depth = $\log_b n$

$5\sum_{i=0}^{\log_b n} c$ $\qquad$ $\sum_{i=0}^{\log_b n} a^i \cdot c$ $\qquad$ depth = $\log_3 n$

Part 1. Asymtotic Analysis

1. $w(n) = 2w(n/3) + 1$

$w(n) = aw(\frac{n}{b}) + c$

2^0 $\quad n$ $\quad$ $w(n)$ ------ 1

2^1 $\quad 2$ $\quad$ $w(\frac{n}{3})$ $\quad$ $w(\frac{n}{3})$ ----- $2 \times 1 = 2$

2^2 $\quad 4$ $\quad$ $w(\frac{n}{9})$ $\quad$ $w(\frac{n}{9})$ $\quad$ $w(\frac{n}{9})$ $\quad$ $w(\frac{n}{9})$ --- $2^2 \times 1 = 4$

$\vdots$

2^i $\quad i$ $\quad$ $w(\frac{n}{3^i})$ -------- $2^i \cdot 1 = 2^i$

$\frac{n}{3^i}$ $\qquad n = 3^i$ $\qquad i = \log_3 n$

$\sum_{i=0}^{\log_3 n} 2^i \cdot 1$

$2^{\log_3 n}$ $\qquad$ $W(n) = O(2^{\log_3 n})$

$\log 2^n$

helped by TA.

2. $W(n) = 5W(\frac{n}{4}) + n$

5^0 1 $W(n)$ ---------- $5^0 \cdot n = n$

5^1 5 $W(\frac{n}{4})$ $W(\frac{n}{4})$ $W(\frac{n}{4})$ $W(\frac{n}{4})$ $W(\frac{n}{4})$ ---- $5^1 \cdot \frac{n}{4} =$

5^i i $W(\frac{n}{4^i})$ ---- $\frac{n}{4^i} \times 5^i$

$\frac{n}{4^i}$ $n \sum_{i=0}^{\log_4 n} \frac{n}{4^i} \times 5^i$

$n = 4^i$

$i = \log_4 n$ $n \sum_{i=0}^{\log_4 n} (\frac{5}{4})^i$ ∴

$W(n) = O(n \log n)$

3. $W(n) = 7W(\frac{n}{7}) + n$

7^0 7 $W(n)$ ---------- $7^0 \cdot n = n$

7^1 7 $W(\frac{n}{7})$ $W(\frac{n}{7})$ $W(\frac{n}{7})$ $W(\frac{n}{7})$ $W(\frac{n}{7})$ $W(\frac{n}{7})$ $W(\frac{n}{7})$ $7^1 \cdot \frac{n}{7} = n$

7^i i $W(\frac{n}{7^i})$

$i = \log_7 n$ $n \sum_{i=0}^{\log_7 n} \frac{n}{7^i} \times 7^i$ balanced dorm x level ---- $7^i \times \frac{n}{7^i} = n$

"n" log n

$n \sum_{i=0}^{\log_7 n} \frac{7^i n}{7^i} = n$ $W(n) = O(n \log n)$

4. $W(n) = 9W(\frac{n}{3}) + n^2$

$w(n)$ ———— $9^0 \cdot n^2 = n^2$

9^0 1

9^1 9 $w(\frac{n}{3})^2$ $w(\frac{n}{3})^2$ $w(\frac{n}{3})^2$ $w(\frac{n}{3})^2$ $w(\frac{n}{3})^2$ $w(\frac{n}{3})^2$ $w(\frac{n}{3})^2$ $w(\frac{n}{3})^2$ $w(\frac{n}{3})^2$ ———— $9^1(\frac{n}{3})^2 = n^2$

9^i $\frac{n}{3^i}$ $w(\frac{n}{3^i})^2$

$i = \log_3 n$ $\quad n^2 \sum_{i=0}^{\log_3 n} (\frac{n}{3^i})^2 \cdot 9^i$ $\qquad (\frac{n}{3^i})^2 \cdot 9^i = n^2$

$W(n) = O(n^2 \log n)$ balanced

5. $8W(\frac{n}{2}) + n^3$

$w(n)$ ———— $8^0 \cdot n^3 = n^3$

9^0 1

9^1 2 $w(\frac{n}{2})^3$ $w(\frac{n}{2})^3$ $w(\frac{n}{2})^3$ $w(\frac{n}{2})^3$ $w(\frac{n}{2})^3$ $w(\frac{n}{2})^3$ $w(\frac{n}{2})^3$ $w(\frac{n}{2})^3$ $\quad 8^1(\frac{n}{2})^3 = n^3$

