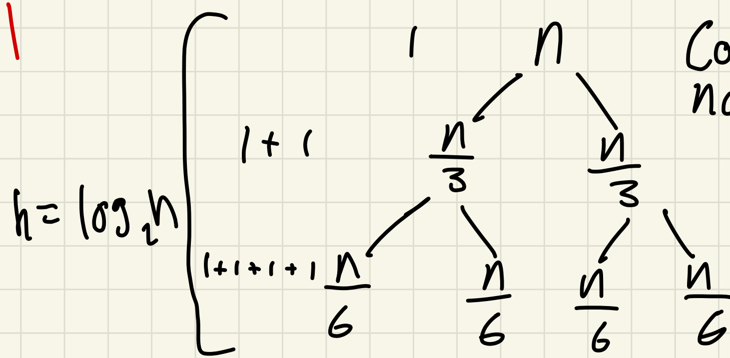
Work for assignment 2

1) $W(n) = 2W(n/3) + 1$

$$\boxed{2W(n/3)+1 \in O(n)}$$

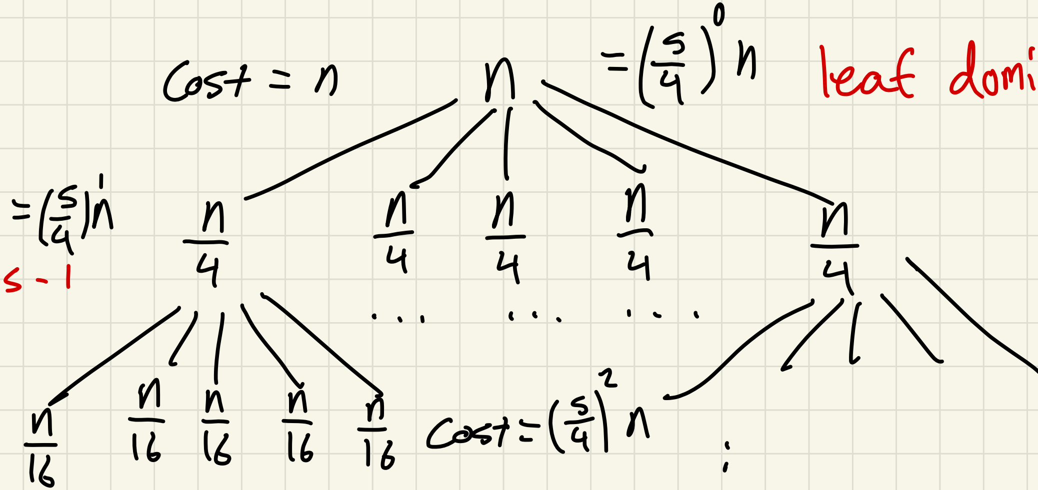


Cost of each node = 1

Number of nodes = n

2) $W(n) = 5W(n/4) + n$

leaf dominated



Cost $= \left(\frac{5}{4}\right)^1 n$

Cost of leaf level:

