CMPS 2200 Assignment 2
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1.) a) $W(n) = 2W(n/3) + 1$

$$W(n) = \sum_{i=0}^{\log_2 n} (2^i c_1) < n c_1$$

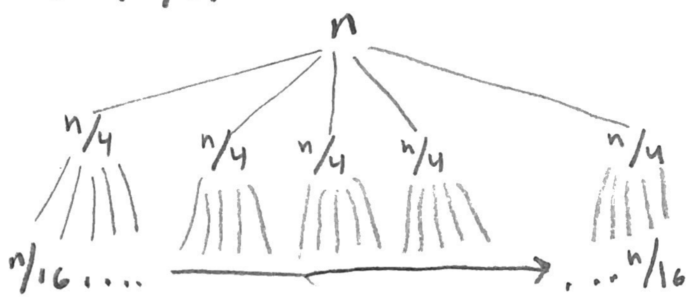
$\log_3 n$

$1(c_1)$
$2(c_1)$
$4(c_1)$

$W(n) \in O(n^{\log_3 2})$

b) $W(n) = 5W(n/4) + n$

$\log_4 n$



$c_1 n + c_2$
$5(n/4\, c_1 + c_2)$
$25(n/16\, c_1 + c_2)$

$$w(n) = \sum_{i=0}^{\log_4 n} \left(5^i\left(\frac{n}{4^i} c_1 + c_2\right)\right) < n c_2 \log_5 n + \frac{5}{4} n c_2$$

$w(n) \in O(n^{\log_4 5})$

c) $W(n) = 7W(n/7) + n$

Depth: $\log_7 n$

lvl 1: $c_1 n + c_2$
lvl 2: $7(n/7\, c_1 + c_2) = nc_1 + 7c_2$
lvl 3: $49(n/49\, c_1 + c_2) = nc_1 + 49c_2$

$$w(n) = \sum_{i=0}^{\log_7 n} (nc_1 + 7^i c_2) < n c_2 \log_7 n + 7c_2 \in O(n \log_7 n)$$

$w(n) \in O(n \log_7 n)$

d) $W(n) = 9W(n/3) + n^2$

Depth $= \log_3 n$

lvl 1: $c_1 n^2 + c_2$
lvl 2: $9(c_1(n/3)^2 + c_2) = n^2/3\, c_1 + 9c_2$
lvl 3: $81(c_1(n/9)^2 + c_2) = n^2/9\, c_1 + 81c_2$

$$w(n) = \sum_{i=0}^{\log_3 n} \frac{n^2}{3^i} + 9^i c_2 < 3n^2 c_1 + 9n c_2 \in O(n^2)$$

$W(n) \in O(n^2 \log_3 n)$

e.) $W(n) = 8W(n/2) + n^3$

lvl 1 : $c_1 n^3 + c_2$

lvl 2 : $8(c_1(n/2)^3 + c_2) = n^3/2\, c_1 + 8c_2$

lvl 3 : $64(c_1(n/4)^3 + c_2) \quad n^3/4\, c_1 + 64c_2$

Depth : $\log_2 n$

$$W(n) = \sum_{i=0}^{\log n} n^3/2^i\, c_1 + 8^i c_2 < 2n^3 c_1 + 8n c_2 \in O(n^3)$$

$W(n) \in O(n^3 \log_2 n)$

f.) $w(n) = 49(n/25) + n^{\frac{3}{2}} \log n$

lvl 1: $n^{3/2} \log n + c_2$

lvl 2: $49\left(\frac{n^{3/2}}{25} \log(n/25) c_1 + c_2\right)$

$= \frac{49^{3/2}}{25} \log(n/25) c_1 + 49c_2$

Depth : $\log_{25} n$

$$w(n) = \sum_{i=0}^{\log_{25} n} \left(\frac{n^{3/2}}{25^i} \log\left(\frac{n}{25^i}\right) c_1 + 49^i c_2\right) = \sum_{i=0}^{\log_{25} n}\left(\frac{n^{3/2}}{25^i} \log\left(\frac{n}{25^i}\right) c_1\right) + \sum_{i=0}^{\log_{25} n}\left(49^i c_2\right)$$

$w(n) < 25n^{3/2}(\log^2 n + \log n) + 49nc_2$

$< 25n^{3/2}(\log^2 n + \log n)$

$w(n) \in O(n^{3/2} \log n)$

g.) $w(n) = w(n-1) + 2$

$w(n) = \sum_{i=1}^{n} 2 = n(n-1) \in O(n)$

$w(n) \in O(n)$

h.) $w(n) = W(n-1) + n^c$

$w(n) = \sum_{i=1}^{n} i^c = \frac{n^c(n^c - 1)}{2} \in O(c^n)$

$w(n) \in O(n^{c+1})$

i.) $w(n) = w(\sqrt{n}) + 1$

$w(n) = w(n^{1/2}) + 1$

$w(n) \in O(\log(\log(n)))$

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2.) Algorithm A: $W(n) = 5W(n/2) + n$
Algorithm B: $W(n) = 2W(n-1) + 1$
Algorithm C: $W(n) = 9W(n/3) + O(n^2)$

Algorithm A would have $W(n) \in O(n^{\log_2 5})$
Algorithm B would have $w(n) \in O(2^n)$
Algorithm C would have $W(n) \in O(n^2 \log_3 n)$

work proven in #1:
1b, 1g, 1e

Comparing W(n) A to W(n) C: $n^{\log_2 5} \stackrel{?}{=} n^2 \log_3 n$ meaning
$\log n < n^{\log 5/4}$
$W(n)A < W(n)C$

W(n) C is less work and therefore the best choice.