B: $W(n) = 2W(n-1) + O(1)$

$W(n) = 2(2W(n-2) + O(1)) + O(1)$

$= 2^2 W(n-2) + 2O(1) + O(1)$

$= 2^3 W(n-3) + 2^2 O(1) + 2O(1) + O(1)$

$W(n) = 2^n W(0) + \sum_{i=0}^{n-1} 2^i O(1)$

$\frac{2^n - 1}{2-1} = 2^n - 1 = 2^n$

$W(n) = O(2^n)$

C: $W(n) = 9W(\frac{n}{3}) + O(n^2)$

From earlier: $W(n) = O(n^2 \log n)$

3b.

parens_update: $W(n) = W(n-1) + O(1)$

$= W(n-2) + O(1) + O(1)$

$= W(n-3) + O(1) + O(1) + O(1)$

$= W(n) = O(n)$

No parallelism: span $= S(n) = O(n)$

3d.

$W(n) = O(n)$

$S(n) = O(\log n)$

3f.

$W(n) = 2W(\frac{n}{2}) + O(1)$ $\quad W(n) = O(n)$

$S(n) = S(\frac{n}{2}) + O(1)$ $\quad S(n) = O(\log n)$

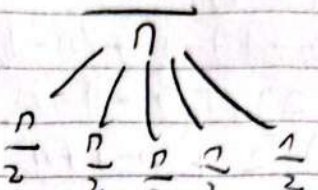
Assignment 2 part 2.

A: $W(n) = 5W(\frac{n}{2}) + O(n)$
B: $W(n) = 2W(n-1) + O(1)$
C: $W(n) = 9W(\frac{n}{3}) + O(n^2)$

$W(n) = 5W(\frac{n}{2}) + O(n)$

i	size	cost	# nodes	work
0	n	n	1	n
1	$\frac{n}{2}$	$\frac{n}{2}$	5	$5 \cdot \frac{n}{2}$
2	$\frac{n}{4}$	$\frac{n}{4}$	25	$25 \cdot \frac{n}{4}$
i	$\frac{n}{2^i}$	$\frac{n}{2^i}$	5^i	$5^i \cdot \frac{n}{2^i}$

Tree



work at level i: $5^i \cdot O(\frac{n}{2^i}) = n \cdot (\frac{5}{2})^i$

$1 = \frac{n}{2^i} \to 2^i = n \to \log_2 n = i$

$(\frac{5}{2})^{\log_2 n} = n^{\log_2 5}$

Work: $W(n) = O(n^{\log_2 5})$

9. $W(n) = W(\sqrt{n}) + 1$ $\qquad \sqrt{n} = n^{\frac{1}{2}}$

$= W(n^{\frac{1}{4}}) + 1 + 1$ $\qquad \sqrt{\sqrt{n}} = n^{\frac{1}{2} \cdot \frac{1}{2}} = n^{\frac{1}{4}}$

$= W(n^{\frac{1}{4}}) + 2$

$= W(n^{\frac{1}{8}}) + 3$

~~$\Rightarrow W(n) = \sqrt[\frac{k}{2}]{n} +$~~

Base case: $\sqrt[k]{n} = 1 \rightarrow n^{\frac{1}{k}} = 1$

$n^{\frac{1}{2}^{k}} = 1$

$\log_2 (n^{\frac{1}{2}^{k}}) = \log_2 1$

$k = \log_2 \log_2 n$

$\boxed{W(n) = O(\log \log n)}$

7. $W(n) = W(n-1) + 2$

i	size	cost	work	# node
0	n	2	2	1
1	$n-1$	1		1

tree

2

1

$2-1=1$

$W(n) = W(n-1) + 2$

$= W(n-1)-1 + 2 + 2$

$= W(n) = W(n-k) + 2k$

Base case: $n-k=1$

$k = n-1$

$W(n) = W(1) + 2(n-1) \rightarrow$ constant

Work: $O(n)$

8. $W(n) = W(n-1) + n^c, \; c \geq 1$

$= W(n-1) - 1 + (n-1)^c + n^c$

$= (W(n-2) + (n-1)^c) + n^c$

$= W(n-3) + (n-2)^c + (n-1)^c + n^c$

$W(n) = W(1) + \sum_{k=1}^{n} k^c$

$\sum_{k=1}^{n} k^c = O(n^{c+1})$

$c=1 \rightarrow W(n) = O(n^2)$

$c=2 \rightarrow W(n) = O(n^3)$

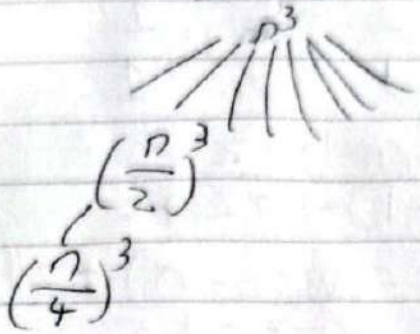
$c=3 \rightarrow W(n) = O(n^4)$

$\boxed{W(n) = O(n^{c+1})}$

$W(n) = 8W\left(\frac{n}{2}\right) + n^3$

size	cost	# nodes	work
n	n^3	1	n^3
$n/2$	$n^3/2$	8	$(n/2)^3$
$n/4$	$n^3/4$	64	$(n/4)^3$
$n/2^i$	$n^3/2^i$	8^i	$(n/2^i)^3$

