$W(n) = aW\left(\frac{n}{b}\right) + f(n)$

Leaf dominated: When $f(n) < n^{\log_b a}$ $\quad O(n) = n^{\log_b a}$

Balanced $O(n) = (n^{(\log_b a)}) \cdot \log_b n$ $\quad \# B = \frac{n}{B}$

Root dominated: $O(n) = f(n)$

$A=2$ $\quad B=3$ $\quad f(n)=1$

1. $W(n) = 2W(n/3) + 1$ $\quad 1 \quad n^{\log_3 2} \quad n^0$ $\quad = O(\log_3 n) \cdot 1 = O(\log_3 n)$

$O(n) = n^{\log_3 2}$

$a=5 \quad b=4 \quad f(n)=n$

2. $W(n) = 5W\left(\frac{n}{4}\right) + n$

$O(n) = n^{\log_4 5}$

$n$

$\frac{n}{4} \quad \frac{n}{4} \quad \frac{n}{4} \quad \frac{n}{4} \quad \frac{n}{4}$

$5\left(\frac{n}{4}\right)$ $\quad O(\log n)$

$25\left(\frac{n}{20}\right)$

$125\left(\frac{n}{100}\right)$

3. $W(n) = 7W\left(\frac{n}{7}\right) + n$ $\quad n^{\log_7 7} \cdot \log_7 n = n \log_7 n$ $\quad O(n) = n \log n$

$n$

$\frac{n}{7}$ $\quad 7\left(\frac{n}{7}\right)$

$49\left(\frac{n}{49}\right)$

4. $W(n) = 9W\left(\frac{n}{3}\right) + n^2$

$n^2 \quad n^{\log_3 9} \cdot \log_3 n = O(n) = n^2 \log n$

$\left(\frac{n}{3}\right)^2$ $\quad 9\left(\frac{n}{3}\right)^2 \quad \cancel{9}\frac{n^2}{\cancel{9}} = n^2$

$81\left(\frac{n}{9}\right)^2 = n^2$

5. $8W\left(\frac{n}{2}\right) + n^3$ $\quad n^{\log_2 8} \cdot \log_2 n = O(n) = n^3 \cdot \log_2 n$

$n^3$

$\left(\frac{n}{2}\right)^3$ $\quad 8\left(\frac{n}{2}\right)^3 \quad \cancel{8}\left(\frac{n^3}{\cancel{8}}\right) = n^3$

6

$a=49 \quad b=25$ $\quad \log_{25} 49$ $\quad b^a = c \quad a = \log_b c$ 24

$W(n) = 49W\left(\frac{n}{25}\right) + n^{\frac{3}{2}} \log n$ $\quad \log_b a^c = c \cdot \log_b a$

$\log_b a = \log_{25} 49 = \log_{25} 7^2 = 2\log_{25} 7$ $\quad \log_{a^d} b = \frac{1}{d} \log_a b$

$= 2\log_{5^2} 7 = 2 \cdot \frac{1}{2} \log_5 7 = \log_5 7$

$a = 49$

$b = 25$

$c = \frac{3}{2}$ $\quad c = \frac{3}{2}$

$k = 1$

$n^{\frac{3}{2}} \log n$

$\frac{n}{25}$ $\quad = 49\left(\frac{n}{25}\right)^{\frac{3}{2}} \log n = 49 \frac{\left(n^{\frac{3}{2}}\right)}{25^{\frac{3}{2}}} \quad 49\left(\frac{n^{\frac{3}{2}}}{125}\right)$

when $f(n) = \Theta\left(n^c \log^k n\right)$

if $b = a^2$ && $f(n) = \Theta(n^{\frac{1}{2}})$, then $T(n) = \Theta(n^{\frac{1}{2}} \log n)$

else if

$b = a^2$ && $f(n) = \Theta(n^{\frac{1}{2}} \log n)$, $T(n) = \Theta(n^{\frac{1}{2}} \log^2 n)$

$T(n) = \Theta(n^c \log^{k+1} n)$

$\Theta(n) = n^{\frac{3}{2}} \log^2 n$

Brick Method

$W(n) = W(n-1) + 2$

$W(n-1) = W(n-2) + 2$

$W(n) = \left(\left[W(n-2) + 2\right]\right) + 2$ $\quad W(n-2) = W(n-3) + 2$

$W(n) = W(n-2) + 2 + 2$

$W(n-2) + 4$ $\quad W(n) = W(n-1) + 2$

$W(n) = \left[W(n-3) + 2\right] + 4$ $\quad = W(n-2) + 2 \times 2$

$W(n) = W(n-3) + 6$ $\quad = W(n-3) + 2 \times 3$

$W(1) = 1$ $\quad = W(n-k) + 2 \times k$

We want $n-k=1$. So, $k=n-1$. Plug in.

$W(n) = W(n-(n-1)) + 2(n-1)$

$W(n-n+1) + 2n-2$ $\quad O(n) = 2n-1 \quad = O(n)$

$W(1) + 2n - 2$

$1 + 2n - 2 = 2n - 1$

$W(n) = W(n-1) + n^c$, with $c \geq 1$

$W(n-1) = W(n-2) + (n-1)^c$

$W(n) = W(n-2) + (n-1)^c + n^c$

$W(n-2) = W(n-3) + (n-2)^c$

$W(n) = W(n-3) + (n-2)^c + (n-1)^c + n^c$

$W(n) = W(n-K) + n^c + (n-1)^c + \ldots + (n-K+1)^c$

Let $K = n-1$

$1 = n-K$

$n^c \geq (n-1)^c$

$= W(1)$

$= 1 + n^c + (n-1)^c + \ldots + 2^c$

$= 1 + n^c + n^c + \ldots$

$= 1 + n^c \times n$

$= 1 + n^{c+1}$

$O(n^{c+1})$

2. Algorithm A

A: $W(n) = 5W\left(\frac{n}{2}\right) + n$ $\qquad O(n^{\log_2 5})$ $\qquad n \quad \frac{n}{2} \quad 5\left(\frac{n}{2}\right) \quad 25\left(\frac{n}{4}\right)$

B: $W(n) = 2W(n-1) + 1$ $\qquad O(2^n)$

C: $W(n) = 9W\left(\frac{n}{3}\right) + n^2$ $\qquad O(n^2 \log_3 n)$ $\qquad n^2 \quad \frac{n}{3} \quad 9\left(\frac{n}{3}\right)^2 = 9\frac{n^2}{9}$

$W(n) = 2W(n-1) + 1$

$W(n-1) = 2W(n-2) + 1$

$W(n) = 2\left(2W(n-2) + 1\right) + 1$

$2^2 W(n-2) + 2 + 1$

$2^k W(n-k) + k + (k-1) + 1$

$k = n-1$

$2^{n-1} W(1) + k + (k-1) \ldots + 1$

$2^n$