

$$w(n) = a w\left(\frac{n}{b}\right) + f(n)$$

Leaf dominated: When  $f(n) < n^{\log_b a}$   $O(n) = n^{\log_b a}$

Balance:  $O(n) = (n^{\log_b a}) \cdot \log_b n$   $a = b$

Root dominated:  $O(n) = f(n)$

(A) (B) (C)  $f(n) = 1$

$$1. w(n) = 2w\left(\frac{n}{5}\right) + 1$$

a=2 b=5 c(n)=1

1

$n^{\log_5 2}$

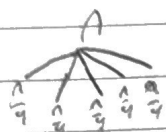
$n^0$

$$= O(\log_5 n) \cdot 1 = O(\log_5 n)$$

(1)

$$O(n) = n^{\log_5 2}$$

$$2. w(n) = 5w\left(\frac{n}{2}\right) + n$$



$5\left(\frac{n}{2}\right)$

$O(\log n)$

$25\left(\frac{n}{4}\right)$

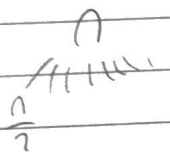
$125\left(\frac{n}{8}\right)$

$$O(n) = n^{\log_2 5}$$

$$3. w(n) = 7w\left(\frac{n}{3}\right) + n$$

$$n^{\log_3 7} \cdot \log_3 n = n^{\log_3 7}$$

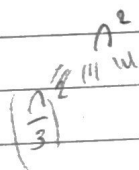
$$O(n^{\log_3 7})$$



$7\left(\frac{n}{3}\right)$

$49\left(\frac{n}{9}\right)$

$$4. w(n) = 9w\left(\frac{n}{3}\right) + n^2$$



$9\left(\frac{n}{3}\right)^2$

$\frac{n^2}{3} = n^2$

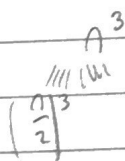
$81\left(\frac{n}{9}\right)^2 = n^2$

$$n^{\log_3 9} \cdot \log_3 n = n^2$$

$$O(n^2)$$

$$5. 8w\left(\frac{n}{2}\right) + n^3$$

$$n^{\log_2 8} \cdot \log_2 n = O(n^3 \cdot \log_2 n)$$



$8\left(\frac{n}{2}\right)^3$

$8\left(\frac{n}{2}\right)^3 = n^3$

$$a=49 \quad b=25$$

$$\log_{25} 49$$

$$b^a = c \quad a = \log_b c$$

$$W(n) = 49W\left(\frac{n}{25}\right) + n^{\frac{3}{2}} \log n$$

$$\log_b a^c = c \cdot \log_b a$$

$$\log_b a = \log_{25} 49 = \log_{25} 7^2 = 2 \log_{25} 7$$

$$\log_a b = \frac{1}{a} \log_a b$$

$$= 2 \log_{25} 7 = 2 \cdot \frac{1}{2} \log_5 7 = \log_5 7$$

$$n^{\frac{3}{2}} \log n$$

$$c = \frac{3}{2}$$

$$a = 49$$

$$b = 25$$

$$c = \frac{3}{2}$$

$$k = 1$$

$$\frac{n}{25}$$

$$= 49 \left(\frac{n}{25}\right)^{\frac{3}{2}} \log n$$

$$= 49 \frac{n^{\frac{3}{2}}}{25^{\frac{3}{2}}}$$

$$49 \left(\frac{n^{\frac{3}{2}}}{125}\right)$$

$$\text{when } f(n) = \Theta(n^c \log^k n)$$

$$\text{if } b = a^2 \text{ and } f(n) = \Theta(n^{\frac{1}{2}}), \text{ then } T(n) = \Theta(n^{\frac{1}{2}} \log n)$$

else if

$$b = a^2 \text{ and } f(n) = \Theta(n^{\frac{1}{2}} \log n), \text{ then } T(n) = \Theta(n^{\frac{1}{2}} \log^2 n)$$

$$T(n) = \Theta(n^c \log^{k+1} n)$$

$$[O(n) = n^{\frac{3}{2}} \log^2 n]$$

Brick method

$$W(n) = W(n-1) + 2$$

$$W(n-1) = W(n-2) + 2$$

$$W(n) = [W(n-2) + 2] + 2$$

$$W(n-2) = W(n-3) + 2$$

$$W(n) = W(n-2) + 2 + 2$$

$$W(n-2) + 4$$

$$W(n) = W(n-1) + 2$$

$$W(n) = [W(n-3) + 2] + 4$$

$$= W(n-2) + 2 \times 2$$

$$W(n) = W(n-3) + 6$$

$$= W(n-3) + 2 \times 3$$

$$W(1) = 1$$

$$= W(n-k) + 2 \times k$$

We want  $n-k=1$ . So,  $k=n-1$ . Plug in.

$$W(n) = W(n-(n-1)) + 2(n-1)$$

$$W(n-n+1) + 2n-2$$

$$W(1) + 2n-2$$

$$1 + 2n-2 = 2n-1$$

$$O(n) = 2n-1$$

$$= O(n)$$

$$W(n) = W(n-1) + n^c, \quad \text{with } c \geq 1$$

$$W(n-1) = W(n-2) + (n-1)^c$$

$$W(n) = W(n-2) + (n-1)^c + n^c$$

$$W(n-2) = W(n-3) + (n-2)^c$$

$$W(n) = W(n-3) + (n-2)^c + (n-1)^c + n^c$$

$$W(n) = W(n-K) + n^c + (n-1)^c + \dots + (n-K+1)^c$$

$$\text{Let } K = n-1$$

$$1 = n-K$$

$$n^c \geq (n-1)^c$$

$$= W(1)$$

$$= 1 + n^c + (n-1)^c + \dots + 2^c$$

$$= 1 + n^c + n^c + \dots$$

$$= 1 + n^c \times n$$

$$= 1 + n^{c+1}$$

$$O(n^{c+1})$$

## 2. Algorithm A

A:  $w(n) = 5w(\frac{n}{2}) + 1$

$O(n^{\frac{1}{2} \log_2 5})$

$\frac{1}{2} \log_2 5$   
 $5(\frac{n}{2})$   
 $25(\frac{n}{4})$

B:  $w(n) = 2w(n-1) + 1$

$O(2^n)$

C:  $w(n) = 9w(\frac{n}{3}) + n^2$

$O(n^2 \log n)$

$n^2$   
 $\frac{1}{3} \log_3 9$   
 $9(\frac{n}{3})^2 = 9 \frac{n^2}{9}$

$w(n) = 2w(n-1) + 1$

$w(n-1) = 2w(n-2) + 1$

$w(n) = 2(2w(n-2) + 1) + 1$

$2^2 w(n-2) + 2 + 1$

$2^k w(n-k) + k + (k-1) + 1$

$k = n-1$

$\frac{2^{n-1} w(1) + k + (k-1) + 1}{2^n}$