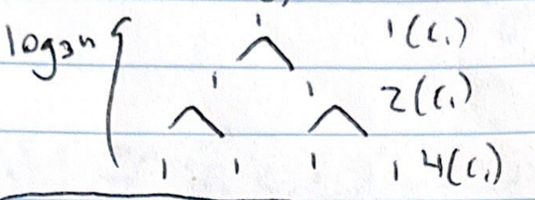
Tanner Martz
3/5/23

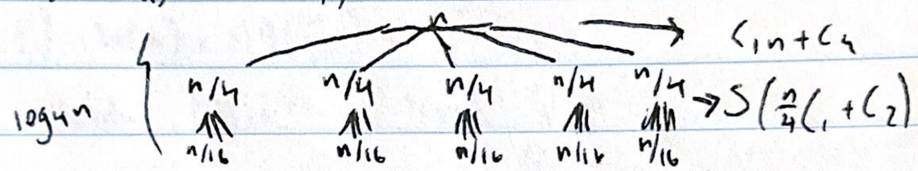
Assigment 2

1) a) $W(n) = 2W(\frac{n}{3}) + 1$



$W(n) \in O(n^{\log_3 2})$ $\quad W(n) = \sum_{i=0}^{\log_3 n} (2^i c_1) < nc_1$

b) $W(n) = 5W(\frac{n}{4}) + n$



$25(\frac{n}{16}c_1 + c_2)$

$w(n) = \sum_{i=0}^{\log_4 n} (5^i (\frac{n}{4^i} c_1 + c_2)) < nc_2 \log_3 n + \frac{5}{4} nc_2$

$V(n) \in O(n^{\log_4 5})$

c) $W(n) = 7W(\frac{n}{7}) + n$

level:
① $c_1 n + c_2$

Depth $\log_7 n$

② $7(\frac{n}{7}c_1 + c_2) = nc_1 + 7c_2$

③ $49(\frac{n}{49}c_1 + c_2) = nc_1 + 49c_1$

$V(n) = \sum_{i=0}^{\log_7 n} (nc_1 + 7c_2) < nc_2 \log_7 n + 7c_2 \in O\, n\log_7 n$

$V(n) \in O(n \log_7 n)$

d) $W(n) = 9w(\frac{n}{3}) + n^2$

level
① $c_1 n^2 + c_2$

Depth $\log_3 n$

② $(c_1(\frac{n}{3})^2 + c_2)9 = \frac{n^2}{3}c_1 + 9c_2$

③ $(c_1(\frac{n}{9})^2 + c_2)81 \quad \frac{n^2}{9}c_1 + 81c_2$

$W(n) = \sum_{i=0}^{\log_3 n} \frac{n^2}{3^i} + 9^i c_2 < 3n^2 c_1 + 9nc_2 \in O(n^2)$

$W(n) \in O(n^2 \log_3 n)$

Depth $\log_2 n$

e) $W(n) = 8W\left(\frac{n}{2}\right) + n^3$

level:

① $c_1 n^3 + c_2$

② $8\left(c_1\left(\frac{n}{2}\right)^3 + c_2\right) = \frac{n^3}{2}c_1 + 8c_2$

③ $64\left(c_1\left(\frac{n}{4}\right)^3 + c_2\right) = \frac{n^3}{4}c_1 + 64c_2$

$W(n) = \sum_{i=0}^{\log n} \frac{n^3}{8^i}c_1 + 8^i c_2 < 2n^3 c_1 + 8nc_2 \in O(n^3)$

$O(n^3 \log_2 n)$

f) $W(n) = 49\left(\frac{n}{25}\right) + n^{\frac{3}{2}} \log n$

Depth $\log_{25} n$

level:

① $n^{\frac{3}{2}} \log n + c_2$

② $49\left(\frac{n^{3/2}}{25}\log\left(\frac{n}{25}\right)c_1 + c_2\right) = \frac{49^{3/2}}{25}\log\frac{n}{25}c_1 + 49c_2$

$W(n) = \sum_{i=0}^{\log_{25} n}\left(\frac{n^{\frac{3}{2}}}{25^i}\log\left(\frac{n}{25^i}\right)c_1 + 49^i c_2\right) = \sum\left(\frac{n^{\frac{3}{2}}}{25^i}\log\left(\frac{n}{25^i}\right)c_1\right) + \sum\left(49^i c_2\right)$

$w(n) < 25n^{\frac{3}{2}}\left(\log^2 n + \log n\right) + 49nc_2 \Rightarrow 25n^{\frac{3}{2}}\left(\log^2 n + \log n\right)$

$W(n) \in O\left(n^{\frac{3}{2}} \log n\right)$

g) $W(n) = W(n-1) + 2$, $W(n) = \sum_{i=1}^{n} 2 = n(n-1) \in O(n)$

$W(N) \in O(n)$

h) $W(n) = W(n-1) + n^c$

$W(n) = \sum_{i=1}^{n} i^c = \frac{n^c(n^c-1)}{2} \in O(c^n)$ $W(n) \in O(n^{c+1})$

i) $W(n) = W(\sqrt{n}) + 1 \Rightarrow W(n^{1/2}) + 1$ $w(n) \in O(\log(\log(n)))$

2) A: $W(n) = 5w_1\left(\frac{n}{2}\right) + n$

B: $W(n) = 2w(n-1) + 1$

C: $W(n) = 9w\left(\frac{n}{3}\right) + O(n^2)$

1b demonstrates $W(n)$ for A is $\in O(n^{\log_2 5})$

1g demonstrates $W(n)$ for B is $\in O(n^2)$

1e demonstrates $W(n)$ for C is $\in O(n^2 \log_3 n)$

Choose Algorithm C due to $W(n)A$ and $W(n)B$ are dominated by $W(n)C$.