David Webster

Assignment 1

1a. $\lim_{n\to\infty} \frac{2^{n+1}}{2^n} = 2 \lim_{n\to\infty} \frac{2^n}{2^n} = 2$

Thus $2^{n+1} \in \Theta(2^n)$

$\rightarrow 2^{n+1} \in O(2^n)$ ✓

1b. $\lim_{n\to\infty} \frac{2^{2^n}}{2^n} \rightarrow \lim 2^{2^n - n} = \infty$

since $2^n > n$ for $n \geq 4$

so $\lim_{n\to\infty} 2^n - n = \infty$

$2^{2^n} \notin O(2^n)$, $2^{2^n} \in \Omega(2^n)$

1c. $\lim_{n\to\infty} \frac{n^{1.01}}{(\log n)^2} \rightarrow \lim \frac{1.01 n^{\frac{1}{100}}}{2(\log n)\cdot \frac{1}{n}} \rightarrow \frac{1.01 n^{1.01}}{2\log n}$

apply l'hopital   apply l'hopital again.

$\rightarrow \frac{2.02 n^{0.01}}{2\cdot\frac{1}{n}} \rightarrow 1.01 n^{1.01} = \infty$

thus, $n^{1.01} \notin O(\log^2 n)$

1.d based on the previous question since the limit method produced ∞,

$n^{1.01} \in \Omega(\log^2 n)$ is true.

1e.

$$\lim_{n\to\infty} \frac{\sqrt{n}}{(\log n)^3} \to \frac{\frac{1}{2} n^{-\frac{1}{2}}}{3(\log n)^2 \cdot \frac{1}{n}} \to \frac{n^{\frac{1}{2}}}{6(\log n)^2}$$

apply lhopital   apply lhopital

$$\to \frac{\frac{1}{2} n^{-\frac{1}{2}}}{12(\log n)\cdot\frac{1}{n}} \to \frac{n^{\frac{1}{2}}}{24 \log n} \to \frac{\frac{1}{2} n^{-\frac{1}{2}}}{24 \frac{1}{n}} \to \frac{\sqrt{n}}{48} = \infty$$

apply lhopital

Since the limit method produces ∞ ~~the~~

$\sqrt{n} \notin O((\log n)^3)$,

but

1f. $\sqrt{n} \in \Omega((\log n)^3)$

2.3. The function is a recursive function ~~with~~ base case of $x \le 1$, return x, it then ~~calculates~~ calls the function on the two previous numbers $x-1$, $x-2$ and assigns them to ra, rb respectively, Then adds them together.