HW #2

$2w(\frac{n}{3}) + 1$

$c(root) = 1$

$c(level\ 1) = 1(\frac{n \cdot 1}{3}) + 1(\frac{n}{3})$

1) a) $O(n^{\log_3 2})$

b) $w(n) = 5w(\frac{n}{4}) + n$

$O(root) = n$

$c(level) = (\frac{n}{4}) + (\frac{n}{4}) + (\frac{n}{4}) + (\frac{n}{4}) + (\frac{n}{4})$

$O(n^{\log_4 5})$

c) $7w(\frac{n}{7}) + n$

$c(root) = n$

$c(level 1) = 7(\frac{n}{7}) = n$

$O(n \log_b a \cdot \log_b 7) = O(n \log n)$

d) $w(n) = 9w(\frac{n}{3}) + n^2$

$c(root) = n^2$

$c(level\ 1) = 9(\frac{n}{3})^2 \rightarrow 9(\frac{n^2}{9}) = n^2$

$O(n \log_b a \cdot \log_b n) = O(n \log_3 9 \cdot \log_3 n)$

$= O(n^2 \log n)$

$= O(n^2 \log n)$

e) $w(n) = 8w(\frac{n}{2}) + n^3$

$c(root) = n^3$

$c(level\ 1) = 8(\frac{n}{2})^3 = 8(\frac{n^3}{8}) = n^3$

$O(n^{\log_2 8} \cdot \log_2 n)$

$O(n^3 \log_2 n)$

$O(n^3 \log n)$

f) $w(n) = 49w(\frac{n}{25}) + n^{3/2} \log n$

$c(root) = n^{3/2} \log n$

$c(level\ 1) = 49(\frac{n}{25})^{3/2} \log_{25}(\frac{n}{25})$

$f(n) = n^{3/2} \log n$

$b: 25$

$a: 49$

$f(n) = \Theta(n^c \log^k n)$ $c = \log_{25} 49$

so; by master theorem

$w(n) \in O(n^{\log_{25} 49} \log^2 n)$

g) $w(n-1) + 2 = w(n)$

$w(n-1) = w(n-2) + 2$

$w(n-2) = w(n-3) + 2$

$w(n) = 2k + w(n-k)$

$k = n-1$

$w(n) = 2(n-1) + 1$

$\in O(n)$

h) $w(n-1) + n^c = w(n) \quad c \geq 1$

$w(n-1) = w(n-2) + (n-1)^c$

$w(n) = w(n-2) + (n-1)^c + n^c$

$w(n) = w(n-3) + (n-2)^c + (n-1)^c + n^c$

$= w(n-k) + \ldots + (n-k+1)^c$

becomes 2

Let $k = n-1$

$= w(1) + n^c + (n-1)^c + \ldots (n-k+1)^c$

$= 1 + n^c + (n-1)^c + \ldots + 1^c$

$\leq 1 + n^c + n^c + \ldots + n^c$

$= 1 + n^{c+1}$

$\in O(n^{c+1})$ for any $c \geq 1$

2)

i) $w(n) = w(\sqrt{n}) + 1$

$w(2) = 1$

$w(4) = w(\sqrt{4}) + 1 = w(2) + 1 = 2$

$w(16) = w(\sqrt{16}) + 1 = w(4) + 1$

$w(2^2) = w(2) + 1$

$w(2^4) = w(2^2) + 1$

$w(2^8) = w(2^4) + 1 \rightarrow w(2^{2^k}) = w(2^{2^{k-1}}) + 1$

Let $n = 2^{2^k}$

$w(n) = w(\sqrt{n}) + 1$

$n = 2^{2^k}$

$\log n = 2^k$

$\log(\log n) = k$

$O(\log(\log(n)))$

2) Alg A: $5W(\frac{n}{2}) + n = W(n)$
Alg B: $2W(n-1) + 1 = W(n)$
Alg C: $9W(\frac{n}{3}) + n^2 = W(n)$

A: $W(n) \in O(n^{\log_2 5})$
B: $W(n) \in O(2^n)$
C: $W(n) \in \Theta(n^2 \log n)$

$$\frac{n^{\log_2 5}}{n^2 \log n} = \frac{n^{\log_2 5/4}}{\log n} \rightarrow \lim_{n \to 0} \frac{\log_2 5/4 \, n^{\log_2 5/4 - 1}}{1/n} \rightarrow \infty$$

I would choose algorithm c because $n^2 \log n < n^{\log_2 5} < 2^n$ for sufficiently large n