$$n\left(\frac{5}{4}\right)^{(\log_4 n - 1)}$$ ← num levels - 1

$$= n^{\log_4\left(\frac{5}{4}\right)}$$

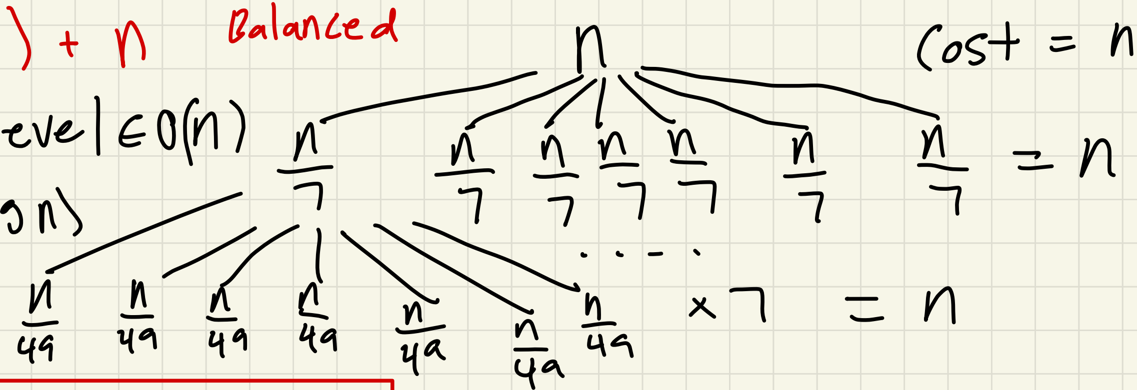
$$n(n-1) \in O(n^2)$$

$$\boxed{5W(n/4)+n \in O(n^2)}$$

3) $W(n) = 7W(\frac{n}{7}) + n$ Balanced

Cost of each level $\in O(n)$

num levels $\in O(\log n)$



$$\boxed{W(n) = 7W(\frac{n}{7}) + n \in O(n \log n)}$$

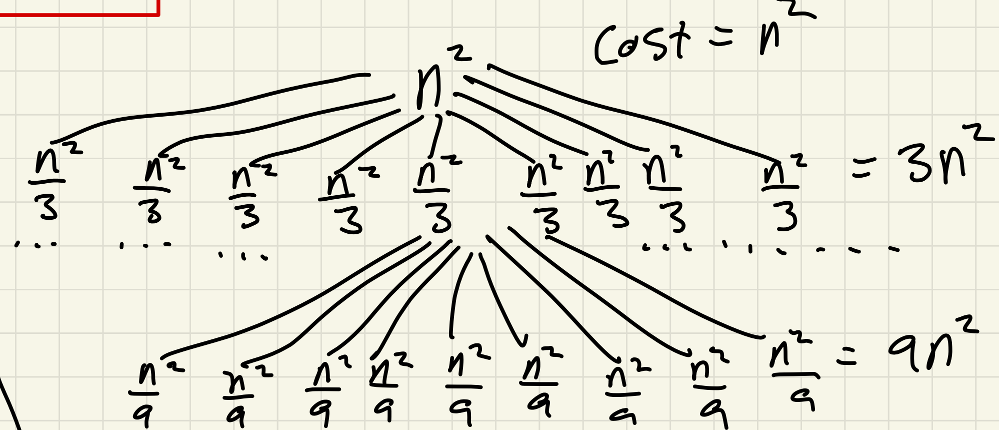
4) $W(n) = 9W(\frac{n}{3}) + n^2$

leaf dominated

leaf level =

$$3^{(\log_3 n - 1)} \cdot n^2$$

$$(n-1)(n^2) \in \boxed{O(n^3)}$$



5) $W(n) = 8\,W\left(\frac{n}{2}\right) + n^3$

leaf dominated

last level $= 4^{i-1} \cdot n^3$

$i = \log_2 n$

$4^{\log_2 n - 1} \Rightarrow n^{\log_2 4 - 1}$

$= n^{2-1} = n$ $n \cdot n^3 = n^4$

$n^3 \quad = n^3$

$\frac{n^3}{2} \quad \frac{n^3}{2} \quad \frac{n^3}{2} \quad \frac{n^3}{2} \quad \frac{n^3}{2} \quad \frac{n^3}{2} \quad \frac{n^3}{2} \quad \frac{n^3}{2} \quad = 4n^3$

$\log_2 n$

$\vdots$

$= 16n^3$

$W(n) = 8W\left(\frac{n}{2}\right) + n^3 \in O(n^4)$

6) $W(n) = 49W(n/25) + n^{3/2}\log n$

last level $i = \log_{25} n$

$\left(\frac{49}{25}\right)^{\log_{25} n}$

$= n^{\log_{25}\left(\frac{49}{25}\right)} \cdot n^{3/2}\log n$

$\approx n^{.2} \cdot n^{3/2}\log n$

$\in O(n^2 \log n)$

$n^{3/2}\log n$

$\downarrow$

$\frac{49}{25}\, n^{3/2}\log n$

$\downarrow$

$\left(\frac{49}{25}\right)^2$

$\vdots$

$\left(\frac{49}{25}\right)^i n^{3/2}\log n$

7) $W(n) = W(n-1) + 2$

$= 2n \in O(n)$

$$n\left\{\begin{array}{ll} n = 2 & \\ | & + \\ n-1 = 2 & \\ | & + \\ n-2 = 2 & \\ \vdots & + \end{array}\right.$$

8) $W(n) = W(n-1) + n^c$ where $c \geq 1$

$\in O(n^{1+c})$

$$n\left\{\begin{array}{l} n = n^c \\ \downarrow \\ n-1 = (n-1)^c \\ \downarrow \\ n-2 = (n-2)^c \\ \vdots \end{array}\right.$$

9) $W(n) = W(\sqrt{n}) + 1$

$i =$

$O(\log(\log n))$

for any number $\leq 2^i$ it takes i steps to reach base case

ex) $2^4 = 16$ $\quad \sqrt{16} = 4$ $\quad \sqrt{4} = 2$

2) Algorithm A: $W(n) = 5W(n/2) + n$

Algorithm B: $W(n) = 2W(n-1) + 1$

Algorithm C: $W(n) = 9W(\frac{n}{3}) + n^2$

A:

$\log_2 n$ levels:

$n \quad = n$

$\frac{n}{2} \quad \frac{n}{2} \quad \frac{n}{2} \quad \frac{n}{2} \quad \frac{n}{2} \quad = \frac{5n}{2}$

$\cdots \quad \cdots \quad = \left(\frac{5}{2}\right)^2 n$

leaf dominated

$$\left(\frac{5}{2}\right)^{i-1} n = (n)^{\log_2 \frac{5}{2}} \cdot n$$

$$n^{2\log \frac{5}{2}} = n^{\log \frac{25}{4}}$$

$$\in O(n^3)$$

B:

n levels:

$n+1 \quad 2^0$

$n-1 \quad n-1 \quad 2 \quad 2^1$

$\cdots \quad \cdots \quad \cdots \quad 4 \quad 2^{i-1}$

$$2^{n-1}$$

$$\in O(2^n)$$

C: $W(n) = 9W\left(\frac{n}{3}\right) + n^2$, already did this, $\in O(n^3)$