Tree



work at level i: $(n/2^i)^3 \cdot 8^i$

$2^n = 8$

$\frac{n^3}{2^{3i}} \cdot 2^{3i} = \boxed{n^3}$

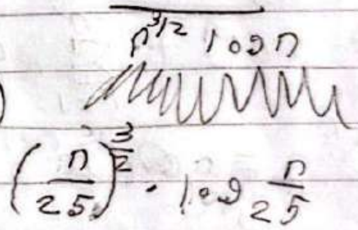
Base case: $1 = \frac{n}{2^i} \rightarrow 2^i = n \rightarrow \log_2 n = i$

$n^3 \cdot \log_2 n = \boxed{\Theta(n^3 \log n)}$

6. $W(n) = 49\left(\frac{n}{25}\right) + n^{\frac{3}{2}} \log n$

i	size	cost	# nodes	work
0	n	$n^{3/2}\log n$	1	$n^{3/2}\log n$
1	$\frac{n}{25}$		49	$\left(\frac{n}{25}\right)^{\frac{3}{2}} \cdot \log\left(\frac{n}{25}\right)$
i	$n/25^i$		49^i	

Tree



work at level i: $49^i \cdot \left(\frac{n}{25^i}\right)^{\frac{3}{2}} \cdot \log\left(\frac{n}{25^i}\right)$

$1 = \frac{n}{25^i} \rightarrow 25^i = n \rightarrow \log_{25} n = i$

$49^i \cdot O\left(\left(\frac{n}{25^i}\right)^{\frac{3}{2}} \cdot \log\left(\frac{n}{25^i}\right)\right) \cdot O(\log_{25} n)$

$= ???$

3. $W(n) = 7W\left(\frac{n}{7}\right) + n$ Very Balanced

i	size	Tree	Cost	# nodes
0	n	n	n	1
1	$\frac{n}{7}$	$\frac{n}{7}$ $\frac{n}{7}$ $\frac{n}{7}$	$\frac{n}{7}$	7
2	$\frac{n}{49}$		$\frac{n}{49}$	49
$\vdots$				
i	$\frac{n}{7^i}$	$\frac{n}{49}$	$\frac{n}{7^i}$	7^i

$1 = \frac{n}{7^i} \rightarrow 7^i = n \rightarrow \log_7 n = i$

$$\sum_{i=0}^{\log_7 n} O(n) = \boxed{O(n \log n)}$$

4. $W(n) = 9W\left(\frac{n}{3}\right) + n^2$ Leaf dominated

i	size	Cost	# nodes	Work
0	n	n^2	1	n^2
1	$n/3$	$n^2/3$	9	$(n/3^1)^2$
i	$n/3^i$	$n^2/3^i$	9^i	$(n/3^i)^2$

Tree: n^2, $\left(\frac{n}{3}\right)^2$

Work at level i: $9^i \cdot O(n/3^i)^2$

$$= 9^i \cdot \left(\frac{n}{3^i}\right)^2 = (3^2)^i \cdot \frac{n^2}{3^{2i}} = 3^{2i} \cdot \frac{n^2}{3^{2i}} = O(n^2)$$

$1 = \frac{n}{3^i} \rightarrow 3^i = n \rightarrow i = \log_3 n$

$$O(n^2) \cdot \sum_{i=0}^{\log_3 n} 9^i = O(n^2) \cdot O(9^{\log_3 n})$$

$$= O(n^2) \cdot O(n^{\log_3 9}) = \boxed{O(n^2 \log n)}$$

1 —

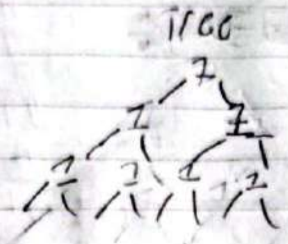
Algorithms Assignment 2 3/11/25

Derive asymptotic upper bounds of work

1. $W(n) = 2W(\frac{n}{3}) + 1$

i	size
0	n
1	$n/3$
2	$\frac{n}{3^2}$
3	
i	$\frac{n}{3^i}$

Tree



Cost	Nodes
1	1
2	2
4	4
2^i	2^i

Leaf dominated

Base case: $\frac{n}{3^i} = 1$

$n = 3^i$

$\log_3 n = i$

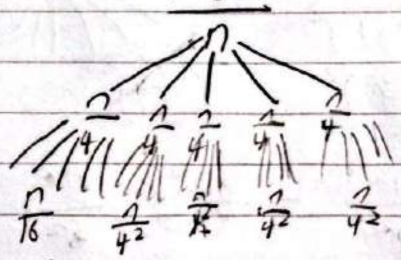
$\sum_{i=0}^{\log_3 n} 2^i$

$\boxed{\text{work: } O(n^{\log_3 2})}$

3. $W(n) = 5W(\frac{n}{4}) + n$

i	size
0	n
1	$\frac{n}{4}$
2	$\frac{n}{4^2}$
i	$\frac{n}{4^i}$

Tree



Cost	# nodes
n	1
$\frac{n}{5}$	5
$\frac{n}{25}$	25
$\frac{n}{5^i}$	5^i

$1 = \frac{n}{4^i} \rightarrow 4^i = n \rightarrow \log_4 n = i$

$\sum_{i=0}^{\log_4 n} 5^i = O(5^{\log_4 n}) = \boxed{O(n^{\log_4 5})}$

Leaf dominated