1. $W(n) = 2W(n/3)+1$

$c(\text{root}) = 1$

$c(\text{lvl } 1): 1/3 + 1/3 = 2/3$, $1 > 2/3$, balanced

# of levels: $\log_3 n+1$, $1 \cdot (\log_3 n+1) \to O(\log n)$

$W(n) = 5W(n/4) + n$

$c(\text{root}) = n$

$c(\text{lvl } 1): \frac{n}{4} + \frac{n}{4} + \frac{n}{4} + \frac{n}{4} + \frac{n}{4} = \frac{5n}{4} > n$ leaf dominated

# of leaves $= 5^{\log_4 n} = n^{\log_4(5)}$ $\quad W(n) = O(n^{\log_4 5})$

$W(n) = 7W(n/7) + n$

$c(\text{root}): n$

$c(\text{lvl } 1) = \frac{7n}{7} = n$ balanced

# of leaves $= 7^{\log_7 n} = n^{\log_7(7)} = n = O(n)$

$W(n) = 9W(n/3) + n^2$

$c(\text{root}) = n^2$

$c(\text{lvl } 1) = \frac{9n^2}{3} = 3n^2 > n^2$

# of leaves $= 9^{\log_3 n} = n^{\log_3 9} = n^2$, $O(n^2)$

$W(n) = 8W(n/2) + n^3$

$c(\text{root}) = n^3$

$c(\text{lvl } 1) = \frac{8n^3}{2} = 4n^3 > n^3$

# of leaves $= n^{\log_2 8} = n^3$ $\quad W(n) = O(n^3)$

$W(n) = 49W(n/25) + n^{3/2}\log n$

$c(\text{root}) = n^{3/2}\log n$

$c(\text{lvl } 1) = \frac{49(n^{3/2}\log n)}{25} > n^{3/2}\log n$, leaf dominated

# of leaves $= n^{\log_{25}(49)}$, $W(n) = O(n^{\log_{25}(49)})$

$W(n) = W(n-1) + 2$ Balanced

$W(n) = 2n \in O(n)$

$W(n) = W(n-1) + n^c$ Balanced

$W(n) = n \cdot n^c = n^{c+1} \in O(n^c)$

$W(n) = W(2\sqrt{n}) + 1$ Root dominated

Root: 1 $W(n) = O(1)$

Level 1: $\sqrt{1}$

2.

$A = 5A(n/2) + O(n) = 5A(n/2) + c_1 n + c_2$

$B = 2B(n-1) + O(1) = 2B(n-1) + c_1 1 + c_2$

$C = 9C(n/3) + O(n^2) = 9C(n/3) + c_1 n^2 + c_2$

$A = 5A(n/2) + c_1 n + c_2$

$C(\text{Root})$ $c_1 n + c_2$

$C(\text{Lvl } 1)$: $\frac{5(c_1 n + c_2)}{2} > c_1 n + c_2$, leaf dominated

leaves $= n^{\log_2(5)}$ $O(n^{\log_2 5})$

$B = 2B(n-1) + c_1 \cdot 1 + c_2$ Balanced

n levels, largest level is $c_1 \cdot 1 + c_2$, $O(1)$

$C = 9C(n/3) + c_1 n^2 + c_2$

$C(\text{root}) = c_1 n^2 + c_2$

$C(\text{lvl } 1) = \frac{9(c_1 n^2 + c_2)}{3} = 3(c_1 n^2 + c_2) > n^2$, leaf dominated

# of leaves $= n^{\log_3 9} = n^2$, $O(n^2)$

Alg B, since its constant time