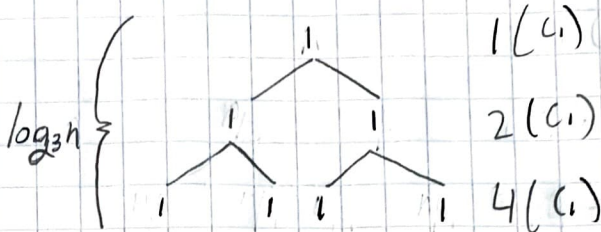


Assignment 2

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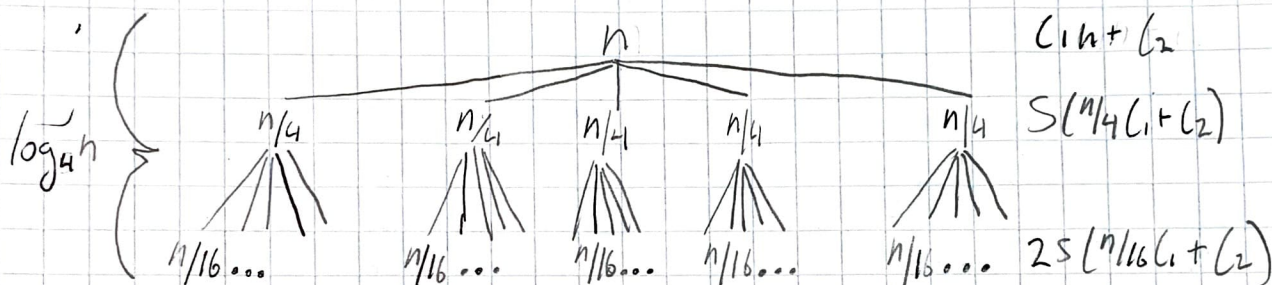
1) a) $W(n) = 2W(n/3) + 1$



$$W(n) = \sum_{i=0}^{\log_3 n} (2^i c_1) < n c_1 \in O(n)$$

$$W(n) \in O(n^{\log_3 2})$$

b) $W(n) = 5W(n/4) + n$



$$W(n) = \sum_{i=0}^{\log_4 n} \left(5^i \left(\frac{n}{4^i} (c_1 + c_2) \right) \right) < n c_2 \log_5 n + \frac{5}{4} n c_2 \quad \boxed{W(n) \in O(n^{\log_4 5})}$$

c) $W(n) = 7W(n/7) + n$

level 1: $c_1 n + c_2$

Depth = $\log_7 n$

level 2: $7(n/7(c_1 + c_2)) = n c_1 + 7 c_2$

level 3: $49(n/49(c_1 + c_2)) = n c_1 + 49 c_2$

$$W(n) = \sum_{i=0}^{\log_7 n} (n c_1 + 7^i c_2) < n c_2 \log_7 n + 7 c_2 \in O(n \log_7 n)$$

$$\boxed{W(n) \in O(n \log_7 n)}$$

$$d) \quad W(n) = 9W(n/3) + n^2 \quad \text{level 1: } C_1 n^2 + C_2$$

$$\text{Depth} = \log_3 n$$

$$\text{level 2: } (C_1 (n/3)^2 + C_2) 9 = \frac{n^2}{3} C_1 + 9C_2$$

$$\text{level 3: } (C_1 (n/9)^2 + C_2) 81 = \frac{n^2}{9} C_1 + 81C_2$$

$$W(n) = \sum_{i=0}^{\log_3 n} \frac{n^2}{3^i} + 9^i C_2 < 3n^2 C_1 + 9n C_2 \in O(n^2) \quad \boxed{W(n) \in O(n^2 \log_3 n)}$$

$$e) \quad W(n) = 8W(n/2) + n^3 \quad \text{level 1: } C_1 n^3 + C_2$$

$$\text{Depth} = \log_2 n$$

$$\text{level 2: } 8(C_1 (n/2)^3 + C_2) = \frac{n^3}{2} C_1 + 8C_2$$

$$\text{level 3: } 64(C_1 (n/4)^3 + C_2) = \frac{n^3}{4} C_1 + 64C_2$$

$$W(n) = \sum_{i=0}^{\log_2 n} \frac{n^3}{2^i} C_1 + 8^i C_2 < 2n^3 C_1 + 8n C_2 \in O(n^3) \quad \boxed{O(n^3 \log_2 n)}$$

$$f) \quad W(n) = 49(n/25) + n^{3/2} \log n$$

$$\text{Depth} = \log_{25} n$$

$$\text{level 1: } n^{3/2} \log n + C_2$$

$$\text{level 2: } 49 \left(\frac{n^{3/2}}{25} \log(n/25) C_1 + C_2 \right) = \frac{49^{3/2}}{25} \log\left(\frac{n}{25}\right) C_1 + 49C_2$$

$$W(n) = \sum_{i=0}^{\log_{25} n} \left(\frac{n^{3/2}}{25^i} \log\left(\frac{n}{25^i}\right) C_1 + 49^i C_2 \right) = \sum \left(\frac{n^{3/2}}{25^i} \log\left(\frac{n}{25^i}\right) C_1 \right) + \sum (49^i C_2)$$

$$W(n) < 25n^{3/2} (\log^2 n + \log n) + 49n C_2 \Rightarrow 25n^{3/2} (\log^2 n + \log n) \quad \text{* reduce constant *$$

$$\boxed{W(n) \in O(n^{3/2} \log n)}$$

g) $W(n) = W(n-1) + 2$

$$W(n) = \sum_{i=1}^n 2 = n(n-1) \in O(n)$$

$$\boxed{W(n) \in O(n)}$$

h) $W(n) = W(n-1) + n^c$

$$W(n) = \sum_{i=1}^n i^c = \frac{n^{c+1} - 1}{c+1} \in O(n^{c+1})$$

$$\boxed{W(n) \in O(n^{c+1})}$$

i) $W(n) = W(\sqrt{n}) + 1 \Rightarrow W(n^{1/2}) + 1$

$$\boxed{W(n) \in O(\log(\log(n)))}$$

2)

A: $W(n) = 5W(n/2) + n$

B: $W(n) = 2W(n-1) + 1$

C: $W(n) = 9W(n/3) + O(n^2)$

- 1b shows that the $W(n)$ for algo A is $\in O(n^{\log_2 5})$

- 1g shows that the $W(n)$ for algo B is $\in O(n^2)$

- 1c shows that $W(n)$ for algo C is $\in O(n^2 / \log_3 n)$

I would choose Algo C because $W(n)$ A and $W(n)$ B are strictly dominated by $W(n)$ C.