9^i i $w(\frac{n}{2^i})^3$ ———— $8^i(\frac{n}{2^i})^3 = n^3$

$i = \log_2 n$ $\quad n^3 \sum_{i=0}^{\log_2 n} 8^i (\frac{n}{2^i})^3$ $\qquad W(n) = O(n^3 \log n)$ balanced

depth $= \log_2 n$

6. $49W(\frac{n}{25}) + n^{3/2} \log n$

49^0 1 $w(n)$ ———— $n^{3/2} \log n$

49^1 2 $w(\frac{n}{25})^{3/2} \log(\frac{n}{25})$ ... ———— $49(\frac{n}{25})^{3/2} \log(\frac{n}{25}) = n^{3/2} \log n$

$\frac{n}{25^i}$ $w(\frac{n}{25^i})^{3/2} \log(\frac{n}{25^i})$ ———— $49^i(\frac{n}{25^i})^{3/2} \log(\frac{n}{25^i}) = n^{3/2} \log(\frac{n}{25^i})$

$i = \log_{25} n$

$$n^{3/2} \log n \sum_{i=0}^{\log_{25} n} 49^i \left(\frac{n}{25^i}\right)^{3/2} \cdot \log\left(\frac{n}{25^i}\right)$$

$W(n) = O(n^{3/2} \log n)$

7. $W(n) = W(n-1) + 2$

1^0 0 $W(n)$ - - - - - 2

1^1 1 $W(n-1)$ - $W(n-1)$ - - - 2

1^i n-1 $W(n-i)$ 2

i = n-1

$$\sum_{i=0}^{n-1} 2 = 2(n-1) \qquad W(n) = 2n = 2$$

$$W(n) = O(n)$$

8. $W(n) = W(n-1) + n^c$ with $c \geq 1$

n 0 $W(n)$ — n^c

n-1 1 $(n-1)^1$ $(n-1)^1$ — — $(n-1)^c$

n-i i $(n-i)^1$ — — $(n-i)^c$

depth = n-1

$$W(n) = \sum_{(n-i)=1}^{n-1} (n-i)^c$$

$$W(n) = \sum_{(n-i)=1}^{n} (n-i)^c = O(n^{c+1}) \qquad W(n) = O(n^{c+1})$$

9. $W(n) = W(\sqrt{n}) + 1$ depth | $\log\log n$

$\sqrt{n} = n^{1/2}$

1^0 1 $W(n)$ --- $1^0 \cdot 1 = 1$ depth $\log_2 n$

1^1 2 $W(n^{1/2})$ ---- $1^1 \cdot n^{1/2} = n^{\frac{1}{2}} \cdot 1$

$$\log_2 n \sum_{i=0}^{\log_2 n} (n^{\frac{1}{2^i}})^{\frac{1}{\log}}$$

1^2 $W(n^{\frac{1}{2}})^2$ --- $1^2 \cdot n^{1/4} = n^{\frac{1}{4}} \cdot 1$

1^i i $W(n^{\frac{1}{2^i}})^{\frac{1}{\log}}$ --- $(n^{\frac{1}{2^i}})^{\frac{1}{\log}} \cdot 1^i \cdot 1$

$$W(n) = O(\log\log n)$$

Part 2. Algorithm comparison

A. $W(n) = 5W(\frac{n}{2}) + n$

5^0 0 $W(n)$ ———— $5^0 \cdot n = n$

5^1 1 $W(\frac{n}{2})$ .... ———— $5^1(\frac{n}{2}) = \frac{5n}{4}$

5^i i $W(\frac{n}{2^i})$ ———— $5^i(\frac{n}{2^i})$

$\frac{n}{2^i}$ $i = \log_2 n$ $n^{\log_2 5}$

$n = 2^i$

$$n \sum_{i=0}^{\log_2 n} 5^i \left(\frac{n}{2^i}\right)$$ $W(n)\ (5^{\log_2 n})$ leaf dominates

B. $W(n) = 2W(n-1) + 1$

2^0 0 0^n $W(n)$ ———— 1

2^1 1 $W(n-1)$ $W(n-1)$ ———— 2,

2^i 2^i $W(n-i)$ ... ———— 2^i

$$\sum_{i=0}^{n} a^i = \frac{a^{n+1} - 1}{a-1} < \frac{a}{a-1} \cdot a^n$$

$$\sum_{i=0}^{n-1} 2^i = \frac{2^{n-1+1} - 1}{2-1} = \frac{2^n - 1}{1}$$ $W(n) = O(2^n)$

C. $W(n) = 9W(\frac{n}{3}) + n^2$

9^0 0 $W(n)$ ———— n^2

9^1 1 $W(\frac{n}{3})^2$ ... ———— $9^1(\frac{n}{3})^2 = n^2$

9^i i $W(\frac{n}{3^i})^2$ ———— $9^i(\frac{n}{3^i})^2 = n^2$

$i = \log_3 n$

$$n^2 \sum_{i=0}^{\log_3 n} 9^i \left(\frac{n}{3^i}\right)^2$$ $W(n) = O(n^2 \log n)$

$n =$

The third algorith is better than the rest
C is the best