1. $w(n) = 2w(n/3) + 1$

1

$\frac{2n}{3}$ $\qquad 2^i \frac{n}{3^i} \qquad - \sum_{i=0}^{\log_3 n} 2^i \Rightarrow n^{\log_3 2}$

$\frac{4n}{9}$

$\vdots$

$$\boxed{O\left(n^{\log_3 2}\right)}$$

2. $w(n) = 5w(n/4) + n$

n

$5 \quad \frac{5n}{4} \qquad 5^i \frac{n}{4^i} \Rightarrow n \sum_{i=0}^{\log_4 n} \left(\frac{5}{4}\right)^i = \frac{\frac{5}{4}}{\frac{1}{4}} \cdot \left(\frac{5}{4}\right)^{\log_4 n}$

$\cancel{10} \quad \frac{10n}{16} \qquad 5 \cdot n^{\log_4 5}$

$n^{\log_4 5}$

$$\boxed{O\left(n^{\log_4 5}\right)}$$

3. $w(n) = 7(w(n/7) + n$

balanced.

n

$n \quad - \sum_{i=0}^{\log_7 n} n \Rightarrow n \sum_{i=0}^{\log_7 n} 1 \Rightarrow n \log n$

$\vdots$

$$\boxed{O(n \log n)}$$

4. $w(n) = 9(w(n/3) + n^2$

$\rightarrow 9^i \frac{n^2}{3^i} \rightarrow \quad 9^i \cdot \left(\frac{n}{3^i}\right)^2 \quad \sum_{i=0}^{\log_3 n} \frac{9^i}{(3^i)^2} \cdot n^2$

$n^2 \cdot \frac{9^{\log_3 n}}{(3^{\log_3 n})^2} \sim \frac{n^{\log_3 9}}{(n^{\log_3 3})^2} \quad \frac{n^{\log_3 9}}{n^2} = 1$

$n^2 \sum_{i=0}^{\log} 1 \rightarrow n^2 \log n$

$$\boxed{O(n^2 \log n)}$$

$W(n) = 8(n/2) + n^2$

$\rightarrow 8^i \left(\frac{n}{2^i}\right)^3 \quad \cancel{8^i} \frac{n^3}{\cancel{8^i}} \rightarrow n^3 \rightarrow \sum_{i=0}^{\log_2 n} n^2 = n^3 \log n$

$\boxed{O(n^3 \log n)}$

$W(n) = 49W(n/25) + n^{3/2} \log n$

$49^i \left(\frac{n}{25^i}\right)^{3/2} \log\left(\frac{n}{25^i}\right) \rightarrow n^{3/2} \left(\frac{49}{25^3}\right)^i \left(\log n - \log 25^i\right)$

$n^{3/2} \left(\frac{a^i}{5^2}\right)^i \cdot \left(\log n - \log 25^i\right)$

first term

$49 \rightarrow 45$

$n^{3/2} \left(\left(\frac{9}{25}\right)^i \log n - \left(\frac{9^i}{25}\right) \log(25^i)\right)$

$n^{\frac{3}{2}} \left(\left(\frac{9}{25}\right)^i \log n - \left(\frac{9}{25}\right)^i \cdot i \log 25\right)$

$\rightarrow n^{3/2} \left(\left(\frac{49}{125}\right)^i \left(\log n - i \log 25\right)\right)$

brick roots $\rightarrow$ root dominated

$\boxed{O(n^{\frac{3}{2}} \log n)}$

$W(n-1) + 2 = W(n)$

$2 \rightarrow$ balanced

$2 \rightarrow \sum_{i=0}^{n} 2 \quad 2 \sum_{i=0}^{n} 1 \le 2n$

$\boxed{O(n)}$

$W(n) = W(n-1) + n^c$, $c \geq 1$

n^c

n^c

$\vdots$

balance? $\sum_{i=0}^{n} n^c \rightarrow n^c \sum_{i=0}^{n} 1 < n^c n$

n^{c+1}

$\boxed{O(n^{c+1})}$

$W(n) = W(\sqrt{n}) + 1$

1

1

1

balance: $\sum_{i=0}^{\sqrt{n}} 1$ $\sum_{i=0}^{n^{\frac{1}{2}}} 1$

$x: \sqrt{n} \Rightarrow \underbrace{(n/2)(2)(2)\ldots}_{x}$

$n^{\frac{1}{2}}$

$n^{\frac{1}{2^i}}$

$x_i = n^{\frac{1}{2^i}} \Rightarrow \underbrace{((n/2)/2)(2)}_{x_0}\underbrace{/2)}_{x_1}/2) \ldots$

$n \rightarrow 1$ $(n/2^{i_1})/2^{i_2}) \ldots$

from these sequences we see

the length is log of log. nested log

$\sum_{i=0}^{\log\log n} 1 < \log \log n$

$\boxed{O(\log(\log n))}$

A, $W(n) = 5(n/2) + n$

$\left(\frac{5}{2}\right)^i n$ $n\left(\frac{5}{2}\right)^{\log_2 n}$ $n\left(\frac{5^{\log n}}{n}\right) = n^{\log 5}$

$\boxed{O(n^{\log_2 5})}$

B $W(n) = 2W(n-1) + 1$

1

2

4

2^i $\sum_{i=0} 2^i \Rightarrow 2^n$

$\boxed{O(2^n)}$

C. $W(n) = 9C(n/3) + n^2$

$9\left(\frac{n}{3}\right)^2 \quad n \quad \sum_{i=0}^{\log_3 n} n^2$

$n^2 \log n$

$\boxed{O(n^2 \log n)}$

$n^2 \log n < n^{\log_2 5} < 2^n$

thus algorithm C is best

$\boxed{\text{choose } C}$